How the McGucken Principle of a Fourth Expanding Dimension Derives the Results of Hawking's

"Particle Creation by Black Holes" (1975)

$dx_4/dt = ic$ as the Physical Mechanism Underlying Hawking Radiation, the Hawking Temperature, the Bekenstein–Hawking Formula $S = A/4$, the Refined Generalized Second Law, and Black-Hole Evaporation

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Light Time Dimension Theory

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"More intellectual curiosity, versatility and yen for physics than Elliot McGucken's I have never seen in any senior or graduate student. Originality, powerful motivation, and a can-do spirit make me think that McGucken is a top bet."

— Dr. John Archibald Wheeler, Joseph Henry Professor of Physics, Princeton University

Abstract

Starting from a single geometric postulate, $dx_4/dt = ic$, this paper derives Hawking's five central 1975 results as explicit propositions of that postulate: thermal radiation from black holes, the Hawking temperature $T_H = \hbar\kappa/(2\pi c k_B)$, the Bekenstein–Hawking entropy $S_{BH} = k_B A/(4\ell_P^2)$ with coefficient 1/4, the mass-loss law $dM/dt \propto -1/M^2$, and the refined Generalized Second Law. The derivation uses five pieces of standard general-relativistic and Euclidean-QFT machinery — the Rindler near-horizon form, the Wick-rotated Schwarzschild "cigar," the KMS condition, the Einstein–Hilbert plus Gibbons–Hawking–York action, and the Stefan–Boltzmann law — and each one of these is itself derivable from $dx_4/dt = ic$ rather than imposed as an independent assumption. §II.5 works through each derivation in enough detail that the present paper is self-contained: the Minkowski metric follows from substituting $x_4 = ict$ into the four-dimensional Euclidean line element; the Rindler form follows from the four-speed-budget analysis of the McGucken Proof applied to hyperbolic worldlines; the Wick rotation is the physical removal of the i from x_4 [MG-Wick]; the KMS condition is the statistical-mechanical

shadow of that removal; the Einstein–Hilbert action follows from local x_4-phase invariance [MG-SM], with the Gibbons–Hawking–York boundary term required for a well-posed variational principle on any manifold with boundary; and the Stefan–Boltzmann law follows from the second law [MG-HLA] applied to the Planck-quantized x_4-stationary mode count [MG-Constants]. The same postulate had already been shown to yield Bekenstein's entropy and the classical area/entropy laws in the companion work [MG-Bekenstein]; taken together, the two papers provide a simpler, unified geometric foundation for black-hole thermodynamics — the expansion of the fourth dimension at the velocity of light — with every tool used in the derivation tracing back to the same source.

Stephen Hawking's 1975 paper *Particle Creation by Black Holes*, published in *Communications in Mathematical Physics* 43, 199–220, is the sequel to Bekenstein's 1973 founding of black-hole thermodynamics and the most consequential single paper in the semiclassical theory of gravity. In it, Hawking established — by applying quantum field theory in a curved Schwarzschild background and tracking the scattering of vacuum modes across gravitational collapse — that a black hole is not black. It emits thermal radiation at temperature $T_H = \hbar\kappa/(2\pi c k_B)$, where κ is the surface gravity of the horizon. The thermal character of the emission fixes the entropy coefficient at $\eta = 1/4$, giving the modern Bekenstein–Hawking formula $S_{BH} = k_B A/(4\ell_P^2)$. The emission rate causes the black hole to lose mass; for a Schwarzschild black hole the evaporation time is $\tau \sim (M/M_\odot)^3 \cdot 10^{67}$ yr, with primordial black holes of mass less than about 10^{15} g having evaporated by the present epoch. The classical area theorem — which had stood since Hawking's own 1971 result that horizon area never decreases — is violated by the quantum radiation, which shrinks the horizon. But a refined Generalized Second Law is preserved: $S_{\text{ext}} + A/4$ never decreases. The paper turned the Bekenstein entropy conjecture into physics: black holes must be thermodynamic objects because they genuinely radiate.

This paper establishes that each one of Hawking's 1975 central results follows as a theorem of a single geometric postulate:

> *The McGucken Principle of a Fourth Expanding Dimension, $dx_4/dt = ic$, derives Hawking's thermal-radiation spectrum, the Hawking temperature $T_H = \hbar\kappa/(2\pi c k_B)$, the exact coefficient $\eta = 1/4$ in the Bekenstein–Hawking formula $S_{BH} = k_B A/(4\ell_P^2)$, the evaporation rate $dM/dt \propto -1/M^2$, and the refined Generalized Second Law that preserves thermodynamic consistency through evaporation. The Wick rotation that produces each of these results is, in the McGucken framework, a physical transformation — the removal of the i from $dx_4/dt = ic$ — not a formal computational device.*

Five formal Propositions prove Hawking's five central results from the McGucken Princi-

ple. Proposition III.1 establishes Hawking radiation as x_4-stationary mode emission from the horizon (result H-1), with the thermal spectrum following from the periodicity of the Euclidean near-horizon geometry after the McGucken Wick rotation. Proposition IV.1 derives the Hawking temperature $T_H = \hbar\kappa/(2\pi c k_B)$ (result H-2) from the Euclidean-time periodicity $\beta = 2\pi/\kappa$, which in the McGucken framework is the period imposed when the i is removed from x_4. Proposition V.1 derives the exact coefficient $\eta = 1/4$ in the Bekenstein–Hawking formula (result H-3) from the Gibbons–Hawking–York Euclidean action of the Schwarzschild cigar, with the 1/4 emerging from the explicit GHY boundary-term evaluation. Proposition VI.1 derives the evaporation rate $dM/dt \propto -1/M^2$ (result H-4) from the horizon's blackbody emission. Proposition VII.1 derives the refined Generalized Second Law (result H-5) by extending the global McGucken second law to include x_4-stationary-mode emission.

The present paper is the sequel to the author's previous derivation of Bekenstein's 1973 results from the same principle [MG-Bekenstein]. Where Bekenstein established the classical area/entropy laws that fall out of the McGucken framework without invoking the Wick rotation, Hawking established the quantum radiation and the exact coefficient, which require the McGucken Wick rotation as the key additional tool. §IX extends the framework from black-hole horizons to general null hypersurfaces, deriving the 't Hooft–Susskind holographic principle, the AdS/CFT correspondence, and FRW/de Sitter cosmological holography as theorems of the same geometric postulate — with a sharp empirical signature $\rho^2(t_{\rm rec}) \approx 7$ distinguishing McGucken cosmological holography from Hubble-horizon holography at recombination [MG-AdSCFT, MG-CosHolo].

Keywords: McGucken Principle; fourth expanding dimension; $dx_4/dt = ic$; Hawking radiation; Hawking temperature; Bekenstein–Hawking formula; black-hole evaporation; Generalized Second Law; Wick rotation; Euclidean near-horizon geometry; cigar geometry; surface gravity; Light Time Dimension Theory.

1. Introduction: Hawking 1975 and What It Changed

1.1. Context and the Bekenstein challenge

When Bekenstein's 1973 paper appeared in *Physical Review D*, the dominant reaction among relativists was skepticism. Hawking himself was openly critical: if black holes genuinely carry entropy, then by standard thermodynamics they must have a temperature, and if they have a temperature they must radiate. But classically they do not — the event horizon is a one-way causal membrane, and no signal can escape it. Hawking's initial response [BCH73], co-authored with Bardeen and Carter, was that the analogy between black-hole mechanics and thermodynamics was formal only; the "temperature" playing the role of T in the first-law analogy was merely proportional to surface gravity κ, with no actual emission mechanism.

Hawking then set out, over the course of 1973–1974, to demonstrate that Bekenstein was wrong — that black holes do not truly have thermodynamic entropy because they do not truly radiate. The calculation he performed to disprove Bekenstein instead proved Bekenstein correct. Applying quantum field theory in a curved spacetime to the Schwarzschild background, Hawking tracked the scattering of vacuum modes across the formation of the black hole by gravitational collapse. The modes at past null infinity, which define the "in" vacuum, do not coincide with the modes at future null infinity, which define the "out" vacuum. The Bogoliubov coefficients relating the two sets of modes produce a non-trivial particle-creation spectrum. For an asymptotic observer at future null infinity, the black hole appears to emit thermal radiation at temperature $T_H = \hbar\kappa/(2\pi c k_B)$. The radiation is genuine: it carries energy, it causes the black hole to lose mass, and it eventually exhausts the black hole entirely. Bekenstein's entropy conjecture was not merely consistent — it was demanded by the quantum field theory of collapse. Hawking published the short announcement in *Nature* in January 1974 [Haw74] and the full derivation in *Communications in Mathematical Physics* in 1975 [Haw75].

1.2. Hawking's five central results

Hawking 1975 establishes, across its twenty-two pages and four sections, five central results that together transformed black-hole thermodynamics from a formal analogy into a physical theory:

Result H-1 (Thermal radiation from black holes). A black hole formed by gravitational collapse emits, at late times after the collapse settles to stationarity, a thermal flux of particles of every species. For an observer at future null infinity, the flux has the Planckian spectrum of blackbody radiation.

Result H-2 (The Hawking temperature). The temperature of the emitted radiation is

$$T_H = \hbar\kappa/(2\pi c k_B),$$

where κ is the surface gravity of the horizon. For a Schwarzschild black hole of mass M, $\kappa = c^4/(4GM)$, giving $T_H \approx 6.17 \times 10^{-8}\ \mathrm{K} \cdot (M_\odot/M)$.

Result H-3 (The Bekenstein–Hawking formula). The thermal character of the radiation fixes the entropy coefficient exactly at $\eta = 1/4$. The modern Bekenstein–Hawking formula is

$$S_{BH} = k_B A/(4\ell_P^2) = k_B c^3 A/(4\hbar G).$$

Result H-4 (Black-hole evaporation). The thermal emission carries energy away at rate

$$dM/dt \propto -1/M^2,$$

by the Stefan–Boltzmann law applied to the horizon blackbody. A Schwarzschild black hole evaporates completely in time $\tau \sim (M/M_\odot)^3 \cdot 10^{67}$ yr. Primordial black holes of mass $\lesssim 10^{15}$ g would have fully evaporated by the present epoch, with a final explosion releasing $\approx 10^{35}$ erg.

Result H-5 (The refined Generalized Second Law). Although quantum radiation violates the classical area theorem, a refined Generalized Second Law is preserved:

$$dS_{\text{ext}}/dt + dS_{BH}/dt \geq 0,$$

with $S_{BH} = k_B A/(4\ell_P^2)$ including the quantum-corrected horizon area.

1.3. What Hawking's derivation left open

Hawking's 1975 derivation is one of the most remarkable calculations in the history of physics. It is also, at the conceptual level, incomplete in ways that have motivated fifty years of subsequent work. Five open problems can be identified:

Open problem HK-1 (Where does the radiation physically come from?). Hawking's derivation is a calculation of Bogoliubov coefficients — a formal mode-matching between "in" and "out" vacuum states. Several competing intuitions appear in the literature: virtual-pair production near the horizon, vacuum-polarization effects, the tunneling picture of Parikh–Wilczek [PW]. None is definitive. What physical process actually produces the radiation?

Open problem HK-2 (Why Euclidean methods work). The alternative derivation of Hawking temperature via the Gibbons–Hawking Euclidean path integral [GH77] works with remarkable efficiency, but why does the Euclidean prescription work? What is the physical meaning of the periodic identification in Euclidean time?

Open problem HK-3 (Why exactly $\eta = 1/4$?). Hawking's thermodynamic derivation pins the coefficient at exactly 1/4 by matching the first law $dM = T\,dS$ with $dM = (\kappa c^2/8\pi G)\,dA$. The physical reason why the coefficient is 1/4 rather than, say, 1/3 or $1/(4\pi)$ is not transparent.

Open problem HK-4 (The information paradox). If a black hole evaporates completely by emitting thermal radiation, the information that fell into it must either be lost (violating quantum unitarity) or encoded in subtle correlations within the radiation.

Open problem HK-5 (The trans-Planckian problem). Hawking's derivation traces late-time radiation modes back to very early times where they had exponentially shorter wavelengths — formally, below the Planck length near the horizon.

Each of these five open problems has been the subject of decades of work. The present paper resolves them simultaneously via the McGucken Principle.

1.4. The thesis of the present paper

Under the physical identification $dx_4/dt = ic$ — the fourth dimension is expanding at the velocity of light relative to the three spatial dimensions [MG-Proof, F1, MG-Mech] — all five of Hawking's 1975 central results follow as theorems, and each of the five open problems admits a resolution. Hawking radiation is x_4-stationary mode emission from the horizon, thermalized by the Euclidean cigar geometry obtained by removing the i from x_4 (Proposition III.1, resolving HK-1). The Hawking temperature is the inverse angular period of the cigar (Proposition IV.1, resolving HK-2). The coefficient 1/4 follows from the Gibbons–Hawking–York Euclidean action of the Schwarzschild manifold (Proposition V.1, resolving HK-3). Black-hole evaporation is Stefan–Boltzmann emission from the horizon's hot surface (Proposition VI.1). The refined GSL follows from the global McGucken second law applied to the evolving partition during evaporation (Proposition VII.1, resolving HK-4 via six-sense locality, HK-5 via Planck quantization).

2. Preliminaries: The McGucken Principle and the Wick Rotation

The derivations that follow rest on five prior results from the LTD corpus: (i) the formal proof of $dx_4/dt = ic$ [MG-Proof, F1]; (ii) the derivation of the second law of thermodynamics from $dx_4/dt = ic$ [MG-HLA]; (iii) the Planck-scale quantization of x_4-oscillation [MG-Constants]; (iv) the Compton coupling of massive particles to x_4 [MG-Dirac, MG-Born]; and (v) the Wick rotation as a physical transformation [MG-Wick]. For the Hawking derivations, the fifth of these — the McGucken Wick rotation — is the central tool.

2.1. The McGucken Proof of $dx_4/dt = ic$

The McGucken Principle is not a postulate introduced ad hoc; it is a theorem [MG-Proof, F1, MG-EinMink] following from three physical axioms plus one structural assumption, all of them standard special relativity.

Axiom 1 (four-dimensional manifold). Spacetime is a four-dimensional differentiable manifold with three spatial coordinates x_1, x_2, x_3 and a fourth coordinate x_4.

Axiom 2 (Minkowski identity). The fourth coordinate is related to coordinate time by Minkowski's 1908 identity $x_4 = ict$. This is the standard relativistic form; the i records the perpendicularity of x_4 to the three spatial coordinates in the sense that it produces the Lorentzian signature when substituted into the four-dimensional Euclidean line element $d\ell^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$.

Axiom 3 (invariant four-speed). Every physical system moves through the four-dimensional manifold with invariant four-speed magnitude c. Equivalently, the four-velocity u^μ satisfies $u_\mu u^\mu = -c^2$, which is the standard relativistic mass-shell condition.

Structural assumption M1 (physical reality of x_4). The fourth coordinate x_4 is a physical geometric axis, not merely a notational device. This is the ontological move parallel to Einstein's 1905 promotion of Planck's $E = hf$ from mathematical trick to physical law. Minkowski in his 1908 Cologne lecture wrote that space and time would "fade away into mere shadows" before a four-dimensional union of the two; the McGucken Principle takes Minkowski's union as physically real rather than as formal notation.

From these, the proof proceeds in six steps:

Step 1 (Four-speed budget). By Axiom 3, every system has $|u|^2 = u_{\text{spatial}}^2 + u_{x_4}^2 = c^2$. This is a budget equation: the invariant four-speed of magnitude c is shared between spatial motion and x_4-motion. A system at spatial rest has all of its four-speed carried by x_4; a system moving spatially loses x_4-speed by exactly the amount of spatial speed it gains.

Step 2 (Photon limit). For a photon with spatial speed $|v| = c$, the spatial budget is exhausted: $u_{\text{spatial}} = c$ and therefore $u_{x_4} = 0$. Photons are stationary in x_4. This is the familiar statement that photons have zero proper time; in the McGucken reading, the reason they do not age is that they carry no x_4-advance.

Step 3 (Photons as x_4-tracers). A photon moving at c spatially but stationary in x_4 traces a null worldline on a constant-x_4 hypersurface. The cross-section of this hypersurface with three-dimensional space is a 2-sphere expanding at c — the observed photon wavefront.

Step 4 (Observed wavefronts encode x_4's geometry). The universal isotropic spherical expansion of light at rate c, observed in every experiment since Huygens, is therefore the three-dimensional cross-section of x_4's advance.

Step 5 (The expansion rate). By Step 4, x_4 advances relative to the three spatial dimensions at the rate c. Writing this as a differential relation: $dx_4/dt = ic$, where the i

restores x_4's perpendicularity in the four-dimensional manifold. The i is not an abstract bookkeeping symbol; it is the algebraic signal that x_4's advance is perpendicular to all three spatial directions simultaneously.

Step 6 (Alternative direct proof). Differentiating Axiom 2 directly: from $x_4 = ict$ one obtains $dx_4/dt = ic$ immediately. This is an identity at the level of differential calculus; the physical content is the promotion (M1) of x_4 to a real geometric axis, which converts the identity from a notational convenience into a statement about how the fourth dimension is physically advancing.

The conclusion — $dx_4/dt = ic$ as a physical law of nature — follows from these six steps [MG-Proof, Theorem 5.4]. The primary-source publication of this derivation is McGucken's 2008 FQXi essay [F1], time-stamped and archived at `forums.fqxi.org/d/238` since August 25, 2008.

2.2. The McGucken Wick rotation

The derivation in [MG-Wick] establishes the Wick rotation as a physical transformation rather than a formal computational device. This is the central tool for the Hawking derivations.

The standard Wick rotation. In standard quantum field theory, the Wick rotation is the substitution $t \to -i\tau$, converting Lorentzian time to Euclidean "imaginary time." Under this substitution, the Minkowski metric $ds^2 = -c^2dt^2 + d\mathbf{x}^2$ becomes the Euclidean metric $ds_E^2 = c^2d\tau^2 + d\mathbf{x}^2$, Lorentzian oscillating phases $e^{iS/\hbar}$ become Euclidean decaying weights $e^{-S_E/\hbar}$, the Feynman path integral becomes the Gibbs partition function, and quantum mechanics becomes Euclidean statistical mechanics. The trick is computationally powerful: many QFT integrals that diverge in Lorentzian signature converge in Euclidean signature. But the physical meaning of the rotation is obscure in standard treatments. What is "imaginary time"? Why does it work?

The McGucken reading. In the McGucken framework, the Wick rotation has a direct physical interpretation. Recall that $dx_4/dt = ic$ means $x_4 = ict$, where the i records x_4's physical perpendicularity. Writing $x_4 = ict$, the Wick rotation $t \to -i\tau$ becomes:

$$x_4 = ict \ \to \ ic(-i\tau) = c\tau.$$

In other words, **the Wick rotation removes the i from x_4**. The Euclidean "imaginary time" τ is simply the real spatial-like coordinate that x_4 becomes when its perpendicularity (the i) is removed. The Euclidean geometry is not imaginary; it is the geometry that would obtain if x_4 were aligned with — rather than perpendicular to — the three spatial dimensions.

Every consequence of the Wick rotation follows from this collapse. Oscillating phases become decaying weights because the i that marks x_4's perpendicularity has been removed. Quantum mechanics becomes statistical mechanics because the McGucken reading of the path integral [MG-PathInt] identifies the quantum-mechanical sum over x_4-oscillation histories with the statistical-mechanical sum over thermal configurations when the i is removed. The $+i\varepsilon$ causal prescription becomes the Euclidean regularization because the i that selects forward x_4-propagation is collapsed onto the real axis. The McGucken Wick rotation is not a formal trick; it is a physical transformation — the collapse of x_4's perpendicularity.

2.3. The near-horizon Euclidean geometry: the cigar

For a black-hole horizon, the McGucken Wick rotation produces a specific geometric structure that is the central ingredient in the Hawking derivations below.

The Lorentzian near-horizon geometry. Near a non-extremal black-hole horizon, in coordinates adapted to the horizon, the Schwarzschild metric has the Rindler form:

$$ds^2 = -(\kappa^2\rho^2/c^2)\, c^2dt^2 + d\rho^2 + d\Omega^2,$$

where ρ is the proper distance from the horizon, $\kappa = c^4/(4GM)$ is the surface gravity, and $d\Omega^2$ is the transverse spherical metric. The horizon is at $\rho = 0$.

The McGucken Wick rotation of the near-horizon geometry. Applying $t \to -i\tau$, the metric becomes:

$$ds_E^2 = (\kappa^2\rho^2/c^2)\, c^2d\tau^2 + d\rho^2 + d\Omega^2.$$

In (ρ, τ) coordinates with angular variable $\theta = \kappa\tau/c$, this is flat polar coordinates on a two-dimensional plane. For the geometry to be regular at $\rho = 0$ (no conical singularity at the horizon), the angular coordinate θ must have range $0 \le \theta < 2\pi$. This forces a periodic identification of the Euclidean time τ with period

$$\beta = 2\pi c/\kappa,$$

or $\beta = 2\pi/\kappa$ in standard units. The resulting Euclidean near-horizon geometry is a two-dimensional cigar: a smooth disk opening up away from the horizon at $\rho = 0$, with τ identified periodically. The horizon is the tip of the cigar.

The periodicity as inverse Hawking temperature. In Euclidean quantum field theory, periodic identification of Euclidean time with period β corresponds to thermal equilibrium at temperature $T = \hbar/(k_B\beta)$. Applied to the black-hole near-horizon geometry:

$$T_H = \hbar/(k_B\beta) = \hbar\kappa/(2\pi c k_B).$$

This is the Hawking temperature. The derivation is originally Gibbons–Hawking 1977 [GH77]; in the McGucken framework the Euclidean-time periodicity is not a formal trick but a geometric consequence of removing the i from x_4 at the horizon.

2.4. Summary of preliminaries

The key result is the McGucken Wick rotation as a physical transformation: removing the i from $dx_4/dt = ic$ collapses x_4's perpendicularity and converts Lorentzian to Euclidean geometry. Applied to a black-hole horizon, the resulting near-horizon Euclidean geometry is the two-dimensional cigar with angular period $\beta = 2\pi/\kappa$. The Hawking temperature $T_H = \hbar\kappa/(2\pi c k_B)$ is the inverse of this angular period, expressed in thermodynamic units. Every derivation in §III–§VII follows from this single geometric structure.

2.5. The machinery used in this paper is itself derivable from $dx_4/dt = ic$

The Hawking derivations in §III–§VII use five pieces of standard general-relativistic and Euclidean-QFT machinery: the Rindler near-horizon form, the Wick-rotated Schwarzschild cigar, the KMS condition, the Einstein–Hilbert plus Gibbons–Hawking–York action, and the Stefan–Boltzmann law. Each of these is standardly treated as a given from the GR and QFT literatures. In the McGucken framework, each is itself derivable from $dx_4/dt = ic$. This section works through the five derivations in enough detail that the present paper is self-contained on this point, with references to where each is developed in full in the corpus.

2.5.1. The Minkowski metric and Rindler near-horizon form

Step 1: The Minkowski metric from $dx_4/dt = ic$. Define the auxiliary four-dimensional Euclidean line element $d\ell^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$. This is a definition, not an independent physical postulate; it specifies the quantity whose transformation properties will be examined. By Axiom 2 of the McGucken Proof, $x_4 = ict$, so $dx_4 = ic\,dt$, so $dx_4^2 = (ic)^2 dt^2 = -c^2 dt^2$. Substituting:

$$d\ell^2 = dx_1^2 + dx_2^2 + dx_3^2 - c^2 dt^2 = ds^2.$$

The Minkowski metric $ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$ is therefore a theorem of $dx_4/dt = ic$, not an independent postulate. This is Theorem 1 of [MG-CosHolo] §3. The Lorentzian signature — the minus sign in front of c^2dt^2 — is the algebraic shadow of x_4's perpendicularity, recorded by the i in $x_4 = ict$.

Step 2: The four-speed budget defines uniform proper acceleration. By Axiom 3 of the McGucken Proof (Step 1 of §II.1), every physical system satisfies $|u|^2 = u^2_{\text{spatial}} + u^2_{x_4} = c^2$. This budget equation also constrains what counts as "uniform proper acceleration." An observer with constant magnitude of spatial acceleration $|a|$ but rotating direction in the (x, x_4) plane has a four-velocity that pseudo-rotates in that plane while preserving $|u|^2 = c^2$. The worldline's three-dimensional projection is hyperbolic: $x^2 - c^2t^2 = c^4/a^2$. This is the standard definition of a uniformly accelerated observer in special relativity, now seen as a pseudo-rotation orbit in the perpendicular plane where x_4 lives.

Step 3: Rindler coordinates from hyperbolic worldlines. Consider a family of uniformly accelerated observers with proper accelerations $a(\xi) = c^2/\xi$, labeled by the Rindler spatial coordinate $\xi > 0$. Each observer's worldline is a hyperbola of constant ξ; the orthogonal time slices are Rindler time η. The coordinate transformation $(t, x) \to (\eta, \xi)$ adapted to this family gives the Rindler line element:

$$ds^2 = -\xi^2\, d\eta^2 + d\xi^2 + dy^2 + dz^2.$$

The surface $\xi = 0$ is the Rindler horizon — a null hypersurface, because $u_{x_4} = 0$ there (by the four-speed-budget argument: observers with diverging proper acceleration approach the null cone and exhaust their spatial speed budget at c). This is the same $u_{x_4} = 0$ condition that characterizes black-hole horizons (Proposition III.1 of [MG-Bekenstein]). The Rindler horizon and the Schwarzschild horizon are two instances of the same x_4-stationary hypersurface; the near-horizon equivalence of Schwarzschild to Rindler that §II.3 uses is therefore not a coincidence, it is the fact that both horizons have $u_{x_4} = 0$ and any such hypersurface, in its local adapted frame, looks like Rindler.

Full expansion of this derivation of the Rindler form is a direct corollary of the McGucken Proof [MG-Proof] and the Minkowski-metric derivation of [MG-CosHolo, Theorem 1], combined with the standard special-relativistic coordinate transformation to accelerated frames.

2.5.2. The Wick rotation

Developed in full in §II.2 above and in the dedicated paper [MG-Wick]. Briefly: the Wick rotation $t \to -i\tau$, applied to $x_4 = ict$, gives $x_4 = ic(-i\tau) = c\tau$, which is the removal of the i from x_4. The Euclidean "imaginary time" τ is the real spatial-like coordinate that x_4 becomes when its perpendicularity is collapsed. This is a physical transformation, not a formal trick; every consequence of the Wick rotation (Lorentzian oscillating phases $\to$ Euclidean decaying weights, path integral $\to$ partition function, QM $\to$ statistical mechanics, $+i\varepsilon \to$ Euclidean regularization) follows from this single physical collapse. The Wick rotation is therefore a theorem of $dx_4/dt = ic$, applied to any geometry where one wishes to convert Lorentzian to Euclidean signature.

2.5.3. The KMS condition

In standard finite-temperature QFT, the KMS (Kubo–Martin–Schwinger) condition states that a quantum field on a Euclidean manifold with periodic time direction of period β is in thermal equilibrium at temperature $T = \hbar/(k_B\beta)$. This is standardly treated as an axiom of thermal field theory. In the McGucken framework it is a theorem of the Wick rotation combined with the second law [MG-HLA].

The derivation: under the Wick rotation, the quantum path integral $\int \mathcal{D}\phi\, e^{iS/\hbar}$ becomes the Euclidean partition function $\int \mathcal{D}\phi\, e^{-S_E/\hbar}$. If the Euclidean time τ is periodic with period β, the partition function is the trace $Z = \mathrm{Tr}(e^{-\beta H})$ where H is the Hamiltonian. This is exactly the Gibbs partition function at temperature $T = \hbar/(k_B\beta)$. The KMS condition is therefore the statement that Euclidean periodicity equals thermal equilibrium, which in the McGucken framework is the statement that periodic compactification of the direction x_4 becomes, after the Wick rotation, periodic compactification of the τ direction. The "why" is geometric: periodic identification of τ means each field configuration returns to itself after advance by β, and the weighting of configurations that accumulate action along the way is $e^{-\beta E}$. The second law [MG-HLA] enforces that this weighting is the thermal distribution, because any non-thermal distribution would imply a gradient of phase-space density that the McGucken expansion of x_4 would immediately isotropize.

The KMS condition is therefore a corollary of the Wick rotation and [MG-HLA], itself a theorem of $dx_4/dt = ic$.

2.5.4. The Einstein–Hilbert action and the Gibbons–Hawking–York boundary term

In standard GR, the Einstein–Hilbert action $S_{\mathrm{EH}} = (c^3/16\pi G)\int d^4x\,\sqrt{-g}\,R$ is the unique (up to constants) gravitational action whose variation yields Einstein's field equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}/c^4$. It is standardly treated as a postulate from which GR is derived. In the McGucken framework it is derived from local x_4-phase invariance in the full Standard-Model-plus-gravity paper [MG-SM], which extends the local x_4-phase invariance that generates U(1) [MG-QED] to include invariance under local reparameterizations of x_4, which generates the diffeomorphism invariance of GR and the Einstein–Hilbert action as the lowest-order invariant action. The full derivation is the content of [MG-SM, §§5–7];

it parallels the derivation of Yang–Mills theory from local gauge invariance, now with the gauge structure being the local reparameterizations of x_4 rather than internal phase rotations.

The Gibbons–Hawking–York boundary term $S_{\mathrm{GHY}} = (c^3/8\pi G)\oint d^3x\,\sqrt{|h|}\,(K - K_0)$ is the boundary contribution required for the variational principle of S_{EH} to be well-posed when the manifold has a boundary ∂M with fixed induced metric h_{ij} [York72, GH77]. Without S_{GHY}, the variation δS_{EH} produces boundary terms proportional to $\delta(\partial g)$ that do not vanish under the boundary condition "fix g on ∂M" (rather than the unphysical "fix g and ∂g on ∂M"). The GHY term exactly cancels these boundary contributions, restoring a well-posed variational problem. This is not an additional physical postulate; it is the unique consistent boundary term required once S_{EH} and fixed-metric boundary conditions are specified. Since S_{EH} is a theorem of $dx_4/dt = ic$ (via [MG-SM]), and S_{GHY} is determined uniquely by S_{EH} plus boundary consistency, the full action $S_{\mathrm{EH}} + S_{\mathrm{GHY}}$ used in Proposition V.1 is a theorem of $dx_4/dt = ic$.

In the cosmological-holography paper [MG-CosHolo, §8], this is made fully explicit: the McGucken horizon surface term $S_{\mathrm{surf}}[g; R_4]$ is the GHY boundary action evaluated on the McGucken horizon, and its variation (together with the bulk S_{EH} variation) produces the emergent Einstein-type equation $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T^{\mathrm{eff}}_{\mu\nu}$. The same machinery used there is what Proposition V.1 uses for the black-hole case.

2.5.5. The Stefan–Boltzmann law

In standard thermodynamics, the Stefan–Boltzmann law states that a blackbody of area A at temperature T radiates energy at rate $dE/dt = \sigma A T^4$, where $\sigma = \pi^2 k_B^4/(60\hbar^3 c^2)$. The law is derived in standard statistical mechanics by integrating the Planck blackbody spectrum over frequency. In the McGucken framework, this derivation goes through unchanged once the second law [MG-HLA] and the Planck-scale quantization of x_4-oscillation [MG-Constants] are in place:

Step 1: Mode count on a blackbody surface. By [MG-Constants], one independent x_4-oscillation mode occupies a minimum area ℓ_P^2 on any two-dimensional hypersurface. The mode density on a surface of area A is bounded by A/ℓ_P^2. At temperature T far below the Planck scale, only modes of frequency $\omega \lesssim k_B T/\hbar$ are populated.

Step 2: Blackbody spectrum from thermal equilibrium. The second law [MG-HLA] enforces that the modes on the surface are thermally distributed at T — this is the same argument that the KMS subsection uses, now applied to the modes on a radiating surface. The Planck spectrum follows: the mean number of photons in a mode of frequency ω is $n(\omega, T) = 1/(e^{\hbar\omega/k_B T} - 1)$, and the spectral energy density is $(\hbar\omega^3/\pi^2 c^3)\cdot$

$n(\omega, T)$.

Step 3: Integration. Integrating the Planck spectrum over all frequencies gives the total energy flux per unit area: $dE/(dt\, dA) = \sigma T^4$, with $\sigma = \pi^2 k_B^4/(60\hbar^3 c^2)$. This is the Stefan–Boltzmann law.

Every step is either the McGucken mode count [MG-Constants], the McGucken second law [MG-HLA], or standard Bose–Einstein statistical mechanics (itself a corollary of the Wick rotation applied to the mode ensemble). The Stefan–Boltzmann constant σ is a specific combination of c, $\hbar$, and k_B; each of c and $\hbar$ is set by the McGucken Principle [MG-Constants], and k_B is the Boltzmann conversion convention between information-theoretic entropy (bits) and thermodynamic entropy. The Stefan–Boltzmann law used in Proposition VI.1 is therefore a theorem of $dx_4/dt = ic$ together with the fundamental-constant values it sets.

2.5.6. Summary

All five pieces of machinery used in §III–§VII — Rindler near-horizon form, Wick-rotated Schwarzschild cigar, KMS condition, Einstein–Hilbert plus Gibbons–Hawking–York action, Stefan–Boltzmann law — are theorems of $dx_4/dt = ic$, developed either in this section or in the corpus papers cited: [MG-Wick] for the Wick rotation, [MG-SM] for the Einstein–Hilbert action, [MG-HLA] for the second law, [MG-Constants] for the Planck-scale mode quantization, [MG-CosHolo] for the GHY surface term, and [MG-Proof] plus [MG-CosHolo, Theorem 1] for the Minkowski metric and Rindler form. The present paper is therefore derivable from $dx_4/dt = ic$ at every level: the postulate produces the tools, and the tools produce the five Hawking results. No tool is borrowed from outside the McGucken framework; every piece of the derivation descends from the same geometric postulate.

3. Hawking Radiation as x_4-Stationary Mode Emission: Result H-1

3.1. What Hawking 1975 established for H-1

Hawking's 1975 derivation of thermal radiation from black holes proceeds by mode analysis. The vacuum of a quantum field is defined relative to a choice of positive-frequency modes. For a black hole formed by gravitational collapse, the natural "in" vacuum is defined by modes at past null infinity that have positive frequency with respect to the asymptotic Minkowski time; the natural "out" vacuum is defined similarly at future null infinity. These two vacuum states do not coincide.

The Bogoliubov coefficients α_{ij} and β_{ij} relate the "in" and "out" mode sets. The coefficient β_{ij}, which mixes positive-frequency "in" modes with negative-frequency "out" modes, produces particle creation: the expected number of outgoing particles in mode j is $\langle N_j \rangle = \sum_i |\beta_{ij}|^2$. Hawking computed β_{ij} explicitly for the Schwarzschild collapse geometry and found that the outgoing flux has a Planckian thermal spectrum at temperature $T_H = \hbar\kappa/(2\pi c k_B)$. The calculation is technically impressive but physically opaque. What physical process produces the particle creation?

3.2. Proposition III.1 (Hawking radiation as x_4-stationary mode emission)

Proposition 3.1 (Thermal radiation from x_4-stationary horizon modes)**.** *By Proposition III.1 of [MG-Bekenstein], the black-hole horizon is an x_4-stationary hypersurface populated by x_4-stationary modes. In the Lorentzian geometry, these modes propagate at c along the null horizon generators but cannot escape to future null infinity. In the Euclidean geometry obtained by the McGucken Wick rotation, the horizon becomes the tip of the cigar, and the horizon modes acquire Euclidean-time periodicity $\beta = 2\pi/\kappa$. By the standard KMS condition, a field with Euclidean-time periodicity β is thermally distributed at temperature $T = \hbar/(k_B\beta)$. The x_4-stationary horizon modes therefore form a thermal ensemble at temperature $T_H = \hbar\kappa/(2\pi c k_B)$. Upon analytic continuation back to Lorentzian signature, the thermal ensemble manifests as outgoing thermal radiation at future null infinity. This is Hawking's result H-1, with the physical mechanism identified: the radiation is x_4-stationary mode emission from the horizon's McGucken-Sphere-analogue, thermalized by the Euclidean cigar-geometry periodicity.*

Proof. Four steps.

Step 1: The horizon supports x_4-stationary modes (from [MG-Bekenstein] Proposition III.1). The event horizon is a null hypersurface. By Proposition IV.1 of [MG-Twistor], null hypersurfaces are exactly the hypersurfaces on which physical excitations are x_4-stationary. The horizon is therefore populated by x_4-stationary modes of every quantum field.

Step 2: These modes are thermalized by the cigar periodicity. Applying the McGucken Wick rotation, the Lorentzian near-horizon geometry becomes the Euclidean cigar with angular period $\beta = 2\pi/\kappa$. Euclidean periodicity in τ with period β is the Kubo–Martin–Schwinger (KMS) condition for thermal equilibrium at temperature $T = \hbar/(k_B\beta) = \hbar\kappa/(2\pi c k_B)$.

Step 3: Analytic continuation back to Lorentzian produces outgoing thermal radiation.

Analytic continuation of the Euclidean equilibrium distribution back to Lorentzian signature gives a real-time ensemble of horizon modes in thermal equilibrium at T_H. By the same x_4-advance mechanism that carries any x_4-stationary mode outward at rate c, these thermal horizon modes propagate outward along null geodesics toward future null infinity.

Step 4: The mechanism is geometric, not formal. Unlike Hawking's Bogoliubov-coefficient calculation, which is mode-matching between two formal vacuum states, the McGucken derivation identifies the physical origin: the horizon supports x_4-stationary modes, these are thermalized by the Euclidean cigar periodicity, and x_4's outward expansion carries them to infinity. The "in" and "out" vacuum mismatch of Hawking's derivation is the formal mode-theoretic signature of the underlying geometric fact. ■

> **Meaning.** Hawking radiation is real because the horizon has a real thermal population of x_4-stationary modes and the x_4 expansion carries them outward. The mode population exists because the horizon is a null hypersurface and null hypersurfaces are x_4-stationary. The thermal distribution exists because the Euclidean near-horizon geometry is a cigar with angular period $\beta = 2\pi/\kappa$, and cigar periodicity is thermal equilibrium. The radiation escapes because x_4 expands at rate c and carries x_4-stationary modes outward. Each step is a geometric consequence of $dx_4/dt = ic$.

3.3. Resolution of HK-1 (where does the radiation come from?)

The physical-origin question that Hawking 1975 left open receives its answer in Proposition III.1: the radiation is the x_4-stationary mode population of the horizon, thermalized by the cigar-geometry periodicity, carried outward by x_4's expansion. Three intuitive pictures of Hawking radiation are prominent in the standard literature; a brief comparison orients the result:

Virtual-pair production. The most widely cited intuitive picture has a vacuum fluctuation near the horizon producing a virtual particle-antiparticle pair, one member of which falls into the black hole while the other escapes. The picture's appeal is its simplicity; its failure is that it has no clean technical backing. In the McGucken framework, the "virtual pair" is the pair of x_4-stationary modes on the Euclidean cigar, one on the forward-τ branch and one on the backward-τ branch. The Wick rotation joins these into a single continuous mode on the cigar; the Lorentzian projection splits them back into a "virtual pair." The picture is not wrong, but it is a Lorentzian shadow of the geometrically unified Euclidean mode.

Parikh–Wilczek tunneling. Parikh and Wilczek [PW] derived Hawking radiation as a semiclassical tunneling process: a particle on a classically forbidden trajectory tunnels through the horizon by WKB amplitude. In the McGucken framework, the "tunneling" is the crossing from the horizon null hypersurface to the outgoing null direction, which is not a tunneling event but the natural propagation of an x_4-stationary mode outward at rate c. The two pictures agree on the spectrum because they compute the same thermal weighting, but the McGucken picture has no barrier and no tunneling — it has null-geodesic emission thermalized by the cigar.

Unruh effect generalization. The third common picture identifies Hawking radiation as a gravitational version of the Unruh effect: a uniformly accelerated observer in flat Minkowski space detects a thermal flux at $T_U = \hbar a/(2\pi c k_B)$; the near-horizon Schwarzschild geometry is Rindler, and a near-horizon static observer has proper acceleration κ, so detects a thermal flux at T_H. This picture is correct and the cleanest of the three; the McGucken framework supplies its underlying mechanism. The Unruh effect itself is, in the McGucken reading, a consequence of the accelerated observer's time coordinate not matching x_4's expansion direction.

The three pictures are complementary Lorentzian shadows of a single Euclidean geometric fact: the near-horizon geometry is a cigar whose angular period is the inverse Hawking temperature. The McGucken framework identifies the common mechanism they all shadow: x_4-stationary-mode emission thermalized by the cigar, carried outward by x_4's expansion, without barriers, tunneling, or virtual-pair-production gymnastics.

4. The Hawking Temperature: Result H-2

4.1. What Hawking 1975 established for H-2

Hawking's result for the radiation temperature is $T_H = \hbar\kappa/(2\pi c k_B)$, obtained via Bogoliubov-coefficient calculation. The alternative via the Gibbons–Hawking Euclidean path integral [GH77] is far cleaner: Wick-rotate the Schwarzschild metric, impose regularity at the horizon by periodic identification in Euclidean time with period $\beta = 2\pi/\kappa$, read off T_H. The Euclidean calculation takes three lines; the Bogoliubov calculation takes dozens of pages. Yet the Euclidean calculation's physical meaning is obscure in standard treatments.

4.2. Proposition IV.1 (The Hawking temperature from the cigar period)

Proposition 4.1 (T_H from the McGucken cigar geometry)**.** *By §II.3, the McGucken Wick rotation applied to the Schwarzschild near-horizon geometry produces a two-dimensional Euclidean cigar with angular period $\beta = 2\pi/\kappa$. By the standard relation between Euclidean-*

time periodicity and thermal equilibrium, the corresponding temperature is $T = \hbar/(k_B\beta) = \hbar\kappa/(2\pi c k_B)$. This is the Hawking temperature. The derivation requires no Bogoliubov coefficients, no "in"-"out" vacuum matching, and no analytic continuation of scattering amplitudes; only the McGucken Wick rotation and the cigar-geometry period.

Proof. Three steps. *Step 1:* By §II.2, removing the i from x_4 converts the Lorentzian near-horizon Schwarzschild geometry to the Euclidean cigar geometry, with angular period $\beta = 2\pi/\kappa$ forced by regularity at the horizon. *Step 2:* In finite-temperature QFT, a quantum field on a Euclidean manifold with a periodic time direction of period β is in thermal equilibrium at temperature $T = \hbar/(k_B\beta)$; this is the KMS condition. *Step 3:* Putting them together, $T_H = \hbar/(k_B\beta) = \hbar\kappa/(2\pi c k_B)$. For Schwarzschild with $\kappa = c^4/(4GM)$, this gives $T_H = \hbar c^3/(8\pi G M k_B) \approx 6.17 \times 10^{-8}$ K $\cdot (M_\odot/M)$. ■

> **Meaning.** Hawking's temperature is not a mysterious quantum-gravity result; it is the angular period of the cigar. The McGucken Wick rotation produces the cigar geometry from the Lorentzian near-horizon Schwarzschild metric by collapsing x_4's perpendicularity. That period, in thermodynamic units, is the Hawking temperature. The Gibbons–Hawking Euclidean-path-integral derivation works in the standard literature because it unknowingly exploits exactly this cigar geometry; the McGucken framework identifies the physical meaning of the rotation.

4.3. Resolution of HK-2 (why Euclidean methods work)

The open problem of why Euclidean methods work for black-hole thermodynamics dissolves under the McGucken Principle. The Wick rotation is not a formal computational device; it is the physical collapse of x_4's perpendicularity to produce the Euclidean geometry. The cigar geometry is the real Euclidean near-horizon manifold that obtains when the collapse is performed. Its angular period is the Hawking temperature. The "why" is geometric: periodicity of the cigar corresponds to periodicity in Euclidean time, which corresponds to thermal equilibrium.

5. The Bekenstein–Hawking Formula $S = A/4$: Result H-3

5.1. What Hawking 1975 established for H-3

Bekenstein's 1973 coefficient $\eta = (\ln 2)/(8\pi)$ was a heuristic estimate. Hawking's 1975 result fixed the coefficient exactly at $\eta = 1/4$ by thermodynamic consistency: the first law of black-hole mechanics is $dM = (\kappa c^2/8\pi G)\, dA$, and the thermodynamic first law is

$dM = T\,dS$. Matching these with $T = T_H$ gives

$$S_{BH} = k_B c^3 A/(4\hbar G) = k_B A/(4\ell_P^2),$$

fixing $\eta = 1/4$. The derivation is thermodynamically unambiguous; but the geometric origin of the factor 1/4 is not transparent. Why 1/4?

5.2. Proposition V.1 ($\eta = 1/4$ from the Euclidean action of the cigar)

Proposition 5.1 (The factor 1/4 from the Euclidean Einstein–Hilbert action on the Schwarzschild cigar)**.** *The McGucken Wick rotation of the Schwarzschild geometry produces a Euclidean manifold that is smooth everywhere — the "cigar" near the horizon, extending to an asymptotically flat region at spatial infinity. The total Euclidean gravitational action, evaluated on this manifold with the Gibbons–Hawking–York boundary term included, is*

$$I_E = \beta M c^2/2,$$

where $\beta = 2\pi/\kappa$ is the Euclidean-time period. The Euclidean partition-function relation $S = k_B\beta\langle E\rangle - k_B I_E/\hbar$ with $\langle E\rangle = Mc^2$ yields

$$S_{BH} = k_B\beta M c^2/(2\hbar) = k_B A/(4\ell_P^2).$$

The factor 1/4 is exactly the numerical factor appearing in $I_E = \beta M c^2/2$, and it has a direct geometric origin: the Euclidean action of the Schwarzschild cigar is half of the on-shell energy integral because the horizon contributes a single Gibbons–Hawking–York boundary term whose coefficient is 1/2 in the standard normalization.

Proof. Four steps.

Step 1: The full Euclidean Schwarzschild geometry. The Schwarzschild line element in the exterior $r > 2GM/c^2$,

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2,$$

under the McGucken Wick rotation $t \to -i\tau$ becomes

$$ds_E^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 d\tau^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

Near the horizon, this reduces to the cigar geometry of §II.3 with $\beta = 2\pi/\kappa$, $\kappa = c^4/(4GM)$; far from the horizon, it approaches Euclidean 4-space. The full Euclidean

Schwarzschild manifold is therefore a cigar at the horizon smoothly joined to asymptotic Euclidean 4-space.

Step 2: The total action $S_{geom} + S_{GHY}$. The Euclidean gravitational action consists of two pieces:

$$I_E = -\frac{c^3}{16\pi G}\int_M d^4x\,\sqrt{g}\,R - \frac{c^3}{8\pi G}\oint_{\partial M} d^3x\,\sqrt{h}\,(K - K_0).$$

Here R is the Ricci scalar, ∂M is the boundary at spatial infinity, h is the induced 3-metric, K is the extrinsic curvature of ∂M, and K_0 is the extrinsic curvature of the same surface embedded in flat Euclidean 4-space. The Gibbons–Hawking–York term is required for the variational principle to be well-posed when the boundary has fixed induced metric [GH77, York72].

Step 3: Evaluation. The Schwarzschild solution is Ricci-flat ($R = 0$), so the bulk term vanishes identically. The entire action comes from the GHY boundary term on the sphere at spatial infinity. Explicit calculation [GH77, Eq. 3.17] gives

$$I_E = \beta M c^2/2.$$

The factor of 1/2 is the geometric consequence of $K - K_0$: the extrinsic curvature of a large sphere in Schwarzschild differs from its flat-space value by a term proportional to M, and the surface integral gives half the period times the mass.

Step 4: Entropy from the partition function. The Euclidean action relates to the partition function by $Z = e^{-I_E/\hbar}$, and thermodynamics gives

$$S_{BH} = k_B(\beta\langle E\rangle - I_E/\hbar) = k_B(\beta M c^2/\hbar - \beta M c^2/(2\hbar)) = k_B \beta M c^2/(2\hbar).$$

Using $\beta = 8\pi G M/c^3$ and $A = 16\pi G^2 M^2/c^4$:

$$S_{BH} = k_B \cdot \frac{Ac^3}{4\hbar G} = k_B \frac{A}{4\ell_P^2},$$

giving $\eta = 1/4$. ■

> **Meaning.** The 1/4 in the Bekenstein–Hawking formula is the numerical factor in $I_E = \beta M c^2/2$ for the Euclidean Schwarzschild geometry. It is the explicit computation of the Gibbons–Hawking–York boundary action, with the subtraction term K_0 (which removes the vacuum-flat-space contribution) generating the 1/2. The Bekenstein $(\ln 2)/(8\pi)$ and the Hawking 1/4 both follow from the McGucken Principle: the former from classical-information one-bit counting per Compton-wavelength absorbed particle [MG-Bekenstein], the latter from the full semiclassical Euclidean-

> action computation presented here. They differ by $2\pi/\ln 2 \approx 9.06$, the ratio of Planckian-thermal mode density to one-bit-per-absorbed-particle counting.

5.3. Resolution of HK-3 (why exactly 1/4?)

The open problem of why the Bekenstein–Hawking coefficient is exactly 1/4 admits the geometric answer: the coefficient is the numerical factor in $I_E = \beta Mc^2/2$, and this factor is the explicit output of the GHY boundary action with the flat-space subtraction K_0. The factor is not mysterious; it is the standard 1977 computation [GH77], performed on a geometry whose physical meaning is now supplied by the McGucken Wick rotation. What was opaque in the standard derivation is resolved by tracing it to the Ricci-flatness of Schwarzschild (which zeroes the bulk integral) and the half-factor in the GHY evaluation at spatial infinity. York's 1972 introduction of the boundary term [York72] and Gibbons–Hawking's 1977 application [GH77] supply the computation; the McGucken framework supplies the physical meaning.

6. Black-Hole Evaporation: Result H-4

6.1. What Hawking 1975 established for H-4

If a black hole emits thermal radiation at temperature T_H, it loses energy at the Stefan–Boltzmann rate:

$$dM/dt = -\sigma A T_H^4/c^2.$$

For a Schwarzschild black hole this gives $dM/dt \propto -1/M^2$, integrating to evaporation time $\tau \sim (M/M_\odot)^3 \cdot 10^{67}$ yr. Primordial black holes of mass $\lesssim 5 \times 10^{14}$ g would be evaporating now. The final stage is explosive: as M decreases, T_H rises, and dM/dt becomes rapid. The thermal emission implies the horizon area decreases, violating the 1971 classical area theorem [Haw71].

6.2. Proposition VI.1 (Evaporation from x_4-stationary-mode emission at Planck resolution)

Proposition 6.1 ($dM/dt \propto -1/M^2$ from blackbody emission)**.** *The black-hole horizon is populated by x_4-stationary modes (Proposition III.1 of [MG-Bekenstein]) thermally distributed at T_H (Proposition III.1 of the present paper). By the McGucken second law [MG-HLA], these modes are carried outward by x_4's expansion at rate c, radiating into the x_4-stationary mode reservoir at infinity. The emission rate is the blackbody rate $\sigma A T_H^4$. For a Schwarzschild black hole with $T_H \propto 1/M$ and $A \propto M^2$, the mass-loss rate* *is* $dM/dt \propto -(M^2)(1/M^4) = -1/M^2$.

Proof. Three steps.

Step 1: Horizon modes are thermal at Planck resolution. By §III and §IV, the x_4-stationary horizon modes form a thermal ensemble at T_H, with mode density one per Planck area ℓ_P^2 on the horizon.

Step 2: Emission rate = Stefan–Boltzmann. x_4-stationary modes are emitted from the horizon at rate c. Total energy flux is $dE/dt = \sigma A T_H^4$, where $\sigma = \pi^2 k_B^4/(60\hbar^3 c^2)$.

Step 3: Integration for Schwarzschild. Using $T_H = \hbar c^3/(8\pi G M k_B)$ and $A = 16\pi G^2 M^2/c^4$, direct substitution gives $dM/dt \propto -1/M^2$ with the correct numerical coefficient. Integrating yields $\tau \propto M_0^3$, with coefficient giving $\tau \approx (M_0/M_\odot)^3 \cdot 2.1 \times 10^{67}$ yr. ■

Meaning. Black-hole evaporation is ordinary blackbody radiation from the horizon. The horizon's hot surface is populated by x_4-stationary modes thermalized by the cigar geometry. x_4's expansion at rate c carries these modes outward. The Stefan–Boltzmann law gives the mass-loss rate. The M^3 evaporation time reflects that small black holes are hot and radiate fast, while large black holes are cold and radiate slow.

6.3. Violation of the classical area theorem

Hawking's 1971 classical area theorem states that horizon area is non-decreasing under any classical evolution. Hawking radiation violates this: the horizon shrinks as the black hole evaporates. In the McGucken framework this is transparent: the area theorem rested on the classical assumption that no null geodesics escape from the horizon, but Hawking radiation is precisely x_4-stationary-mode escape through the quantum emission mechanism. The classical assumption fails at Planck-scale resolution; the classical area theorem is recovered in the $\hbar \to 0$ limit where Hawking radiation vanishes.

7. The Refined Generalized Second Law: Result H-5

7.1. What Hawking 1975 established for H-5

Although the classical area theorem fails under Hawking radiation, a refined Generalized Second Law is preserved:

$$S + k_B A/(4\ell_P^2) \text{ never decreases,}$$

where S is the entropy of matter and radiation outside the horizon (now including the Hawking radiation) and A is the horizon area. As the black hole evaporates, S_{BH} decreases, but the Hawking radiation carries entropy out at the rate required to maintain the overall non-decrease.

7.2. Proposition VII.1 (Refined GSL from the global McGucken second law)

Proposition 7.1 (The refined GSL as the McGucken second law with quantum radiation)**.** *By Proposition VI.1 of [MG-Bekenstein], the Generalized Second Law $dS_{ext} + dS_{BH} \geq 0$ follows from the global McGucken second law [MG-HLA] applied to a spacetime partitioned by an event horizon. Under Hawking radiation, both the exterior and horizon evolve: S_{ext} includes the entropy of the Hawking-emitted thermal radiation, and S_{BH} tracks the horizon area. The McGucken second law still requires $dS_{ext}/dt + dS_{BH}/dt \geq 0$ at every instant, because x_4's expansion continues monotonically regardless of the evaporation. The refined GSL is the same global McGucken second law applied to an evolving partition.*

Proof. By [MG-HLA], $dS_{\text{total}}/dt \geq 0$ for the total entropy of all x_4-stationary and x_4-advancing modes. This includes Hawking-radiation modes. The partition into $S_{\text{total}} = S_{\text{ext}} + S_{BH}$ is time-dependent under evaporation: the horizon shrinks with time, and the partition boundary moves. By the global monotonicity, $dS_{\text{ext}}/dt + dS_{BH}/dt \geq 0$. The balance that Hawking required for thermodynamic consistency is enforced by the global McGucken second law. ■

> **Meaning.** The refined Generalized Second Law is the same global McGucken second law as in [MG-Bekenstein] §VI, applied to an evolving partition where the horizon shrinks under Hawking radiation. The law does not need to be refined; only the partition changes. At every instant, the McGucken second law requires $dS_{\text{total}}/dt \geq 0$, and the horizon shrinkage is compensated by the Hawking-radiation entropy increase.

7.3. Resolution of HK-4 (the information paradox)

Hawking's 1976 paper [Haw76] argued that complete black-hole evaporation destroys information, because the final Hawking radiation is purely thermal and carries no memory of what fell in. This contradicts quantum-mechanical unitarity. The "information paradox" has been the central open problem in black-hole thermodynamics for fifty years.

In the McGucken framework, the resolution is structural. The Hawking radiation in the

Lorentzian picture appears thermal because it is sourced by the cigar-geometry thermalization of horizon modes. But the cigar is only half of the full story: the Euclidean geometry captures the statistical-mechanical ensemble, while the Lorentzian geometry captures the specific mode-by-mode quantum evolution. The modes on the horizon are not independent random thermal excitations; they are x_4-stationary modes that carry information about the interior state.

Six-sense locality preserves the correlations. The horizon is a null hypersurface and, by the six-fold null-surface identity established in [MG-AdSCFT] §2a and used in §IX below, is a geometric locality in six independent mathematical senses simultaneously (foliation, level sets, caustics, contact geometry, conformal geometry, and null-hypersurface cross-section). Every pair of horizon modes shares identity in all six senses. When a mode is emitted as Hawking radiation, it crosses from the horizon null hypersurface to future null infinity via a null geodesic — a trajectory that remains on a null hypersurface the entire way. The six-fold identity of the emitted mode with every other horizon mode is therefore preserved along its outgoing null trajectory. These are not fragile correlations; they are geometric invariants of null-hypersurface propagation.

What this means for unitarity. The apparent thermality of Hawking radiation at any finite time is the result of projecting these six-sense-correlated modes onto a three-dimensional spatial slice, where the correlations appear only as statistical averages. The correlations are not destroyed by the projection; they are hidden by it. As more modes are emitted, more of the six-fold correlation structure becomes accessible at infinity, and the outgoing radiation progressively reveals the interior information. The Page curve — monotonic rise in entanglement entropy until the Page time, then monotonic decline — is the quantitative shadow of this progressive revelation.

The replica-wormhole calculations of Penington [Pen] and Almheiri–Engelhardt–Marolf–Maxfield [AEMM] reproduce the Page curve by identifying "islands" — regions of the interior that contribute to the entanglement wedge of the radiation after the Page time. In the McGucken framework this has a direct interpretation: the island is the set of interior modes whose six-sense partners are in the emitted radiation. The Ryu–Takayanagi formula [RT], the quantum extremal surface prescription [EW], and the island formula [AEMM] all express this same geometric fact in different formalisms. The McGucken framework identifies the common underlying mechanism: six-sense locality of null hypersurfaces preserves bulk-boundary correlations through Hawking emission, and the apparent information loss is only a failure of the three-dimensional projection to resolve those correlations. Unitarity is preserved; the Page curve is a consequence, not a miracle.

7.4. Resolution of HK-5 (the trans-Planckian problem)

Hawking's derivation traces late-time radiation modes back to very early times where they had exponentially shorter wavelengths — formally, below the Planck length near the horizon. The trans-Planckian regime is outside the domain of QFT on curved spacetime, yet the derivation's prediction depends on it.

In the McGucken framework this problem dissolves. By Proposition IV.1 of [MG-Bekenstein] and §II.3 of [MG-Constants], x_4-oscillation is Planck-scale quantized: modes of wavelength shorter than ℓ_P are not independent but represent the same x_4-oscillation state. The trans-Planckian "modes" that appear in the standard Hawking calculation are not physically independent degrees of freedom; they are the same Planck-scale mode viewed in different frequency windows. The physical mode-count on the horizon is A/ℓ_P^2, and the Hawking calculation's extension to arbitrarily short wavelengths is a formal extension that double-counts the same physical modes.

8. What the McGucken-Improved Hawking Programme Looks Like

Assembling the five Propositions, the McGucken-informed reading of Hawking 1975 is: Hawking established five results by Bogoliubov mode matching (H-1, H-2), thermodynamic first-law consistency (H-3), Stefan–Boltzmann blackbody application (H-4), and thought-experiment bookkeeping (H-5). Each was a technical triumph. Under the McGucken Principle $dx_4/dt = ic$, each becomes a theorem:

- **H-1** (thermal radiation): x_4-stationary horizon modes, thermalized by the Euclidean cigar periodicity, carried outward by x_4's expansion.
- **H-2** (Hawking temperature): the angular period $\beta = 2\pi/\kappa$ of the Euclidean cigar obtained by removing the i from x_4.
- **H-3** ($\eta = 1/4$): the numerical factor in $I_E = \beta Mc^2/2$ from the explicit GHY boundary action on the Euclidean Schwarzschild manifold.
- **H-4** (evaporation): Stefan–Boltzmann emission from the horizon's hot surface via x_4's expansion.
- **H-5** (refined GSL): the same global McGucken second law as in [MG-Bekenstein], applied to an evolving partition under evaporation.

The programme as a whole, combining Bekenstein 1973 and Hawking 1975, has all ten central results of foundational black-hole thermodynamics (five from each paper) derived from the single geometric postulate $dx_4/dt = ic$.

9. The Four-Step Chain from Hawking to AdS/CFT and Cosmological Holography

Hawking's 1975 result set the stage for the holographic principle ('t Hooft 1993, Susskind 1995) and its most precise realization in AdS/CFT (Maldacena 1997). In the McGucken framework, the jump from Hawking 1975 to the holographic principle is not a conjecture but a theorem. This section traces the four-step chain, drawing on the dedicated McGucken AdS/CFT paper [MG-AdSCFT] and the McGucken cosmological-holography paper [MG-CosHolo].

9.1. Step 1: The horizon result extends to any null hypersurface

Hawking's $S_{BH} = k_B A/(4\ell_P^2)$ was derived for a black-hole event horizon. The derivation in the present paper invokes only that the horizon is a null hypersurface supporting x_4-stationary modes, thermalized by the Euclidean cigar. These properties belong to *any* null hypersurface. The McGucken Sphere centered on any emission event gives

$$S_{\text{Sphere}}(t) = \pi k_B (ct)^2/\ell_P^2$$

for the entropy on a sphere of age t. The black-hole area law is a special case of the McGucken-Sphere area law [MG-AdSCFT, Proposition 2].

9.2. Step 2: The six-fold null-surface identity of the McGucken Sphere

Why does information on a null hypersurface satisfy an area bound rather than a volume bound? The McGucken AdS/CFT paper establishes that the McGucken Sphere is a geometric locality in six independent mathematical senses [MG-AdSCFT, §2a]: (1) foliation theory (a leaf of a foliation of 3-space by concentric 2-spheres); (2) level sets (equidistant from the origin); (3) caustics and Huygens wavefronts (causal boundary); (4) contact geometry (Legendrian submanifold); (5) conformal/inversive geometry (conformally invariant pencil); (6) null-hypersurface cross-section (causally extremal in Lorentzian geometry). The six-fold identity is decisive: if points on the null surface were causally independent, the boundary would need as many degrees of freedom as the volume. Because the null surface is unified in six senses, the data is correlated, reducing degrees of freedom from volume to area scaling.

9.3. Step 3: The holographic bound as a theorem of $dx_4/dt = ic$

Proposition 9.1 (The holographic bound from McGucken-Sphere mode counting)**.** *Let N be any null hypersurface bounding a bulk region R. By Step 1 and Planck-scale quantization of x_4-oscillation, N supports A/ℓ_P^2 independent x_4-stationary modes, where A is the area of N. By the six-fold null-surface identity (Step 2), the information content of the bulk region R is bounded by*

$$S \leq k_B A/(4\ell_P^2).$$

This is the Bekenstein bound in the 't Hooft–Susskind form. It holds for every null hypersurface in any spacetime — not only black-hole event horizons — and is therefore the content of the holographic principle as a theorem, not a postulate.

9.4. Step 4: AdS/CFT as the specific dual pair of McGucken holography

Maldacena's 1997 conjecture equates type IIB string theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$ with $\mathcal{N} = 4$ super Yang–Mills on the four-dimensional conformal boundary. The present paper does not derive the specific dual pair. What the McGucken framework does establish is the geometric reason *why* a bulk-boundary duality must exist for any consistent theory of quantum gravity on asymptotically anti-de Sitter space:

- Bulk degrees of freedom in any region are bounded by the area of any null hypersurface bounding the region (Proposition IX.1).
- In asymptotically AdS space, the natural null hypersurface is the conformal boundary at spatial infinity.
- The bulk information is therefore encoded on a conformally invariant boundary, and the boundary theory must be conformally invariant — hence a conformal field theory.
- Causal reconstructibility from the conformal boundary ensures bulk physics is reconstructible from boundary data.

Maldacena's construction satisfies these requirements concretely; the McGucken framework establishes they are necessary consequences of $dx_4/dt = ic$ plus asymptotic AdS boundary conditions.

9.5. Cosmological holography: the McGucken horizon in FRW

Our universe is not asymptotically AdS — it is asymptotically de Sitter, with a positive cosmological constant, and has passed through radiation-, matter-, and dark-energy-dominated regimes in spatially flat FRW cosmology. The McGucken framework [MG-CosHolo] provides the holographic screen: the **McGucken horizon**.

Definition IX.2 (McGucken horizon in FRW). Let the cosmological spacetime be spatially flat FRW with scale factor $a(t)$. Define the McGucken embedding map at cosmic time t:

$$X_1 = a(t)r\sin\theta\cos\phi,\ X_2 = a(t)r\sin\theta\sin\phi,\ X_3 = a(t)r\cos\theta,$$

$$X_4 = \sqrt{R_4(t)^2 - a(t)^2r^2}.$$

The embedding is real iff $a(t)r \le R_4(t)$, and the saturation locus $a(t)r_H(t) = R_4(t)$ defines the McGucken horizon, proper radius $R_H(t) = R_4(t) = ct$ in the early-time regime [MG-CosHolo, Theorem 2].

Proposition 9.2 (Cosmological holographic entropy on the McGucken horizon)**.** *The McGucken horizon has proper area $A_{Mc}(t) = 4\pi R_4(t)^2$. The entropy is*

$$S_{Mc}(t) = A_{Mc}(t)/(4\ell_P^2) = \pi R_4(t)^2/\ell_P^2.$$

An explicit Gibbons–Hawking–York surface term on the horizon 3-surface $\Sigma_H(t)$ reproduces this entropy [MG-CosHolo, Theorem 6]:

$$S_{surf}[g; R_4] = \frac{1}{8\pi G}\oint_{\Sigma_H} d^3x\,\sqrt{|h|}\,(K - K_0).$$

Variation of the total action yields $G_{\mu\nu}+\Lambda g_{\mu\nu} = 8\pi G T^{eff}_{\mu\nu}$ with $\Lambda \sim 1/R_4(t)^2$ [MG-CosHolo, Theorem 7].

9.6. A testable prediction: $\rho^2(t) \approx 7$ at recombination

The McGucken horizon in FRW cosmology differs from the standard Hubble horizon. Define $\rho(t) = R_H(t)/R_{\rm Hub}(t) = R_4(t)H(t)/c$. Only in asymptotic de Sitter does $\rho \to 1$. Throughout cosmological history $\rho \neq 1$, and the holographic entropies differ by $\rho^2(t)$.

At recombination ($z \approx 1100$, $t_{\rm rec} \approx 3.8\times10^5$ yr), explicit calculation [MG-CosHolo, §10.5] gives:

- $R_4(t_{\rm rec}) = c \cdot t_{\rm rec} \approx 3.6 \times 10^{21}$ m $\approx 1.2 \times 10^5$ light-years
- $R_{\rm Hub}(t_{\rm rec}) = c/H_{\rm rec} \approx 1.4 \times 10^{21}$ m

- $\rho(t_{\text{rec}}) \approx 2.6$
- $\boldsymbol{S}_{\text{Mc}}/\boldsymbol{S}_{\text{Hub}}$ **at recombination** $\boldsymbol{\approx \rho^2 \approx 7}$**.**

This is a sharp, quantitative prediction. The McGucken horizon area at recombination is ≈ 7 times the Hubble horizon area, and the cosmological holographic entropy differs by the same factor. Unlike AdS/CFT — which has no direct cosmological application because our universe is not AdS — McGucken cosmological holography gives predictions for the universe we actually inhabit.

9.7. The horizon/flatness/homogeneity problem without inflation

A further consequence: the standard horizon problem (why is the CMB so homogeneous given that distant regions were out of causal contact at recombination?) does not arise. The McGucken horizon $R_H(t) = R_4(t) = ct$ is always greater than or equal to the standard causal horizon at early times, because the McGucken null hypersurface advances at c regardless of FRW scale-factor dilation. The universe was in causal contact throughout its early history; no inflation is required [MG-Horizon].

10. Beyond Hawking: What the McGucken Framework Enables

With Bekenstein 1973 and Hawking 1975 both derivable from the McGucken Principle, several further directions become accessible as theorems:

The Bekenstein bound. For an arbitrary system of size R containing energy E, Bekenstein's 1981 bound $S \leq 2\pi k_B RE/(\hbar c)$ is the statement that the maximum x_4-stationary-mode count in a region of radius R with total x_4-coupling energy E is bounded by the McGucken-Sphere area through which the modes must propagate.

The holographic principle and AdS/CFT. Developed in §IX and in [MG-AdSCFT].

Page curves and replica wormholes. The island formula of [Pen, AEMM] identifies the entanglement wedge of the radiation as including the interior of the black hole after the Page time. In the McGucken framework this is the statement that the radiation's x_4-stationary modes share six-sense locality with the interior modes they were emitted from.

The four laws of black-hole mechanics. The Bardeen–Carter–Hawking laws [BCH73] follow from the horizon's role as an x_4-stationary hypersurface with McGucken-Sphere-analogue structure.

11. Conclusion

11.1. In plain terms

Stephen Hawking's 1975 paper turned Bekenstein's conjecture into physics. Applying QFT in a curved Schwarzschild background, Hawking showed that black holes must emit thermal radiation at $T_H = \hbar\kappa/(2\pi c k_B)$, that the entropy coefficient is $\eta = 1/4$, that black holes therefore evaporate over time $\tau \propto M^3$, and that a refined GSL is preserved. Every subsequent development in black-hole thermodynamics — the information paradox, the firewall puzzle, AdS/CFT black-hole thermodynamics, the replica-wormhole resolution of the Page curve — descends from Hawking's 1975 paper.

Each one of Hawking's five central results follows as a theorem of the McGucken Principle $dx_4/dt = ic$, with the McGucken Wick rotation as the single central tool. The horizon's x_4-stationary modes, thermalized by the Euclidean cigar geometry, produce the thermal radiation (Proposition III.1). The angular period of the cigar is the Hawking temperature (Proposition IV.1). The Euclidean Gibbons–Hawking–York action fixes the entropy coefficient at 1/4 (Proposition V.1). Stefan–Boltzmann emission from the horizon's hot surface gives the evaporation rate (Proposition VI.1). The global McGucken second law applied to the evolving partition gives the refined Generalized Second Law (Proposition VII.1).

11.2. The full programme of black-hole thermodynamics, derived

Combining the present paper with [MG-Bekenstein], the ten central results of foundational black-hole thermodynamics — five from Bekenstein 1973 and five from Hawking 1975 — are now theorems of a single geometric postulate. This is the founding programme of the field, complete in its essentials, derived from $dx_4/dt = ic$. No additional postulates are required beyond the McGucken Principle itself.

The two papers treat the classical and quantum halves respectively. Bekenstein's results follow from the horizon's role as an x_4-stationary hypersurface with Planck-scale mode counting. Hawking's results invoke the McGucken Wick rotation to obtain the Euclidean cigar geometry. The classical-to-quantum transition is the transition from Lorentzian to Euclidean geometry, which in the McGucken framework is the physical transformation of removing the i from x_4.

11.3. The far-reaching unifying power of the McGucken Principle

The convergence between Hawking's 1975 programme and the McGucken Principle extends that already established for Bekenstein 1973. The same single postulate $dx_4/dt = ic$ has been shown to underlie Huygens' Principle, the Principle of Least Action, Noether's theorem, and the Schrödinger equation [MG-HLA]; the Born rule [MG-Born]; the canonical commutation relation $[q, p] = i\hbar$ [MG-Commut]; Feynman's path integral [MG-PathInt]; the Dirac equation and spin-$\frac{1}{2}$ [MG-Dirac]; second quantization [MG-SecondQ]; QED and the U(1) gauge structure [MG-QED]; the CKM matrix and the three-generation requirement [MG-Cabibbo, MG-CKM]; the Standard Model Lagrangians and general relativity [MG-SM]; the Wick rotation [MG-Wick]; the holographic principle and AdS/CFT [MG-AdSCFT]; cosmological holography in FRW/de Sitter [MG-CosHolo]; the second law of thermodynamics and the arrow of time [MG-Mech, MG-HLA]; quantum nonlocality and Bell correlations [MG-Nonloc, MG-Equiv]; dark matter [MG-DarkMatter]; the horizon/flatness/homogeneity problems without inflation [MG-Horizon]; the cosmological constant [MG-Lambda]; baryogenesis [MG-Sakharov]; the values of c and $\hbar$ [MG-Constants]; the open problems of Witten's twistor programme [MG-Witten]; Bekenstein's five 1973 results [MG-Bekenstein]; and now Hawking's five 1975 results. The full catalog continues to grow at `elliotmcguckenphysics.com`.

That a single geometric postulate reaches from the Born rule to cosmological holography, from the Dirac equation to dark matter, from the Wick rotation to baryogenesis, and now from Hawking's thermal spectrum to the evaporation rate and the refined Generalized Second Law — this is not overreach. It is the consequence of the McGucken Principle being a foundational statement about the ontology of space and time themselves. All of physics takes place upon the stage of space and time. If the correct foundational statement about that stage has been found, then every branch of physics — quantum, relativistic, thermodynamic, cosmological, particle-physics, and black-hole thermodynamic — is already standing on it. The unifications are not separate achievements to be engineered one by one; they are what a single correct view of spacetime automatically delivers. The fourth dimension is expanding at the velocity of light. Hawking's five results are five more facets of what that one geometric fact requires. That they too fall into place should come as no surprise; it is the expected consequence of a correct foundation.

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