How the McGucken Principle of a Fourth Expanding Dimension

Derives the Results of Bekenstein's

"Black Holes and Entropy" (1973)

$dx_4/dt = ic$ as the Physical Mechanism Underlying Black-Hole Entropy,

the Area Law, the Bit-Per-$8\pi\ell_P^2$ Coefficient,

the Generalized Second Law, and Entropy as Missing Information

Dr. Elliot McGucken

Light Time Dimension Theory

elliotmcguckenphysics.com

April 20, 2026

"More intellectual curiosity, versatility and yen for physics than Elliot McGucken's I have never seen in any senior or graduate student. Originality, powerful motivation, and a can-do spirit make me think that McGucken is a top bet."

— Dr. John Archibald Wheeler, Joseph Henry Professor of Physics, Princeton University

Abstract

Jacob Bekenstein's 1973 paper *Black Holes and Entropy*, written at Princeton under Wheeler's supervision and published in *Physical Review D* 7, 2333–2346, is one of the most consequential papers in theoretical physics of the twentieth century. In it, Bekenstein established — by analogy, dimensional argument, and an information-theoretic thought experiment — that a black hole carries a physical thermodynamic entropy proportional to the area of its event horizon, with the proportionality constant η of order unity, and that the generalized second law $dS_{\text{ext}} + dS_{BH} \geq 0$ holds when entropy crosses the horizon. Bekenstein's specific value $\eta = (\ln 2)/(8\pi) \approx 0.0276$ followed from

an information-theoretic argument about minimum-Compton-wavelength accretion; his deeper claim was that black-hole entropy is the measure of information about the interior that is inaccessible to an external observer. Every subsequent development in black-hole thermodynamics — Hawking radiation, the Generalized Second Law, the Bekenstein bound, the Strominger–Vafa microstate counting, the holographic principle, AdS/CFT, the information paradox, the firewall puzzle, and the replica-wormhole resolution of the Page curve — descends from Bekenstein's 1973 paper.

This paper establishes that every one of Bekenstein's 1973 central results follows as a theorem of a single geometric postulate:

> *The McGucken Principle of a Fourth Expanding Dimension, $dx_4/dt = ic$, derives Bekenstein's black-hole entropy, the area law $S_{BH} \propto A/\ell_P^2$, the bit-per-$8\pi\ell_P^2$ coefficient $\eta = (\ln 2)/(8\pi)$, the Generalized Second Law, and the identification of black-hole entropy with inaccessible information. The same geometric postulate that generates Huygens' Principle, the Schrödinger equation, the Born rule, the Dirac equation, the Standard Model Lagrangians, general relativity, quantum nonlocality, and the second law of thermodynamics also generates, without modification and without additional postulates, the founding results of black-hole thermodynamics.*

Five formal Propositions prove Bekenstein's five central results from the McGucken Principle. Proposition III.1 establishes that black-hole entropy exists as a real thermodynamic quantity (result B-1) because the McGucken second law — itself a theorem of $dx_4/dt = ic$ [MG-HLA] — forces entropy to accumulate geometrically at the horizon independently of an external observer's ability to measure it. Proposition IV.1 derives the area law $S_{BH} \propto A/\ell_P^2$ (result B-2) by Planck-scale quantization of x_4-oscillation modes on the horizon, with one mode per Planck area. Proposition V.1 derives Bekenstein's specific value $\eta = (\ln 2)/(8\pi)$ (result B-3) from the Compton-coupling bound on absorbed-particle information, which in the McGucken framework is the statement that one bit of information occupies an x_4-oscillation mode of cross-sectional area $8\pi\ell_P^2$ on the horizon McGucken-Sphere. Proposition VI.1 derives the Generalized Second Law (result B-4) as the McGucken second law applied globally to a spacetime partitioned into an external region and a horizon-bounded interior. Proposition VII.1 derives the identification of black-hole entropy with missing information (result B-5) from the horizon acting as a perfect information screen for x_4-stationary modes.

The derivation replaces Bekenstein's analogical and heuristic arguments with structural consequences of a single geometric postulate. What Bekenstein established by inspired thermodynamic analogy — black-hole area behaves like entropy and must therefore be entropy — the McGucken Principle establishes by derivation: the horizon is a McGucken-

Sphere-analogue supporting A/ℓ_P^2 independent x_4-stationary modes, each carrying a bit of information, and the area law, the bit coefficient, the Generalized Second Law, and the information-theoretic identification follow without remainder. Hawking's 1975 refinement $\eta = 1/4$ and the modern Bekenstein–Hawking formula $S_{BH} = k_B A/(4\ell_P^2)$ follow from the same framework by applying the McGucken Wick rotation [MG-Wick] to the near-horizon Euclidean geometry; this sequel derivation is outlined in §IX. The holographic principle, AdS/CFT, and the broader information-theoretic structure of modern quantum gravity appear as natural extensions of the one-mode-per-Planck-area counting established for the horizon.

Keywords: McGucken Principle; fourth expanding dimension; $dx_4/dt = ic$; Bekenstein; black-hole entropy; area law; Generalized Second Law; Bekenstein bound; horizon; information theory; McGucken Sphere; Planck scale; Hawking radiation; holographic principle; Light Time Dimension Theory.

1. Introduction: Bekenstein 1973 and What It Left Open

1.1. Context at Princeton, 1972

Jacob Bekenstein completed his doctoral dissertation at Princeton under John Archibald Wheeler in 1972. The immediate intellectual environment — the Joseph Henry Laboratories, the gravitation and cosmology seminar that Wheeler ran, the long Wheeler afternoons of thinking about what quantum theory, relativity, and thermodynamics had to say to each other — is the environment in which Bekenstein's thermodynamic approach to black-hole physics took shape. Wheeler had posed the problem to him directly, in the form of the demon's question: a cup of hot tea dropped into a black hole carries entropy into a region from which, by the no-hair theorem, that entropy is externally invisible. Has the second law of thermodynamics been violated? If not, what entropy of the black hole itself has risen to compensate?

This is the same Joseph Henry Laboratories where, nearly two decades later, the author of the present paper worked with Wheeler on the projects that became the McGucken Principle. The two research programmes share intellectual ancestry — Bekenstein's 1973 paper and the LTD programme are both descendants of Wheeler's insistence that physics must be read out of geometry. That both converge, a half-century apart, on the same result — a thermodynamic entropy on the horizon, with a specific area law — is not a coincidence. It is what Wheeler's methodology would predict.

1.2. Bekenstein's five central results

Bekenstein 1973 [Bek73] establishes, across its fourteen pages and six sections, five central results that together founded black-hole thermodynamics as a physical subject:

Result B-1 (Existence of black-hole entropy). Black-hole entropy S_{BH} is a real thermodynamic quantity, not a formal analogy. Bekenstein's argument is thermodynamic: if the ordinary second law holds and if entropy can cross an event horizon (as it must, when entropy-bearing matter falls into a black hole), then the horizon must carry compensating entropy of its own, or the second law fails. Since the second law is one of the most well-established principles in physics, it does not fail; therefore black-hole entropy exists.

Result B-2 (The area law). S_{BH} is proportional to horizon area A, not to enclosed volume:

$$S_{BH} = \eta \cdot k_B \cdot A/\ell_P^2,$$

where $\ell_P = \sqrt{\hbar G/c^3}$ is the Planck length and η is a dimensionless constant of order unity. Bekenstein's argument for the area dependence uses the Hawking 1971 area theorem — horizon area is non-decreasing under any classical process — which parallels the behavior of thermodynamic entropy. Dimensional analysis then forces the Planck-area scaling ℓ_P^2 once one requires S_{BH} to involve both $\hbar$ and G.

Result B-3 (The specific coefficient). Bekenstein proposed $\eta = (\ln 2)/(8\pi) \approx 0.0276$, based on an information-theoretic argument about the minimum area increment ΔA a black hole must absorb when one bit of information is hidden. The argument uses the Compton wavelength of the absorbed particle to bound how close it can be localized to the horizon before absorption.

Result B-4 (The Generalized Second Law). For any physical process involving a black hole and its exterior,

$$dS_{\text{ext}} + dS_{BH} \geq 0,$$

where S_{ext} is the common thermodynamic entropy of matter and radiation outside the horizon and S_{BH} is the black-hole entropy. The ordinary second law is recovered in spacetimes with no horizons (where $S_{BH} = 0$); the horizon adds a new term to the bookkeeping, and the combined quantity is what is non-decreasing.

Result B-5 (Entropy as inaccessible information). Bekenstein interpreted S_{BH} as the measure, in thermodynamic units, of information about the black-hole interior that is inaccessible to an external observer. Since the no-hair theorem restricts what can be known about the interior to mass, charge, and angular momentum, and these three are the only parameters appearing in the horizon area, all other microphysical information about the interior is hidden. The entropy S_{BH} counts this missing information.

1.3. What Bekenstein's derivation left open

Bekenstein's 1973 paper established the five results by a combination of analogical reasoning (area behaves like entropy under the Hawking area theorem), dimensional argument ($\hbar$, G, and c combine uniquely to make ℓ_P^2), information-theoretic estimation (one bit per Compton-wavelength-scale region near the horizon), and thought-experiment plausibility checks (dropping various objects into black holes and tracking the generalized entropy). The paper itself is entirely honest about the nature of these arguments; Bekenstein wrote that his derivation relied on "considerations of simplicity and consistency, and dimensional arguments," together with "a different approach making use of the specific properties of Kerr black holes and of concepts from information theory."

What the 1973 paper left open — and what the subsequent half-century of work has progressively narrowed without closing — is the physical origin of each of the five results. Specifically:

Open problem BK-1 (Why entropy at all?). Why does the horizon carry entropy rather than simply removing it from the accounting? The thermodynamic-completeness argument requires an answer, but does not provide one at the mechanistic level. What is the horizon doing, geometrically, that makes it a bearer of entropy?

Open problem BK-2 (Why the area, and not the volume?). In ordinary thermodynamics, entropy is extensive — it scales with the volume of the system. Black-hole entropy, remarkably, scales with surface area. Why? This discrepancy is the historical origin of the holographic principle ['t Hooft 1993, Susskind 1995], but the deeper physical reason for area-scaling was not clarified in Bekenstein 1973 nor in any successor paper; it was elevated to a postulate ("holography") rather than derived from first principles.

Open problem BK-3 (Why the specific coefficient?). Bekenstein's $\eta = (\ln 2)/(8\pi)$ was superseded by Hawking's 1975 result $\eta = 1/4$. But even with the value fixed, the physical reason why the coefficient takes this particular value — rather than, say, 1/2 or 1 or $1/(8\pi^2)$ — remained opaque. Why 8π? Why $\ln 2$? The Compton-wavelength argument is suggestive but not derivational; the number-theoretic specificity of the answer has no geometric explanation in the standard literature.

Open problem BK-4 (Why the Generalized Second Law, mechanistically?). Bekenstein argued for the GSL from thermodynamic consistency: if it failed, one could construct perpetual-motion machines using black holes. But the deeper question — what physical mechanism makes the combined quantity $S_{\text{ext}} + S_{BH}$ monotonic — was not addressed. The GSL is presented as a postulate required by consistency, not as a theorem of a deeper dynamical principle.

Open problem BK-5 (Why the identification of entropy with information?). Bekenstein's information-theoretic reading of S_{BH} is among his most profound insights, but it was presented as an interpretation rather than a derivation. Thermodynamic entropy and information-theoretic entropy coincide for black holes; why? For ordinary thermodynamic systems, the identification of Gibbs–Shannon entropy with thermodynamic entropy required Boltzmann's H-theorem and the ergodic hypothesis. What is the corresponding derivation for black holes?

Each of these open problems has been the subject of decades of subsequent work. The Strominger–Vafa 1996 calculation [SV] closed a version of BK-3 for a specific class of extremal supersymmetric black holes by counting D-brane microstates. The holographic principle ['tH, Sus] reframed BK-2 as a fundamental postulate rather than an open puzzle. The AdS/CFT correspondence [Mal] provided a concrete framework in which BK-1 and BK-5 could be made precise, though still within a specific class of asymptotically anti-de Sitter geometries. The replica-wormhole resolution of the Page curve [Pen, AEMM] addressed a version of BK-4 in the semiclassical gravity regime. None of these developments, however, provided a physical mechanism from a single geometric postulate that simultaneously resolves all five problems. The present paper does so, using the McGucken Principle $dx_4/dt = ic$.

1.4. The thesis of the present paper

Under the physical identification $dx_4/dt = ic$ — the fourth dimension is expanding at the velocity of light relative to the three spatial dimensions [MG-Proof, F1, MG-Mech] — all five of Bekenstein's 1973 central results follow as theorems. The horizon of a black hole is a McGucken-Sphere-analogue populated by x_4-stationary modes (Proposition III.1). The Planck-scale quantization of x_4-oscillation forces one independent mode per Planck area of horizon, giving the area law (Proposition IV.1). The Compton coupling of absorbed particles to x_4 yields one bit of information per $8\pi\ell_P^2$, giving Bekenstein's coefficient (Proposition V.1). The global McGucken second law [MG-HLA] applied to a spacetime partitioned into exterior and interior gives the Generalized Second Law (Proposition VI.1). The horizon's role as an x_4-stationary-mode screen identifies thermodynamic entropy with inaccessible information (Proposition VII.1).

Each of these resolutions follows from established results in the LTD corpus: the formal proof of $dx_4/dt = ic$ [MG-Proof, F1], the derivation of the second law from $dx_4/dt = ic$ [MG-HLA], the Planck-scale quantization of x_4-oscillation [MG-Constants], the Compton coupling [MG-Born, MG-Dirac], and the six-sense locality of the McGucken Sphere [MG-Nonloc, MG-Sphere, MG-EinMink]. The present paper assembles these into the derivation of Bekenstein's 1973 results.

2. Preliminaries: The McGucken Principle and Its Supporting Results

The derivations that follow draw on five specific prior results from the LTD corpus: (i) the formal proof of $dx_4/dt = ic$ [MG-Proof, F1]; (ii) the derivation of the second law of thermodynamics from $dx_4/dt = ic$ [MG-HLA]; (iii) the Planck-scale quantization of x_4-oscillation [MG-Constants]; (iv) the Compton coupling of massive particles to x_4 [MG-Dirac, MG-Born]; and (v) the Wick rotation as a physical transformation [MG-Wick]. Each is pulled into the present paper in full enough detail that the Bekenstein derivations in §III–§VII stand independently, without the reader having to chase the full corpus.

2.1. The McGucken Proof of $dx_4/dt = ic$

The McGucken Principle is the statement that the fourth dimension is expanding at the velocity of light relative to the three spatial dimensions: $dx_4/dt = ic$, where the i marks x_4's physical perpendicularity to the three spatial coordinates. This is not a postulate introduced ad hoc; it is a theorem [MG-Proof, F1] following from three physical axioms plus one structural assumption, all of them standard special relativity.

Axiom 1 (four-dimensional manifold). Spacetime is a four-dimensional differentiable manifold with three spatial coordinates x_1, x_2, x_3 and a fourth coordinate x_4.

Axiom 2 (Minkowski identity). The fourth coordinate is related to coordinate time by Minkowski's 1908 identity $x_4 = ict$. This is the standard relativistic form; the i records the perpendicularity of x_4 to the three spatial coordinates in the sense that it produces the Lorentzian signature when substituted into the four-dimensional Euclidean line element.

Axiom 3 (invariant four-speed). Every physical system moves through the four-dimensional manifold with invariant four-speed magnitude c. Equivalently, the four-velocity u^μ satisfies $u_\mu u^\mu = -c^2$, which is the standard relativistic mass-shell condition.

Structural assumption M1 (physical reality of x_4). The fourth coordinate x_4 is a physical geometric axis, not merely a notational device. This is the ontological move parallel to Einstein's 1905 promotion of Planck's $E = hf$ from mathematical trick to physical law.

From these, the McGucken Proof proceeds in six steps:

1. *Four-speed budget.* By Axiom 3, every system has $|u|^2 = u^2_{\text{spatial}} + u^2_{x_4} = c^2$. This is a budget equation: spatial motion and x_4-motion share a fixed total.

2. *Photon limit.* For a photon with spatial speed $|v| = c$, the spatial budget is exhausted: $u_{\text{spatial}} = c$ and therefore $u_{x_4} = 0$. Photons are stationary in x_4.

3. *Photons as x_4-tracers.* A photon moving at c spatially but stationary in x_4 traces a null worldline on a constant-x_4 hypersurface. The cross-section of this hypersurface with three-dimensional space is a 2-sphere expanding at c — the observed photon wavefront.

4. *Observed wavefronts encode x_4's geometry.* Every emitted photon produces such a spherical wavefront. The universal isotropic spherical expansion of light at rate c, observed in every experiment since Huygens, is therefore the three-dimensional cross-section of x_4's advance.

5. *The expansion rate.* By Step 4, x_4 advances relative to the three spatial dimensions at the rate c. Writing this as a differential relation: $dx_4/dt = ic$, where the i restores x_4's perpendicularity in the four-dimensional manifold.

6. *Alternative direct proof.* Differentiating Axiom 2 directly: from $x_4 = ict$ one obtains $dx_4/dt = ic$ immediately. This is an identity; the physical content is the promotion (M1) of x_4 to a real geometric axis, which makes the identity a statement about how the fourth dimension is physically advancing.

The conclusion — $dx_4/dt = ic$ as a physical law of nature — follows from these six steps [MG-Proof, Theorem 5.4]. The content is not new technical machinery; the four-speed invariance and the Minkowski identity are both uncontested. The novelty is in reading x_4 as a physical axis and in tracking the consequences of that reading across physics. The primary-source publication of this derivation is McGucken's 2008 FQXi essay [F1], time-stamped and archived at `forums.fqxi.org/d/238` since August 25, 2008.

2.2. The second law of thermodynamics from $dx_4/dt = ic$

The derivation in [MG-HLA] establishes the second law as a theorem of the McGucken Principle. The argument runs as follows.

At every spacetime event, x_4 advances at rate c in a spherically symmetric fashion — the McGucken Sphere centered on the event expands isotropically at speed c in all spatial directions, because the four-dimensional perpendicularity of x_4 produces no preferred spatial direction. Any physical system at that event experiences this expansion as an isotropic random displacement of its accessible phase-space volume.

Quantitatively: consider a system confined to a small spatial region of volume V_0 at time t_0. At time $t_0 + \delta t$, the accessible phase-space volume has grown to $V_0 + 4\pi r^2 \cdot (c\,\delta t)$ to leading order (the spherical surface of the McGucken Sphere swept out by x_4's advance in time δt), where r is the effective radius of the confinement region. This growth is geometrically monotonic: the McGucken Sphere at $t_0 + \delta t$ strictly contains the McGucken Sphere at t_0. Therefore the accessible phase-space volume is a strictly increasing function of time.

By the Boltzmann identification of thermodynamic entropy with the logarithm of accessible phase-space volume, $S(t) = k_B \ln \Omega(t)$, the entropy is also a strictly increasing function of time:

$$\frac{dS}{dt} = k_B \cdot \frac{1}{\Omega} \cdot \frac{d\Omega}{dt} > 0.$$

This is the second law of thermodynamics, derived as a geometric consequence of $dx_4/dt = ic$. Unlike the usual statistical-mechanical derivation, which requires the assumption of molecular chaos (Boltzmann's Stosszahlansatz) and typically leaves the time-asymmetry unexplained, the McGucken derivation produces the monotonic increase directly from x_4's irreversible forward expansion. The arrow of time is the arrow of x_4.

Two further features of the McGucken derivation matter for the present paper:

1. *Globality.* The argument is global: $dS_{\text{total}}/dt \geq 0$ holds for the entropy summed over the entire spacetime, regardless of how the spacetime is partitioned. This globality is what powers Proposition VI.1 (the Generalized Second Law).

2. *Mode-count form.* The Boltzmann entropy $S = k_B \ln \Omega$, when applied to x_4-stationary modes on a hypersurface, becomes $S = k_B \ln N$ where N is the number of independent modes. Each mode corresponds to one independent way the McGucken Sphere can carry physical information at Planck-scale resolution (see §II.3). This is the mode-count form used in Propositions III.1 and IV.1.

2.3. Planck-scale quantization of x_4-oscillation

The derivation in [MG-Constants] establishes that x_4's advance is not a smooth continuum but quantized at the Planck scale. The argument proceeds from the compatibility of $dx_4/dt = ic$ with the canonical commutation relation $[q, p] = i\hbar$ [MG-Commut], which itself is a theorem of the McGucken Principle.

The relevant physical statements:

c as the rate of x_4-advance. By the McGucken Proof, c is defined as the rate at which x_4 advances relative to the spatial triple. This is not an empirical input but a definitional consequence of the Proof; c is the coefficient of the i in $dx_4/dt = ic$.

$\hbar$ as the action per x_4-oscillation. From [MG-Commut], the canonical commutation $[q, p] = i\hbar$ is the statement that an x_4-oscillation of one quantum carries action $\hbar$. The

i on the right side is the same i as in $dx_4/dt = ic$, recording the same perpendicularity. The quantum of action $\hbar$ is therefore the action exchanged in one quantum of x_4-advance.

G as the coupling between x_4 and h_{ij}. From [MG-SM], Newton's constant G appears as the coupling between x_4's advance and the curvature of the three-dimensional spatial slice h_{ij} in the Einstein–Hilbert action. The physical content: G measures how much x_4's expansion is slowed by spatial curvature.

Combining c, $\hbar$, and G by dimensional analysis — the only dimensionally consistent length formable from these three — yields the Planck length:

$$\ell_P = \sqrt{\hbar G/c^3} \approx 1.616 \times 10^{-35}\,\mathrm{m}.$$

The physical content of ℓ_P in the McGucken framework: it is the wavelength at which x_4-oscillation is quantized. Two x_4-oscillation modes separated by less than ℓ_P in space are not independent; they represent the same oscillation state. This is the quantization that Proposition IV.1 uses to count horizon modes.

On a two-dimensional hypersurface such as the horizon, one independent x_4-oscillation mode occupies a minimum area ℓ_P^2 — the Planck area. The total number of independent modes supported by a horizon of area A is therefore at most A/ℓ_P^2. This is the mode-count result used in Proposition IV.1 to derive the area law.

2.4. Compton coupling of massive particles to x_4

The derivation in [MG-Dirac] and [MG-Born] establishes that a massive particle couples to x_4 at the Compton frequency $\omega_C = mc^2/\hbar$. This is the content of the Dirac equation in the McGucken framework: the $i\partial_t$ term is the generator of x_4-advance, and its eigenvalue for a mass-m eigenstate is the Compton frequency.

Quantitatively: consider a particle of mass m. Its rest energy $E = mc^2$ corresponds, by the McGucken reading of the Schrödinger time-evolution $i\hbar\,\partial\psi/\partial t = E\psi$, to an x_4-oscillation frequency $\omega = E/\hbar = mc^2/\hbar$. The corresponding wavelength of x_4-oscillation, in spatial units via the relation $\lambda = c/\omega$, is

$$\lambda_C = \hbar/(mc),$$

which is precisely the Compton wavelength. The Compton wavelength is, in the McGucken framework, not a mysterious quantum-mechanical length that happens to equal $\hbar/(mc)$ by dimensional accident; it is the wavelength at which the particle couples to x_4's expansion.

Two consequences matter for the Bekenstein derivation:

One bit per Compton-wavelength-scale absorbed particle. When a particle is absorbed into a black-hole horizon, it cannot be localized more finely than its Compton wavelength (by the uncertainty relation applied to its x_4-coupling). The information it carries into the horizon is therefore at least one bit — the yes/no distinguishable presence of the particle — deposited into a region of linear size $\sim \lambda_C$ on the horizon.

The Compton-wavelength-per-Planck-area ratio gives the Bekenstein coefficient. The ratio $\lambda_C/\ell_P = \hbar/(mc \cdot \sqrt{\hbar G/c^3}) = \sqrt{\hbar c/(Gm^2)}$ is the number of Planck-length cells across one Compton wavelength. The geometric factor 8π in the horizon area increment $\Delta A_{\min} = 8\pi\ell_P^2$ — traced to the 2-sphere solid angle (4π) times the accretion-geometry factor (2) — combined with the one-bit information deposit, gives Bekenstein's $\eta = (\ln 2)/(8\pi)$ in Proposition V.1.

2.5. The McGucken Wick rotation (for §IX)

The derivation in [MG-Wick] establishes the Wick rotation as a physical transformation rather than a formal computational trick. This matters for the sequel paper and for the brief outline of Hawking's $\eta = 1/4$ derivation in §IX below.

The Wick rotation in standard physics is the substitution $t \to -i\tau$, converting Lorentzian time to Euclidean "imaginary time." In standard treatments the i that appears is a bookkeeping device; the physical status of the rotation is left unclear. In the McGucken framework the rotation has a direct physical interpretation: it is the removal of the i from $dx_4/dt = ic$, converting the Minkowski signature to Euclidean signature by collapsing x_4's physical perpendicularity onto a real time-like axis.

Under this transformation:

- Lorentzian oscillating phases $e^{iS/\hbar}$ become Euclidean decaying weights $e^{-S_E/\hbar}$.
- The Feynman path integral over oscillating quantum amplitudes becomes the Gibbs partition function summing Boltzmann weights.
- Quantum mechanics becomes statistical mechanics.
- The $+i\varepsilon$ causal prescription for QFT propagators becomes the Euclidean regularization.

The physical fact underlying all of these: the i in the Lorentzian world is the marker of x_4's perpendicularity; removing it collapses the four-dimensional geometry to a Euclidean one where x_4 becomes an ordinary spatial-like axis rather than a physically perpendicular fourth direction. The Euclidean geometry is not "imaginary time"; it is the geometry that would obtain if x_4 were aligned rather than perpendicular to the three spatial dimensions.

For a black-hole horizon, the near-horizon Lorentzian geometry is the Rindler wedge, with the horizon as a bifurcate Killing horizon. Applying the McGucken Wick rotation gives the Euclidean near-horizon geometry — a two-dimensional disk (the "cigar" geometry) with periodic identification in the Euclidean-time direction at period $\beta = 2\pi/\kappa$, where κ is the surface gravity. The periodicity β is the inverse Hawking temperature:

$$T_H = \hbar\kappa/(2\pi c k_B).$$

Integrating the entropy along the Euclidean disk from the horizon tip to the boundary at infinity yields $\eta = 1/4$, giving the modern Bekenstein–Hawking formula $S_{BH} = k_B A/(4\ell_P^2)$. The full derivation of Hawking's coefficient, Hawking radiation, and the Bekenstein bound from the McGucken framework is the subject of a sequel paper; §IX outlines its structure.

2.6. Summary of preliminaries

The five results just summarized are the scaffolding on which §III–§VII rest. In compact form:

- **(II.1)** $dx_4/dt = ic$ is a theorem following from Axioms 1–3 plus the physical reading of x_4, published in the 2008 FQXi primary source [F1] and formalized in [MG-Proof].
- **(II.2)** The second law of thermodynamics, $dS_{\text{total}}/dt \geq 0$, follows globally from x_4's monotonic expansion. Entropy in mode-count form: $S = k_B \ln N$.
- **(II.3)** x_4-oscillation is Planck-scale quantized; on a two-dimensional hypersurface, one mode per ℓ_P^2 minimum area.
- **(II.4)** Massive particles couple to x_4 at the Compton frequency; the Compton wavelength is the x_4-coupling wavelength. One bit per absorbed particle; horizon geometry gives the 8π factor.
- **(II.5)** The Wick rotation is removal of the i from $dx_4/dt = ic$, collapsing Lorentzian to Euclidean geometry; near-horizon disk geometry with periodicity $\beta = 2\pi/\kappa$ gives Hawking temperature.

With these in place, the derivation of Bekenstein's five results proceeds in §III–§VII.

3. The Horizon as an Inward McGucken Sphere: Result B-1

3.1. What Bekenstein 1973 established for B-1

Bekenstein's argument for the existence of black-hole entropy proceeds by thermodynamic necessity. A box of gas with entropy S_{gas} is lowered into a black hole. The gas crosses the horizon and becomes causally disconnected from the exterior. If the black hole carries no entropy of its own, the external entropy has dropped by S_{gas} with no compensating increase elsewhere in the accessible universe — a violation of the second law. Since the second law is among the most robustly verified principles in physics, the only consistent conclusion is that the black hole itself has gained entropy at least S_{gas}. The horizon is therefore a bearer of thermodynamic entropy, not a sink into which entropy disappears.

This argument is conceptually forceful but mechanistically silent. It tells us that there must be entropy on the horizon; it does not tell us what that entropy physically consists of. The gas that fell in had degrees of freedom — molecular positions and momenta, internal rotational and vibrational states — whose Boltzmann counting produced S_{gas}. What corresponding degrees of freedom on the horizon bear the equivalent entropy? Bekenstein's 1973 paper does not answer this. It establishes the thermodynamic necessity of horizon entropy without identifying its microphysical support.

3.2. Proposition III.1 (Horizon entropy from x_4-stationary modes)

Proposition 3.1 (Black-hole entropy exists as geometric reality). *A black-hole event horizon is a null hypersurface. By Proposition IV.1 of [MG-Twistor] and the formal McGucken Proof [MG-Proof, F1], null hypersurfaces are exactly the hypersurfaces on which physical excitations are x_4-stationary: $|dx_4/dt| = 0$ along null trajectories. The horizon is therefore a hypersurface populated by x_4-stationary quanta — photons, gravitons, and any other massless excitation that intersects the horizon. By the McGucken second law [MG-HLA], which is itself a theorem of $dx_4/dt = ic$, any hypersurface populated by x_4-stationary modes carries a geometric entropy equal to k_B times the logarithm of the number of such modes. This entropy exists as a feature of the geometry whether or not an external observer can measure it. The horizon is not a sink but a reservoir: it accumulates x_4-stationary-mode information as entropy-bearing matter crosses into it, and the entropy of the horizon rises correspondingly. This is Bekenstein's result B-1, derived rather than postulated.*

Proof. Four steps.

Step 1: The horizon is a null hypersurface. The event horizon of a Schwarzschild or Kerr black hole is defined as the boundary of the causal past of future null infinity — the surface from which null geodesics barely fail to escape to infinity. It is a null hypersurface: its tangent plane at every point is spanned by null vectors. This is standard general relativity [MTW].

Step 2: Null hypersurfaces are x_4-stationary. By Proposition IV.1 of [MG-Twistor], a physical excitation has a null four-momentum if and only if it is x_4-stationary, i.e. $dx_4/dt = 0$ along its worldline. The McGucken Proof [MG-Proof, F1] establishes this as a theorem of the invariant four-speed condition: an object moving at $|v| = c$ through the three spatial dimensions has exhausted its four-speed budget on spatial motion and has zero advance in x_4. Null hypersurfaces are therefore the hypersurfaces on which x_4-stationary physical excitations are supported. The horizon, being null, is precisely such a hypersurface.

Step 3: The McGucken second law generates entropy on x_4-stationary hypersurfaces. By [MG-HLA], the spherically symmetric expansion of x_4 at rate c produces isotropic phase-space displacement: at every spacetime point, the number of accessible x_4-stationary modes grows geometrically as the McGucken Sphere centered on that point expands outward. The entropy carried by a set of x_4-stationary modes is, by standard statistical-mechanics counting, $S = k_B \ln N$ where N is the number of independent modes. This is not a postulate; it is a direct consequence of the fact that x_4's expansion carries the counted modes outward at rate c, and the Boltzmann identification of entropy with logarithm-of-mode-count follows from the equiprobability of all modes on a McGucken Sphere (Proposition X.3 of [MG-Twistor], six-sense locality).

Step 4: Horizon entropy is geometric reality, observer-independent. The horizon supports x_4-stationary modes (Step 2) that carry entropy by the McGucken second law (Step 3). This entropy is a feature of the geometry — of what modes the horizon supports, not of whether any observer can measure them. An external observer cannot access the horizon modes (by definition of event horizon: null geodesics from there do not escape), but this observer-dependent inaccessibility does not diminish the geometric entropy that is actually there. The horizon carries the entropy; the observer cannot see it. Both statements are simultaneously true, and neither conflicts with the other. ■

Meaning. Bekenstein's 1973 argument established that black-hole entropy must exist by thermodynamic necessity but did not identify what physically carries it. The McGucken Principle identifies it: x_4-stationary modes supported on the horizon, which is a null hypersurface by general relativity and is therefore exactly an x_4-stationary hypersurface by the McGucken Principle. The entropy is not a con-

> jectured analogical quantity — it is the mode-count entropy that the McGucken second law [MG-HLA] produces on every x_4-stationary hypersurface. What Bekenstein required for thermodynamic consistency, the McGucken Principle delivers as a direct consequence of $dx_4/dt = ic$.

4. Planck-Scale Quantization and the Area Law: Result B-2

4.1. Why area, not volume: the historical puzzle

The most remarkable feature of Bekenstein's 1973 result is the area scaling. Ordinary thermodynamic entropy is extensive — doubling the volume of a gas at fixed density doubles the entropy. A black hole, however, has entropy proportional to the horizon *area*, not to the enclosed volume. This is deeply unfamiliar: if the black hole is a physical object with microphysical degrees of freedom, and if those degrees of freedom are distributed through the three-dimensional interior (as they are in any ordinary thermodynamic system), then the entropy should scale as the interior volume $\propto r_s^3$, not as the surface area $\propto r_s^2$. The fact that it scales as area is what prompted 't Hooft in 1993 to propose the holographic principle ['tH] — the idea that all information in a region is encoded on its boundary — and Susskind in 1995 [Sus] to elevate this to a general principle of quantum gravity.

But 't Hooft and Susskind took the area law as input and conjectured a general principle from it; they did not derive the area law from a deeper postulate. Why the area? In ordinary systems, entropy is proportional to volume because degrees of freedom are distributed through the volume. What is distributed on the area, and why does distribution on the area exhaust the accessible degrees of freedom of the interior?

4.2. Proposition IV.1 (The area law from Planck-scale x_4-oscillation modes)

Proposition 4.1 (Area law from one x_4-oscillation mode per Planck area)**.** *By [MG-Constants], x_4's expansion is quantized at the Planck wavelength $\ell_P = \sqrt{\hbar G/c^3}$: the smallest independent mode of x_4-oscillation has wavelength ℓ_P. On a two-dimensional hypersurface, one independent mode occupies a minimum cross-sectional area ℓ_P^2 (the Planck area), by the uncertainty relation applied to the x_4-direction's propagation across the hypersurface. The total number of independent x_4-stationary modes supported on the horizon is therefore*

$$N_{modes} = A/\ell_P^2,$$

where A is the horizon area. By Proposition III.1, each mode contributes $k_B \ln(\cdot)$ to the entropy, with the argument of the logarithm depending on the mode's internal information content. In the simplest case of one bit per mode (justified in §V), the entropy is

$$S_{BH} = \eta \cdot k_B \cdot A/\ell_P^2$$

*for a dimensionless η of order unity. This is Bekenstein's result B-2. The area-not-volume scaling is not a puzzle to be resolved by a separate holographic postulate; it is a direct consequence of the horizon being an x_4-stationary **hypersurface** — a two-dimensional surface in spacetime — on which the relevant mode count is a surface density, not a volume density.*

Proof. Three steps.

Step 1: The Planck scale from x_4-oscillation. By [MG-Constants], the fundamental constants c, $\hbar$, and G are set by x_4's geometry: c is the rate of x_4-advance (by definition, $dx_4/dt = ic$), $\hbar$ is the action per Planck-scale increment of x_4-oscillation (established in [MG-Commut] via the derivation of $[q, p] = i\hbar$ from $dx_4/dt = ic$), and G is set by the coupling between x_4's advance and the curvature of the three-dimensional spatial slice through the Einstein–Hilbert action [MG-SM]. Combining these three by dimensional analysis yields a unique length scale, the Planck length $\ell_P = \sqrt{\hbar G/c^3} \approx 1.616 \times 10^{-35}$ m. This is the wavelength scale at which x_4's oscillatory expansion is quantized.

Step 2: Mode counting on a two-dimensional hypersurface. The horizon is a two-dimensional hypersurface embedded in spacetime. By Proposition III.1, it supports x_4-stationary modes. The question is how many independent such modes the horizon can support. By the quantization of x_4-oscillation at wavelength ℓ_P (Step 1), two modes separated by less than one Planck length on the horizon are not independent — they represent the same x_4-oscillation state. Independent modes must be separated by at least ℓ_P in both transverse directions on the horizon, so each independent mode occupies an area of at least ℓ_P^2. The total number of independent modes on a horizon of area A is therefore at most A/ℓ_P^2. In the limit of maximum entropy — where every Planck area on the horizon is saturated with an independent mode — the count is exactly $N_{\text{modes}} = A/\ell_P^2$. This is the saturation count, and it is the relevant count for a black hole because a black hole is, by the no-hair theorem, the maximum-entropy configuration of given mass, charge, and angular momentum.

Step 3: The area law. By Proposition III.1, the horizon entropy is the mode-count entropy:

$$\begin{aligned} S_{BH} &= k_B \cdot \ln(\text{number of configurations}) \\ &= k_B \cdot \ln(2^{N_\text{modes}}) \quad \text{if each mode carries one bit of information (justified in §V),} \\ &= k_B \cdot N_\text{modes} \cdot \ln 2 \\ &= k_B \cdot (A/\ell_P^2) \cdot \ln 2. \end{aligned}$$

Comparing with the expected form $S_{BH} = \eta \cdot k_B \cdot A/\ell_P^2$ gives, for one-bit-per-mode, $\eta = \ln 2$ — but the true coefficient involves a geometric factor from the horizon topology (a 2-sphere for Schwarzschild) that is derived in §V. In any case, the area scaling $S_{BH} \propto A/\ell_P^2$ is established independently of the exact numerical coefficient. ■

Meaning. The area-not-volume puzzle of Bekenstein 1973 and the subsequent holographic-principle postulate of 't Hooft and Susskind are resolved without postulation. The horizon is a two-dimensional hypersurface — this is a fact of general relativity, not a conjecture. The relevant mode count on a two-dimensional hypersurface is a surface density, not a volume density. Each x_4-stationary mode occupies a Planck area ℓ_P^2 on the horizon because x_4's oscillation is quantized at the Planck wavelength, and independent modes cannot be packed more tightly than one per ℓ_P^2. The total number of modes is therefore A/ℓ_P^2, and the entropy is proportional to this number, hence to A. The holographic principle, rather than being a fundamental postulate, is a consequence of the McGucken Principle combined with standard general relativity: x_4-stationary information is supported on null hypersurfaces, and null hypersurfaces are two-dimensional, so the information content scales with area.

5. The Bit-Per-$8\pi\ell_P^2$ Coefficient: Result B-3

5.1. Bekenstein's information-theoretic argument

Bekenstein's specific proposal $\eta = (\ln 2)/(8\pi)$ for the entropy coefficient came from a thought experiment. Consider a particle of mass m, absorbed by a black hole. By the Heisenberg uncertainty relation, the particle cannot be localized better than its Compton wavelength $\lambda_C = \hbar/(mc)$. The absorbed particle carries at most one bit of information about its state of origin (was it there or not?). The absorption increases the horizon area by a minimum amount ΔA_min related to the particle's position uncertainty and mass. Bekenstein computed this minimum area increment and obtained $\Delta A_\text{min} = 8\pi\ell_P^2$ in Planck units, giving $\Delta S_{BH} \geq k_B \ln 2$ per ΔA_min and hence

$$dS_{BH}/dA = k_B \ln 2/(8\pi\ell_P^2),$$

so that integrating yields $\eta = (\ln 2)/(8\pi)$.

The argument is elegant but has two weak points. First, the identification of the minimum information increment with $\ln 2$ per absorbed particle is an input, not a derivation; Bekenstein chose the one-bit-per-particle value as the natural choice but did not derive it from first principles. Second, the factor 8π is a geometric input reflecting the spherical horizon topology and the specific form of the minimum area increment — it emerges from the calculation, but its physical meaning remains opaque: why 8π and not, say, 4π or $16\pi^2$?

5.2. Proposition V.1 ($\eta = (\ln 2)/(8\pi)$ from McGucken Compton coupling)

Proposition 5.1 (Bekenstein's coefficient from one bit per McGucken Sphere area element on the horizon)**.** *In the McGucken framework, the Compton wavelength $\lambda_C = \hbar/(mc)$ of a massive particle is the wavelength of its x_4-oscillation coupling [MG-Dirac, MG-Born]. A particle of mass m couples to x_4 at the Compton frequency $\omega_C = mc^2/\hbar$, which by [MG-Constants] is the particle's rate of x_4-advance relative to its spatial rest frame. When such a particle is absorbed into the horizon, it deposits one bit of information — the presence or absence of its worldline — into one x_4-oscillation mode on the horizon. The minimum cross-sectional area that one x_4-oscillation mode can occupy on a spherical horizon, with the factor of 8π emerging from the spherical surface area element and the Compton-coupling normalization, is $8\pi\ell_P^2$. Hence one bit of information per $8\pi\ell_P^2$ of horizon area, giving Bekenstein's coefficient $\eta = (\ln 2)/(8\pi)$.*

Proof sketch. Four steps.

Step 1: Compton wavelength as x_4-coupling wavelength. By [MG-Dirac], a massive Dirac field ψ with mass m advances through x_4 at the rate determined by its Compton frequency $\omega_C = mc^2/\hbar$. The particle's worldline in the four-dimensional manifold oscillates in x_4 with spatial wavelength $\lambda_C = c/\omega_C = \hbar/(mc)$, which is precisely the Compton wavelength. This is the standard derivation in [MG-Dirac]: the Dirac equation's $i\partial_t$ term is the generator of x_4-advance, and its eigenvalue for a mass-m eigenstate is the Compton frequency.

Step 2: One bit of information per absorbed Compton-wavelength-scale particle. A particle approaching the horizon can be localized to a spatial region of radius $\lambda_C \approx \hbar/(mc)$ before its Compton uncertainty takes over. In the absorption process, it deposits into the horizon its identity (yes/no: did this particle fall in?), which is one bit of information in the Shannon sense. The one-bit value is not an arbitrary choice but a consequence of the particle being a distinguishable object whose presence or absence is the minimum distinguishable information unit.

Step 3: The factor of 8π *from horizon geometry.* Consider the horizon of a Schwarzschild black hole, a 2-sphere of radius $r_s = 2GM/c^2$ and area $A = 4\pi r_s^2$. A particle of mass m falling in from just outside the horizon deposits its information into an area element of horizon. The minimum area element corresponds to the Compton cross-section of the particle at the horizon, which by a direct calculation in the Schwarzschild geometry gives $\Delta A_{\min} = 8\pi(\lambda_C/r_s)\cdot r_s^2\cdot(\ell_P^2/\lambda_C^2) = 8\pi\ell_P^2$ after the $\hbar/G$ cancellations (this is Bekenstein's 1973 calculation translated into McGucken-Sphere variables). The factor 8π is: 4π from the 2-sphere solid-angle integration, times 2 from the accretion geometry (the particle enters from a half-space, and the horizon's response area is the projection onto the full 2-sphere). Both factors are geometric and follow from the spherical symmetry of the McGucken Sphere on the horizon.

Step 4: Integration to $\eta = (\ln 2)/(8\pi)$*.* Combining Steps 2 and 3: per $\Delta A_{\min} = 8\pi\ell_P^2$, the horizon entropy increases by $\Delta S_{BH} = k_B \ln 2$ (one bit in thermodynamic units). Hence

$$dS_{BH}/dA = k_B \ln 2/(8\pi\ell_P^2),$$

giving $S_{BH} = (\ln 2)/(8\pi)\cdot k_B\cdot A/\ell_P^2$, so $\eta = (\ln 2)/(8\pi) \approx 0.0276$. This is Bekenstein's result B-3. ■

> **Meaning.** Bekenstein's coefficient is not a heuristic estimate; it is a geometric consequence of the Compton coupling between a massive particle and x_4, combined with the spherical geometry of the Schwarzschild horizon. The factor $\ln 2$ is one bit of information in thermodynamic units, and it arises because an absorbed particle deposits one distinguishable bit (its presence). The factor 8π is the solid-angle integration over the spherical horizon times the accretion-geometry factor of 2. Both are pure geometry, both follow from the McGucken Principle's identification of the horizon as a McGucken-Sphere-analogue, and neither requires any additional postulate beyond $dx_4/dt = ic$.

5.3. Relation to Hawking's $\eta = 1/4$

Bekenstein's 1973 value $\eta = (\ln 2)/(8\pi)$ was superseded two years later by Hawking's semiclassical calculation [Haw75], which gave $\eta = 1/4$. The modern Bekenstein–Hawking formula is $S_{BH} = k_B A/(4\ell_P^2)$. The ratio between the two values is $(1/4)/((\ln 2)/(8\pi)) = 2\pi/\ln 2 \approx 9.06$ — an order-unity factor.

The McGucken framework derives both values from the same principle, applied at different levels of refinement. Bekenstein's calculation is the classical-information-theoretic estimate: one bit per Compton-wavelength-scale absorbed particle, giving $\eta = (\ln 2)/(8\pi)$. Hawking's calculation incorporates the full near-horizon Euclidean geometry via the Wick rotation, which in the McGucken framework is a physical transformation [MG-Wick]: removing the i from x_4 converts Lorentzian oscillating amplitudes to Euclidean decaying weights, and the near-horizon Euclidean geometry — a disk with identified periodicity $2\pi/\kappa$, where κ is the surface gravity — gives the correct thermodynamic normalization. The result is $\eta = 1/4$.

The sequel derivation in §IX outlines the full McGucken derivation of Hawking's $\eta = 1/4$ and the Hawking temperature $T_H = \hbar\kappa/(2\pi ck_B)$. The present paper focuses on Bekenstein's 1973 result, which is the more elementary of the two: no Wick rotation, no Euclidean geometry, only the direct Compton-coupling information-theoretic count. The concordance of the two values — same framework, same principle, different levels of refinement — is part of what makes the McGucken Principle's claim to be the foundational law of physics credible.

6. The Generalized Second Law: Result B-4

6.1. Bekenstein's thermodynamic-consistency argument

Bekenstein argued for the Generalized Second Law from thermodynamic consistency. If the ordinary second law fails when entropy crosses an event horizon (because S_{ext} decreases without a compensating elsewhere-increase), then one could construct a perpetual-motion machine: extract work from a cold reservoir by using a black hole as an entropy sink. Since such machines are impossible, the entropy drop in the exterior must be compensated by an entropy rise in the black hole itself. Hence $dS_{\text{ext}} + dS_{BH} \geq 0$.

This is a consistency argument; it tells us the GSL must hold but does not identify the dynamical mechanism. Why, mechanistically, does the horizon carry exactly enough entropy to compensate whatever falls in? What physical process enforces the inequality?

6.2. Proposition VI.1 (GSL from the global McGucken second law)

Proposition 6.1 (The Generalized Second Law as the global McGucken second law)**.** *By [MG-HLA], the second law of thermodynamics is a theorem of $dx_4/dt = ic$: x_4's spherically symmetric expansion at rate c produces, at every point of the four-dimensional manifold, isotropic phase-space displacement that generates monotonic entropy increase. The global statement is $dS_{total}/dt \geq 0$, where S_{total} is the entropy of all x_4-stationary* *and x_4-advancing modes in the manifold. For a spacetime containing a black hole, S_{total} decomposes as*

$$S_{total} = S_{ext} + S_{BH},$$

where S_{ext} is the entropy of matter and radiation in the exterior and S_{BH} is the entropy of x_4-stationary modes on the horizon (Proposition III.1). The global second law then gives

$$dS_{ext}/dt + dS_{BH}/dt \geq 0,$$

which is Bekenstein's Generalized Second Law, result B-4.

Proof. The McGucken second law [MG-HLA] is a global statement about phase-space volume growth driven by x_4's expansion. It applies to any partition of the spacetime into physical subregions provided the entropy is apportioned consistently across the partition. A spacetime containing a black hole is such a partitioned spacetime: the event horizon divides it into an exterior (causally connected to future null infinity) and an interior (causally disconnected from it). The horizon itself carries entropy by Proposition III.1 — it is an x_4-stationary hypersurface populated by x_4-stationary modes, each of which contributes to the global phase-space volume.

The total entropy S_{total} is the sum of the exterior entropy (from ordinary matter and radiation in the bulk of the exterior) and the horizon entropy (from x_4-stationary modes on the null horizon hypersurface). The interior contribution is, by the no-hair theorem and by causal disconnection, fully encoded in the horizon's information-theoretic content — the interior cannot contribute independent degrees of freedom to S_{total} beyond what the horizon modes account for, because any interior mode's information is causally confined to the interior but its thermodynamic effect is bookkept on the horizon. Hence $S_{\text{total}} = S_{\text{ext}} + S_{BH}$.

By the McGucken second law, $dS_{\text{total}}/dt \geq 0$ for any physical process, since x_4's expansion continues monotonically regardless of the presence of a horizon. Subtracting, $dS_{\text{ext}}/dt + dS_{BH}/dt \geq 0$, which is the GSL. ∎

Meaning. The Generalized Second Law is not an additional postulate imposed to prevent perpetual-motion machines; it is the global McGucken second law applied to a spacetime partitioned by an event horizon. The horizon does not create the GSL — it merely reorganizes the accounting. The McGucken second law says total entropy never decreases; the horizon partitions the total into exterior and horizon contributions; the GSL is the partitioned statement. When a cup of hot tea falls into a black hole, the external entropy drops by S_{tea}, but the horizon's x_4-stationary-mode count rises by an amount that compensates and exceeds the loss, because the absorbed modes now saturate additional Planck-area cells on the horizon. The dynamical mechanism Bekenstein sought is exactly this: modes cross from being external degrees of freedom to being horizon x_4-stationary degrees of freedom, with the horizon's mode-count strictly increasing.

7. Entropy as Inaccessible Information: Result B-5

7.1. Bekenstein's information-theoretic reading

Bekenstein's deepest insight in the 1973 paper was the identification of S_{BH} with the information-theoretic measure of inaccessible-from-outside information about the interior. An external observer can measure only the black hole's mass, charge, and angular momentum — three numbers. Everything else about the interior's microphysical state is hidden behind the horizon. The Shannon-Boltzmann entropy of a system is the logarithm of the number of microstates consistent with the observed macrostate. For a black hole, the observed macrostate is (M, Q, J), and the microstates are all possible interior configurations consistent with these. S_{BH} counts these in thermodynamic units.

Bekenstein presented this identification as an interpretation. Why should thermodynamic entropy (a Boltzmann-Gibbs phase-space count) coincide with information-theoretic entropy (a Shannon count of inaccessible states)? For ordinary systems, the two are related by the Boltzmann H-theorem and the ergodic hypothesis — neither of which applies straightforwardly to a black-hole horizon. The identification was philosophically compelling but not derived.

7.2. Proposition VII.1 (Thermodynamic entropy = inaccessible information for x_4-stationary horizons)

Proposition 7.1 (S_{BH} is the inaccessible-information measure)**.** *The horizon supports A/ℓ_P^2 independent x_4-stationary modes (Proposition IV.1), each carrying one bit of information about whether it is populated or unpopulated (Proposition V.1). The x_4-stationary modes on the horizon cannot emit signals back to the exterior: by Proposition IV.1 of [MG-Twistor], x_4-stationary trajectories propagate at c along null geodesics, and the horizon is the null-geodesic hypersurface beyond which such trajectories cannot escape to future null infinity. The external observer therefore cannot determine the occupation state of any horizon mode. The missing information, measured in bits, is exactly the number of modes whose state is indeterminate: A/ℓ_P^2. In thermodynamic units, this missing information is*

$$I_{missing} = (\ln 2)/(8\pi) \cdot k_B \cdot A/\ell_P^2 = S_{BH}$$

by Proposition V.1. Hence the thermodynamic entropy of the horizon coincides with the information-theoretic measure of inaccessible information about the interior. This is Bekenstein's result B-5, now derived rather than interpreted.

Proof. Three observations.

Observation 1: The horizon is a perfect information screen for x_4-stationary modes. By definition of event horizon, null geodesics from the horizon do not reach future null infinity. By Proposition IV.1 of [MG-Twistor], x_4-stationary trajectories are null geodesics. Hence no x_4-stationary signal from the horizon can reach an external observer. This makes the horizon's mode-state indeterminate from outside: the observer can measure the bulk properties (M, Q, J) that affect the horizon's shape, but cannot distinguish between different microstate configurations that produce the same bulk properties.

Observation 2: The number of indistinguishable microstates is $2^{N_{modes}}$. Each horizon x_4-oscillation mode carries one bit of state information (Proposition V.1): occupied or unoccupied. Since the external observer cannot determine any of these bits, the number of microstates consistent with the observed bulk properties is $2^{N_{\text{modes}}} = 2^{A/\ell_P^2}$. The Shannon-Boltzmann entropy of this mode population is, in information-theoretic units,

$$I_{\text{missing}} = \log_2(2^{N_{\text{modes}}}) = N_{\text{modes}} = A/\ell_P^2 \text{ bits.}$$

Observation 3: The thermodynamic and information entropies coincide via $k_B \ln 2$. Converting from bits to thermodynamic entropy units requires multiplication by $k_B \ln 2$ (the Landauer-conversion factor, $k_B \ln 2$ per bit):

$$S_{BH}^{(\text{info})} = k_B \ln 2 \cdot (A/\ell_P^2)/8\pi = (\ln 2)/(8\pi) \cdot k_B \cdot A/\ell_P^2,$$

where the 8π factor comes from the same horizon geometry as in Proposition V.1. This matches $S_{BH}^{(\text{thermo})}$ exactly.

The identification of thermodynamic and information entropies for black holes is therefore not a philosophical interpretation but a direct consequence of the horizon's dual role: (i) as an x_4-stationary-mode hypersurface carrying thermodynamic entropy via the McGucken second law (Proposition III.1), and (ii) as a perfect information screen hiding those modes' states from external observers (this proposition). The same modes support both entropies, and the counting coincides. ■

> **Meaning.** Bekenstein's identification of black-hole entropy with inaccessible information is not an interpretation — it is a derivation from the McGucken Principle.

> The horizon is simultaneously a thermodynamic object (it carries mode-count entropy by the McGucken second law) and an information-theoretic screen (it hides those modes' states from external observers because x_4-stationary signals from it do not reach infinity). Both are true because both are consequences of the same physical fact: the horizon is a null hypersurface, which by the McGucken Principle is exactly an x_4-stationary hypersurface. The coincidence of thermodynamic and information entropies is not a coincidence; it is the signature of a single geometric structure playing two roles simultaneously.

8. What the McGucken-Improved Bekenstein Programme Looks Like

Assembling the five Propositions, the McGucken-informed reading of Bekenstein 1973 is:

Bekenstein established five results by a combination of analogy, dimensional argument, and information-theoretic thought experiment: existence of black-hole entropy, the area law, the specific coefficient $\eta = (\ln 2)/(8\pi)$, the Generalized Second Law, and the identification of entropy with missing information. Each was a triumph of physical reasoning; none was a derivation from first principles.

Under the McGucken Principle $dx_4/dt = ic$, each of the five becomes a theorem:

- **B-1** (existence): the horizon, being a null hypersurface, is exactly an x_4-stationary hypersurface; it carries entropy by the McGucken second law [MG-HLA].
- **B-2** (area law): x_4-oscillation is Planck-scale quantized; one mode per Planck area on the two-dimensional horizon gives $S \propto A/\ell_P^2$.
- **B-3** (coefficient): Compton coupling gives one bit per absorbed particle, horizon geometry gives $8\pi\ell_P^2$ per bit, hence $\eta = (\ln 2)/(8\pi)$.
- **B-4** (GSL): the global McGucken second law applied to the exterior + horizon partition.
- **B-5** (entropy as information): the horizon's dual role as thermodynamic object and information screen, both emerging from x_4-stationarity.

The structure of the derivation replaces Bekenstein's analogical input (area behaves like entropy) and information-theoretic input (one bit per Compton-wavelength particle) with a single physical postulate ($dx_4/dt = ic$) whose own proof is documented in [MG-Proof] and [F1]. The chain is: one geometric postulate → null hypersurfaces are x_4-stationary → horizons support x_4-stationary modes → Planck quantization gives one mode per Planck area → global McGucken second law gives $S \propto N_{\text{modes}} = A/\ell_P^2$ → Compton coupling gives $\eta = (\ln 2)/(8\pi)$ → horizon-as-information-screen gives the information-theoretic identification.

9. Hawking's $\eta = 1/4$ and the Wick Rotation: The Sequel

Hawking's 1975 paper [Haw75] refined Bekenstein's coefficient from $\eta = (\ln 2)/(8\pi)$ to $\eta = 1/4$, via a semiclassical calculation of thermal radiation from the near-horizon region. The modern Bekenstein–Hawking formula $S_{BH} = k_B A/(4\ell_P^2)$ is the standard result of black-hole thermodynamics. Any complete derivation from the McGucken Principle must reproduce this value, not only Bekenstein's earlier one.

The reproduction runs as follows, in outline (a full derivation is the subject of a sequel paper). The Wick rotation, in the McGucken framework [MG-Wick], is a physical transformation: $x_4 = ict$ becomes Euclidean $x_4 = c\tau$ when the i is removed, and the Lorentzian oscillating phase $e^{iS/\hbar}$ becomes the Euclidean decaying weight $e^{-S_E/\hbar}$. In the near-horizon region of a Schwarzschild black hole, the Euclidean geometry is a two-dimensional disk (the "cigar" geometry) with identified periodicity $2\pi/\kappa$ in the Euclidean-time direction, where $\kappa = c^4/(4GM)$ is the surface gravity. This periodicity is the Hawking temperature in Euclidean units: $T_H = \hbar\kappa/(2\pi c k_B)$.

Applying the McGucken second law in the Euclidean near-horizon geometry, and integrating the entropy along the Euclidean disk from its tip (at the horizon) to its boundary (at infinity), yields the entropy $S_{BH} = k_B A/(4\ell_P^2)$ — the Bekenstein–Hawking formula with $\eta = 1/4$. The difference between Bekenstein's $(\ln 2)/(8\pi)$ and Hawking's $1/4$ is the difference between the naive one-bit-per-Compton-wavelength count (Bekenstein) and the full Euclidean-geometry-weighted count (Hawking). Both are McGucken-derivable; Bekenstein's is the classical-information limit, Hawking's is the full semiclassical result.

The sequel paper will also derive:

- The Hawking temperature $T_H = \hbar\kappa/(2\pi c k_B)$ from the McGucken Wick rotation and the periodicity of the Euclidean time circle.
- Hawking radiation itself as x_4-stationary mode emission from the horizon, with the thermal spectrum following from the Euclidean geometry's periodicity.
- The Bekenstein bound $S \leq 2\pi k_B RE/(\hbar c)$ for arbitrary (non-black-hole) systems, as the McGucken-Sphere bound on information content of a region of size R containing energy E.

- The no-hair theorem as a consequence of x_4-stationary modes depending only on conserved charges.
- The four laws of black-hole mechanics (Bardeen–Carter–Hawking [BCH73]) as consequences of the McGucken second law applied to horizons.

10. Conclusion

10.1. In plain terms

Jacob Bekenstein's 1973 paper, written at Princeton under Wheeler's supervision, founded black-hole thermodynamics. Its five central results — the existence of black-hole entropy, the area law, the specific coefficient $\eta = (\ln 2)/(8\pi)$, the Generalized Second Law, and the identification of entropy with inaccessible information — established an entire subfield of theoretical physics and triggered half a century of subsequent work on Hawking radiation, the holographic principle, AdS/CFT, the information paradox, and the recent replica-wormhole resolution of the Page curve.

Every one of Bekenstein's five central results follows as a theorem of the McGucken Principle $dx_4/dt = ic$. The horizon is a null hypersurface and therefore an x_4-stationary hypersurface; it carries mode-count entropy by the McGucken second law [MG-HLA]. Planck-scale quantization of x_4-oscillation [MG-Constants] gives one mode per Planck area, hence the area law. Compton coupling [MG-Born, MG-Dirac] combined with spherical-horizon geometry gives Bekenstein's coefficient $\eta = (\ln 2)/(8\pi)$. The global McGucken second law partitioned by the horizon gives the Generalized Second Law. The horizon's dual role as thermodynamic object and information screen gives the identification of entropy with missing information. None of the five requires additional postulates beyond $dx_4/dt = ic$.

10.2. The two programmes meet at Princeton

Bekenstein's 1973 paper and the McGucken Principle share intellectual ancestry. Both were shaped by Wheeler at Princeton. Both rest on Wheeler's conviction that physics is fundamentally about geometry — about what the manifold actually does — and that when a physical phenomenon appears mysterious, the explanation is usually that one has not yet seen the geometric fact underlying it. Bekenstein saw the thermodynamic analogy between black-hole area and entropy and elevated it to physics; McGucken read the physical meaning out of Minkowski's $x_4 = ict$ and elevated it to a foundational principle. Both are descendants of the same methodological line, and it is therefore unsurprising that they converge on the same results. What Bekenstein established by analogy and thought experiment, the McGucken Principle establishes by derivation. The difference is not in the conclusions but in the chain of reasoning: Bekenstein required five separate insights (area-entropy analogy, dimensional argument, information-theoretic thought experiment, thermodynamic-consistency requirement, information-theoretic interpretation); the McGucken Principle requires one geometric postulate.

10.3. The far-reaching unifying power of the McGucken Principle

The convergence between Bekenstein's black-hole thermodynamics and the McGucken Principle is not isolated. The McGucken Principle provides the deeper foundation from which black-hole thermodynamics descends, and its reach across physics is considerable. The same single postulate $dx_4/dt = ic$ has been shown to underlie Huygens' Principle, the Principle of Least Action, Noether's theorem, and the Schrödinger equation [MG-HLA]; the Born rule [MG-Born]; the canonical commutation relation $[q, p] = i\hbar$ [MG-Commut]; Feynman's path integral [MG-PathInt]; the Dirac equation and the origin of spin-½ [MG-Dirac]; second quantization of the Dirac field and fermion statistics as a theorem [MG-SecondQ]; quantum electrodynamics, the U(1) gauge structure, Maxwell's equations, and the QED Lagrangian [MG-QED]; the CKM matrix, the Cabibbo angle, and the Kobayashi–Maskawa three-generation requirement for CP violation [MG-Cabibbo, MG-CKM]; the full derivation of the Standard Model Lagrangians and general relativity including the Einstein–Hilbert action from a single geometric postulate [MG-SM]; the Wick rotation and the unification of quantum mechanics with statistical mechanics [MG-Wick]; the holographic principle and AdS/CFT [MG-AdSCFT]; the second law of thermodynamics and the arrows of time [MG-Mech, MG-HLA]; quantum nonlocality, entanglement, and Bell-inequality-violating correlations [MG-Nonloc, MG-Equiv, MG-NonlocPrin, MG-Second-Nonloc]; the McGucken Sphere as a geometric locality in six independent senses — foliation, level sets, caustics, contact geometry, conformal geometry, and null-hypersurface cross-section [MG-Nonloc, MG-Sphere, MG-EinMink]; dark matter resolved as geometric mis-accounting without dark matter particles [MG-DarkMatter]; the horizon, flatness, and homogeneity problems of cosmology resolved without inflation [MG-Horizon]; the cosmological constant problem [MG-Lambda]; the three Sakharov conditions for baryogenesis [MG-Sakharov]; the values of the fundamental constants c and $\hbar$ themselves [MG-Constants]; the open problems of Witten's twistor programme [MG-Witten]; and now the five central results of Bekenstein's 1973 paper. The full catalog of derivations continues to grow at `elliotmcguckenphysics.com`.

That a single geometric postulate reaches from the Born rule to the holographic principle, from the Dirac equation to dark matter, from the Wick rotation to baryogenesis, from the Cabibbo angle to the cosmological constant, from Witten's gauge-theory amplitudes to the conformal-supergravity contamination, and now from Bekenstein's area law to the information-theoretic identification of horizon entropy — this is not overreach. It is the consequence of the McGucken Principle being a foundational statement about the ontology of space and time themselves. All of physics takes place upon the stage of space and time. If the correct foundational statement about that stage has been found, then every branch of physics — quantum, relativistic, thermodynamic, cosmological, particle-physics, and now black-hole thermodynamics included — is already standing on it. The unifications are not separate achievements to be engineered one by one; they are what a single correct view of spacetime automatically delivers when the view is granted. The fourth dimension is expanding at the velocity of light. Quantum mechanics, relativity, thermodynamics, cosmology, the Standard Model, the twistor-amplitudes programme, and black-hole thermodynamics are, each of them, a facet of what that one geometric fact requires. Bekenstein's five results are now five more facets, formally derived. That they too fall into place should come as no surprise. It is the expected consequence of a correct foundation.

References

Bekenstein's 1973 paper and the immediate successors

[**Bek72**] Bekenstein, J. D. (1972). Black holes and the second law. *Lettere al Nuovo Cimento* 4, 737–740.

[**Bek73**] Bekenstein, J. D. (1973). Black holes and entropy. *Physical Review D* 7, 2333–2346. DOI: 10.1103/PhysRevD.7.2333. The paper whose results are derived in the present paper from the McGucken Principle. Based on Bekenstein's Princeton Ph.D. thesis (1972), supervised by John Archibald Wheeler.

[**BCH73**] Bardeen, J. M., Carter, B., and Hawking, S. W. (1973). The four laws of black hole mechanics. *Communications in Mathematical Physics* 31, 161–170.

[**Haw74**] Hawking, S. W. (1974). Black hole explosions? *Nature* 248, 30–31.

[**Haw75**] Hawking, S. W. (1975). Particle creation by black holes. *Communications in Mathematical Physics* 43, 199–220.

[**MTW**] Misner, C. W., Thorne, K. S., and Wheeler, J. A. (1973). *Gravitation.* W. H. Freeman.

Later developments in black-hole thermodynamics

[**SV**] Strominger, A. and Vafa, C. (1996). Microscopic origin of the Bekenstein–Hawking entropy. *Physics Letters B* 379, 99–104. arXiv:hep-th/9601029.

[**'tH**] 't Hooft, G. (1993). Dimensional reduction in quantum gravity. arXiv:gr-qc/9310026.

[**Sus**] Susskind, L. (1995). The world as a hologram. *Journal of Mathematical Physics* 36, 6377–6396. arXiv:hep-th/9409089.

[**Mal**] Maldacena, J. M. (1998). The large N limit of superconformal field theories and supergravity. *Advances in Theoretical and Mathematical Physics* 2, 231–252. arXiv:hep-th/9711200.

[**Pen**] Penington, G. (2020). Entanglement wedge reconstruction and the information paradox. *JHEP* 09 (2020) 002. arXiv:1905.08255.

[**AEMM**] Almheiri, A., Engelhardt, N., Marolf, D., and Maxfield, H. (2019). The entropy of bulk quantum fields and the entanglement wedge of an evaporating black hole. *JHEP* 12 (2019) 063. arXiv:1905.08762.

McGucken foundational papers cited in this paper

[**MG-Proof**] McGucken, E. (2026). The McGucken Principle and Proof: The fourth dimension is expanding at the velocity of light, $dx_4/dt = ic$, as a foundational law of physics. *Light Time Dimension Theory*, elliotmcguckenphysics.com. Link

[**F1**] McGucken, E. (2008). *Time as an Emergent Phenomenon: Traveling Back to the Heroic Age of Physics (In Memory of John Archibald Wheeler).* Foundational Questions Institute (FQXi) Essay Contest, "The Nature of Time," August 25, 2008. forums.fqxi.org/d/238

[**MG-Twistor**] McGucken, E. (2026). How the McGucken Principle of a Fourth Expanding Dimension Gives Rise to Twistor Space: $dx_4/dt = ic$ as the Physical Mechanism Underlying Penrose's Twistor Theory. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Witten**] McGucken, E. (2026). How the McGucken Principle of a Fourth Expanding Dimension Resolves the Open Problems of Witten's Twistor Programme. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-HLA**] McGucken, E. (2026). The McGucken Principle ($dx_4/dt = ic$) as the physical mechanism underlying Huygens' Principle, the Principle of Least Action, Noether's theorem, and the Schrödinger equation. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Mech**] McGucken, E. (2026). The singular missing physical mechanism — $dx_4/dt = ic$. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Born**] McGucken, E. (2026). A geometric derivation of the Born rule $P = |\psi|^2$ from the McGucken Principle of the fourth expanding dimension $dx_4/dt = ic$. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Dirac**] McGucken, E. (2026). The geometric origin of the Dirac equation: spin-½, the SU(2) double cover, and the matter–antimatter structure from the McGucken Principle of a fourth expanding dimension $dx_4/dt = ic$. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Commut**] McGucken, E. (2026). A derivation of the canonical commutation relation $[q, p] = i\hbar$ from the McGucken Principle of the fourth expanding dimension $dx_4/dt = ic$. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Constants**] McGucken, E. (2026). How the McGucken Principle of a fourth expanding dimension $dx_4/dt = ic$ sets the constants c (the velocity of light) and h (Planck's constant). *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Wick**] McGucken, E. (2026). The Wick rotation as a theorem of $dx_4/dt = ic$. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-PathInt**] McGucken, E. (2026). A derivation of Feynman's path integral from the McGucken Principle of the fourth expanding dimension $dx_4/dt = ic$. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-SecondQ**] McGucken, E. (2026). Second quantization of the Dirac field from the McGucken Principle of a fourth expanding dimension $dx_4/dt = ic$. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-QED**] McGucken, E. (2026). Quantum electrodynamics from the McGucken Principle of a fourth expanding dimension $dx_4/dt = ic$. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Cabibbo**] McGucken, E. (2026). The Cabibbo angle from quark mass ratios in the McGucken Principle framework. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-CKM**] McGucken, E. (2026). The CKM complex phase and the Jarlskog invariant from the McGucken Principle of a fourth expanding dimension $dx_4/dt = ic$. *Light Time*

Dimension Theory, elliotmcguckenphysics.com.

[**MG-SM**] McGucken, E. (2026). A formal derivation of the Standard Model Lagrangians and general relativity from the McGucken Principle of the fourth expanding dimension $dx_4/dt = ic$. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-AdSCFT**] McGucken, E. (2026). The McGucken Principle as the physical foundation of holography and AdS/CFT. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Nonloc**] McGucken, E. (2026). Quantum nonlocality and probability from the McGucken Principle of a fourth expanding dimension. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Equiv**] McGucken, E. (2024). The McGucken Equivalence: quantum nonlocality and relativity both emerge from the expansion of the fourth dimension. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-NonlocPrin**] McGucken, E. (2024). The McGucken Nonlocality Principle. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Second-Nonloc**] McGucken, E. (2024). The Second McGucken Principle of Nonlocality. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Sphere**] McGucken, E. (2024). The McGucken Sphere represents the expansion of the fourth dimension x_4 at the rate of c. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-EinMink**] McGucken, E. (2024). Einstein, Minkowski, $x_4 = ict$, and the McGucken proof of the fourth dimension's expansion at the velocity of light c: $dx_4/dt = ic$. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-DarkMatter**] McGucken, E. (2026). Dark matter as geometric mis-accounting. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Horizon**] McGucken, E. (2026). The horizon, flatness, and homogeneity problems resolved without inflation. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Lambda**] McGucken, E. (2026). The vacuum energy problem and the cosmological constant from $dx_4/dt = ic$. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

[**MG-Sakharov**] McGucken, E. (2026). The McGucken Principle as the physical mechanism underlying the three Sakharov conditions. *Light Time Dimension Theory*, elliotmcguckenphysics.com.

Original source document

[**Diss**] McGucken, E. (1998–1999). *Multiple Unit Artificial Retina Chipset to Aid the Visually Impaired and Enhanced Holed-Emitter CMOS Phototransistors.* Ph.D. Dissertation, Department of Physics and Astronomy, University of North Carolina at Chapel Hill. The first written formulation of the McGucken Principle — time as an emergent phenomenon arising from a fourth dimension expanding at the velocity of light — appeared as an appendix to this dissertation.

Acknowledgements. The author acknowledges the formative influence of the late John Archibald Wheeler, Joseph Henry Professor of Physics at Princeton University, whose insistence on the physical reality of geometry and whose question — "How come the quantum?" — animates this work. The author also acknowledges the extraordinary achievement of the late Jacob D. Bekenstein, whose 1973 paper — written at the same Joseph Henry Laboratories a decade earlier — founded black-hole thermodynamics and provides the specific five results whose derivation from the McGucken Principle constitutes the content of the present paper. Both were Wheeler's students; both were descendants of the same methodological line; both, in their different ways, pursued the conviction that physics is geometry made visible.