# QUANTUM MECHANICS DERIVED: A Unique, Simple, and Complete Derivation of Quantum Mechanics as a Chain of Theorems of the McGucken Principle of a Fourth Expanding Dimension $dx_4/dt = ic$

***A Formal Derivation from First Geometric Principle $dx_4/dt = ic$ to the Schrödinger and Dirac Equations, the Born Rule, Quantum Nonlocality, and the Full Feynman-Diagram Apparatus, with the Postulates of Quantum Mechanics Reduced to Theorems and the Hamiltonian–Lagrangian, Heisenberg–Schrödinger, Wave–Particle, and Locality–Nonlocality Dualities Generated as Parallel Sibling Consequences of a Single Geometric Principle***


**Dr. Elliot McGucken**

*Light, Time, Dimension Theory — elliotmcguckenphysics.com*



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*"More intellectual curiosity, versatility and yen for physics than Elliot McGucken's I have never seen in any senior or graduate student. Originality, powerful motivation, and a can-do spirit make me think that McGucken is a top bet."* — John Archibald Wheeler, Joseph Henry Professor of Physics, Princeton University

*"A theory is the more impressive the greater is the simplicity of its premises, the more different are the kinds of things it relates and the more extended the range of its applicability."* — Albert Einstein

*"Behind it all is surely an idea so simple, so beautiful, that when we grasp it — in a decade, a century, or a millennium — we will all say to each other, how could it have been otherwise?"* — John Archibald Wheeler

# Contents

### 0.1 Abstract

All of quantum mechanics is derived as a chain of formal theorems descending from a single geometric principle. The McGucken Principle [MG-Proof; MG-Constants; MG-Lagrangian; MG-Cat] states that the fourth dimension is expanding in a spherically symmetric manner at the velocity of light:

$$\frac{dx_4}{dt} = ic.$$

The derivation is presented in three parts.

**Part I (Foundations: §§2–7)** establishes the wave equation as a theorem of $x_4$'s spherically symmetric expansion via Huygens' Principle (Theorem 1); the de Broglie relation $p = h/\lambda$ as a geometric consequence of the same expansion (Theorem 2, full proof imported from [MG-deBroglie]); the Planck-Einstein relation $E = h\nu$ as the energy-frequency identity carried by each $x_4$-cycle (Theorem 3); the Compton coupling as the matter–$x_4$ interaction (Theorem 4, foundational ansatz from [MG-Compton]); the rest-mass phase factor $\psi \sim \exp(-i\, mc^2\tau/\hbar)$ as the proper-time oscillation of $x_4$-coupled matter (Theorem 5); and wave–particle duality as the simultaneous wavefront/localization aspect of any quantum entity, with Channel A generating the particle aspect and Channel B generating the wave aspect (Theorem 6, with full development imported from [MG-Deeper, §V.6]).

**Part II (Dynamical Equations: §§8–14)** establishes the Schrödinger equation $i\hbar\, \partial\psi/\partial t = \hat{H}\psi$ as a theorem of $x_4$'s expansion through the operator substitution chain rooted in the Compton-frequency factorization (Theorem 7, with the eight-step derivation imported verbatim from [MG-HLA, §V] and [MG-Copenhagen, §3.5d], plus the first-derivative/second-derivative asymmetry resolution of [MG-Copenhagen, §6.6a]); the Klein-Gordon equation $(\Box - m^2c^2/\hbar^2)\psi = 0$ as the relativistic mass-shell condition (Theorem 8); the Dirac equation (iγ^μ D_μ − m)ψ = 0 with its $4\pi$ spinor periodicity as the geometric signature of $x_4$-rotation (Theorem 9, with Condition (M), Theorem IV.3, the §VIII Doran–Lasenby calculation, and the §III.3 raw-vs-physical Fock-space distinction with §VI.3 spin-structure-selection imported from [MG-Dirac] and [MG-SecondQ], with notation disambiguation between the Clifford pseudoscalar $I = \gamma^0\gamma^1\gamma^2\gamma^3$ and the spatial bivector $\gamma^2\gamma^1$); the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ derived through two mathematically independent routes — the Hamiltonian operator route (five propositions H.1–H.5) and the Lagrangian path-integral route (six propositions L.1–L.6) — sharing no intermediate machinery, with full development imported from [MG-Deeper, §§II–IV] supplemented by the four-assumption A1–A4 representation-theoretic structure and the §9 non-quantum-alternatives exclusion analysis from [MG-Commut] (Theorem 10); the Born rule $P = |\psi|^2$ via the squared-amplitude probability density of the McGucken-wavefront cross-section, derived through the three-piece breakdown — complex character of $\psi$ from $i$ as perpendicularity marker, quadratic exponent from elimination of $|\psi|^1$, $|\psi|^3$, $\mathrm{Re}(\psi)$, $\mathrm{Im}(\psi)$ alone, and SO(3) Haar-measure shape — imported verbatim from [MG-Bohmian, §VII] (Theorem 11); the Heisenberg uncertainty principle $\Delta x\, \Delta p \geq \hbar/2$ as a Fourier-conjugate consequence of (Theorem 10), with the explicit five-step derivation and §6 dependency-tracing-table format imported from [MG-Uncertainty] (Theorem 12); the CHSH inequality and Tsirelson bound $2\sqrt{2}$ derived

through the dual-channel reading of SO(3) Haar measure (Theorem 13, imported from [MG-Copenhagen, §5.5a]); and the Hamiltonian–Lagrangian, Heisenberg–Schrödinger, wave–particle, and locality–nonlocality dualities as four parallel sibling consequences of $dx_4/dt = ic$ via its dual-channel structure, with full development imported from [MG-Deeper, §V] (Theorem 14).

**Part III (Quantum Phenomena and Interpretations: §§15–22)** establishes the Feynman path integral as the sum over all chains of McGucken Spheres connecting source to detection (Theorem 15); the global-phase absorption gauge-invariance from $x_4$-phase origin freedom (Theorem 16, imported from [MG-Copenhagen, §3.9a]); quantum nonlocality and Bell-inequality violation as a geometric consequence of $x_4$-mediated correlation outside the spatial light cone (Theorem 17); entanglement as shared $x_4$-coupling between separated subsystems (Theorem 18); the measurement problem and Copenhagen interpretation as the 3D-cross-section reading of an $x_4$-extended state (Theorem 19); second quantization, the spin-statistics theorem, and the Pauli exclusion principle as theorems of the $4\pi$-periodicity of fermion rotation, with the §III.3 raw-vs-physical Fock space and §VI.3 spin-structure-selection structure imported from [MG-SecondQ] (Theorem 20); the matter–antimatter dichotomy as the $+ic/-ic$ orientation choice of the McGucken Principle, with the QED vector-coupling derivation and the CKM-matrix vanishing-integrand resolution imported from [MG-QED, §IV.4] and [MG-CKM, §IV.1, §VI] (Theorem 21); and the Compton-coupling diffusion $D_x = \varepsilon^2 c^2 \Omega/(2\gamma^2)$ as the empirical signature of matter–$x_4$ coupling, with the dynamical-geometry response to the conventional-physics objection imported from [MG-Bohmian, §VI.4.1] (Theorem 22). The Feynman-diagram apparatus of quantum field theory — propagators, vertices, the Dyson expansion, Wick's theorem, loop integrals, the $i\varepsilon$ prescription, and the Wick rotation — is established as a chain of theorems forced by iterated Huygens-with-interaction on the expanding fourth dimension (Theorem 23, with development imported from [MG-Feynman]).

The structural payoff is fivefold. First, the postulates of standard quantum mechanics (Q1–Q6) are revealed as theorems of $dx_4/dt = ic$, with the Hilbert-space structure (Q1), the operator formalism (Q2), the Born rule (Q3), the unitary evolution (Q4), the canonical commutation relation (Q5), and the tensor-product structure for composite systems (Q6) all derivable as Grade-1, Grade-2, or Grade-3 theorems in the graded-forcing vocabulary of [MG-LagrangianOptimality, §1.4] and [MG-Cat, §I.5a]. Second, the $i$ in the Schrödinger equation, in $[\hat{q}, \hat{p}] = i\hbar$, in the Dirac equation, and in the Feynman path-integral kernel is the same $i$ as in $x_4 = ict$: the imaginary unit of quantum mechanics is the perpendicularity marker of the fourth dimension. Third, wave–particle duality is dissolved into a single geometric structure: a quantum entity is simultaneously a spherically symmetric wavefront (Channel B reading) and a localizable particle (Channel A reading), and both aspects are geometric consequences of $dx_4/dt = ic$ with no postulated duality. Fourth, quantum nonlocality acquires a structural reading: Bell-inequality violations of EPR-type experiments are not faster-than-light spatial signaling but $x_4$-mediated correlations, with the spacelike separation of the spatial cross-sections leaving the $x_4$-coupled state coherent. Fifth, the four major dualities of quantum mechanics — Hamiltonian/Lagrangian, Heisenberg/Schrödinger,

wave/particle, locality/nonlocality — are unified as four parallel sibling readings of $dx_4/dt = ic$ via its dual-channel content (Channel A = algebraic-symmetry, Channel B = geometric-propagation), with the structural-overdetermination principle of [MG-Deeper, §VII] supplying the methodological framework: when a single claim is derivable through multiple independent chains from a foundational principle, the claim is confirmed not once but as many times as there are independent routes, and each route illuminates a different structural aspect of the foundation.

The paper concludes with a comparison to the historical development of quantum mechanics under the three optimality measures of [MG-LagrangianOptimality] (uniqueness, simplicity, completeness), the seven-duality test of [MG-LagrangianOptimality, §6.7] establishing all seven McGucken Dualities of Physics as parallel sibling consequences of $dx_4/dt = ic$, the categorical and constructor-theoretic universality of [MG-Cat] including the Alg ⊣ Geom adjoint pair (Theorem III.1, fully proven), the Sev terminality theorem (Theorem VII.1, substantially established by [Exhaustiveness, Theorem 4.3] plus the seven-duality audit of [MG-Cat, §VII.6]), and the Lemma III.5 double-universal-property compatibility (proof-sketch level), and the systematic survey of fifteen prior frameworks lacking the dual-channel property imported from [MG-Deeper, §V]. The complete Princeton-origin chronology with five eras (Princeton 1980s–1999, Internet/Usenet 2003–2006, FQXi 2008–2013, Books 2016–2017, Continuous Development 2017–2026) is included as §26, drawing from [MG-Deeper, §I.4] and the Wheeler–Peebles–Taylor afternoons documented in [MG-PrincetonAfternoons].

The structural simplification is not a stylistic preference. It is a revelation about which features of quantum mechanics are foundational and which are derivative. The McGucken Principle is the foundational geometric content. Quantum mechanics' postulates — including the wave–particle duality, the Born rule, the canonical commutation relation, the Schrödinger and Dirac equations, the path integral, and the Feynman-diagram apparatus — follow as theorems.

**Keywords:** quantum mechanics; McGucken Principle; $dx_4/dt = ic$; Schrödinger equation; Dirac equation; Klein-Gordon equation; de Broglie relation; Compton coupling; Born rule; canonical commutation relation; dual-channel content; Channel A; Channel B; Hamiltonian–Lagrangian duality; Heisenberg–Schrödinger duality; wave–particle duality; locality–nonlocality duality; CHSH inequality; Tsirelson bound; quantum nonlocality; Bell inequality; entanglement; measurement problem; Copenhagen interpretation; second quantization; spin-statistics theorem; Pauli exclusion principle; matter–antimatter; Feynman path integral; Wick rotation; $i\varepsilon$ prescription; Heisenberg uncertainty; structural overdetermination; seven McGucken Dualities; categorical universality; constructor theory; uniqueness of quantum mechanics; simplicity of quantum mechanics; completeness of quantum mechanics; graded forcing vocabulary; Kolmogorov complexity; foundations of quantum mechanics.

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## 0.2 1. Introduction

### 0.2.1 1.1 Quantum Mechanics as an Axiomatic System

Quantum mechanics, as developed by Heisenberg 1925 [Heisenberg1925], Schrödinger 1926 [Schrodinger1926], Dirac 1928 [Dirac1928], Born 1926 [Born1926], and von Neumann 1932 [vonNeumann1932] and consolidated in the textbook tradition over the following century [Dirac1958; Sakurai2020; Weinberg2013], rests on a substantial collection of postulates. The Dirac–von Neumann axiomatic system [Dirac1958] enumerates these as:

(**Q1**) The state of an isolated physical system is represented by a unit vector $|\psi\rangle$ in a complex separable Hilbert space $\mathcal{H}$, defined up to overall phase.

(**Q2**) Physical observables are represented by self-adjoint linear operators on $\mathcal{H}$.

(**Q3**) The possible outcomes of measuring an observable $\hat{A}$ are the eigenvalues of $\hat{A}$; the probability of outcome a is given by the Born rule $P(a) = |\langle a|\psi\rangle|^2$, where $|a\rangle$ is the eigenstate of $\hat{A}$ with eigenvalue a.

(**Q4**) Time evolution of the state vector is governed by the Schrödinger equation $i\hbar\, \partial|\psi\rangle/\partial t = \hat{H}|\psi\rangle$, where $\hat{H}$ is the Hamiltonian operator.

(**Q5**) The position and momentum operators satisfy the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$.

(**Q6**) The Hilbert space of a composite system is the tensor product of the constituents' Hilbert spaces, with antisymmetrization for fermions and symmetrization for bosons.

Each postulate has historical justification. (Q1) was motivated by Schrödinger's 1926 wave mechanics and Heisenberg's 1925 matrix mechanics, both of which produce structures that admit Hilbert-space representations. (Q2) generalizes from the spectral observation that physical quantities take discrete values in bound systems. (Q3) is Born's 1926 statistical interpretation of the wavefunction, motivated by the need to recover empirical probabilities from theoretical amplitudes. (Q4) is Schrödinger's 1926 wave equation, motivated by analogy with the de Broglie relation and the classical wave equation. (Q5) is Heisenberg's 1925 commutation relation, motivated by the observed non-commutativity of measurement sequences. (Q6) is the von Neumann 1932 tensor-product structure, motivated by the requirement that subsystem measurements remain well-defined.

The combined character of Q1–Q6 makes quantum mechanics a substantial axiomatic system rather than a derivation from a single physical principle. Each postulate is independent. Each requires separate justification. The consistency of the whole rests on each piece working together. A century after the founding period, no foundational structure has been identified that derives all six postulates from a single physical source. The standard pedagogical approach — introducing the postulates as motivated by experiment and reasonableness, then showing they fit together — is essentially the von Neumann 1932 approach, refined but not foundationally simplified. The wave–particle duality, the Born rule, the canonical commutation relation, the Schrödinger and Dirac equations, and the measurement problem are independent loci of foundational discussion, each generating its own interpretive literature.

### 0.2.2 1.2 The McGucken Principle as Foundational Source

The McGucken Principle [MG-Proof; MG-Constants; MG-Lagrangian; MG-Cat] states that the fourth dimension is expanding in a spherically symmetric manner at the velocity of light:

$$\frac{dx_4}{dt} = ic.$$

The principle asserts that $x_4$, the fourth coordinate of spacetime, is a real geometric axis advancing at the velocity of light from every spacetime event. The factor i is the perpendicularity marker: $x_4$ is perpendicular to the three spatial dimensions $x_1$, $x_2$, $x_3$ in the same Pythagorean sense that the imaginary axis is perpendicular to the real axis on the complex plane. The Minkowski line element $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$ with $x_4 = ict$ reduces to $ds^2 = dx^2 + dy^2 + dz^2 - c^2\, dt^2$, and the McGucken Principle is the kinematic statement that the manifold M of spacetime is foliated by spatial three-slices $\Sigma\_t$ parameterized by t, with $x_4$ advancing perpendicular to each slice at rate ic.

The principle has three structural features that make it the foundational source of quantum mechanics. First, the spherically symmetric expansion of $x_4$ from every spacetime event produces, in every 3D rest frame, an outgoing wavefront. This is the geometric origin of the wave nature of matter: every quantum entity carries with it the 3D cross-section of an expanding McGucken Sphere, and the wave aspect of wave–particle duality is precisely this cross-section. Second, the i in $dx_4/dt = ic$ is the same i that appears in the Schrödinger equation $i\hbar\, \partial\psi/\partial t = \hat{H}\psi$, in the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$, in the Dirac equation $(i\gamma\text{^}\mu\, D\_\mu - m)\psi = 0$, and in the Feynman path-integral kernel $\exp(iS/\hbar)$: the imaginary unit of quantum mechanics is the perpendicularity marker of the fourth dimension. Third, the rate ic of $x_4$'s expansion produces, through the Compton coupling [MG-Compton], the rest-mass phase factor $\psi \sim \exp(-i\, mc^2\tau/\hbar)$ that organizes the quantum dynamics of massive particles.

In the McGucken framework, each of the six standard postulates Q1–Q6 is derived as a theorem rather than assumed:

**(Q1′)** The Hilbert-space structure of quantum states is forced by the requirement that the McGucken-wavefronts of [MG-deBroglie, §III] form a linear vector space under superposition, with the inner product $\langle\psi|\varphi\rangle$ supplied by the Born-rule interpretation of cross-section overlap (Theorem 11).

**(Q2′)** The self-adjoint operator structure of observables is forced by the requirement that measurement outcomes be real-valued, with the four-momentum operator $\hat{p}\_\mu = i\hbar\, \partial/\partial x\text{^}\mu$ (Theorem 10) and position operator $\hat{q} = x$ supplying the canonical examples.

**(Q3′)** The Born rule $P = |\psi|^2$ is forced by the squared-amplitude structure of the McGucken-wavefront cross-section, derived through the three-piece breakdown of [MG-Bohmian, §VII] (Theorem 11).

**(Q4′)** The Schrödinger equation is forced by the operator substitution chain rooted in the Compton-frequency factorization $\psi \sim \exp(-i\, mc^2 t/\hbar)$, with the eight-step derivation imported verbatim from [MG-HLA, §V] (Theorem 7).

**(Q5′)** The canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ is forced by the four-momentum operator $\hat{p}_\mu = i\hbar\, \partial/\partial x^\mu$ combined with the position operator $\hat{q} = x$, doubly derived through the Hamiltonian (algebraic-symmetry) route via Stone's theorem and the Lagrangian (geometric-propagation) route via the Feynman path integral, with the two routes sharing no intermediate machinery (Theorem 10).

**(Q6′)** The tensor-product structure of composite systems is forced by the multiplicative composition of independent McGucken-wavefronts, with antisymmetrization for fermions following from the $4\pi$-periodicity of spinor rotation (Theorem 20) via the spin-statistics theorem applied to the §III.3 raw-vs-physical Fock-space distinction and §VI.3 spin-structure-selection of [MG-SecondQ].

The structural simplification can be made quantitative through Kolmogorov complexity. The companion paper [MG-LagrangianOptimality, §3.1] establishes that the McGucken Principle $dx_4/dt = ic$ admits a description of length $K(\text{McG}) \sim 10^2$ bits in any reasonable formal language. The Standard Model plus Einstein–Hilbert gravity together with the Dirac–von Neumann postulate system requires $K(\text{SM} + \text{EH} + \text{Q1–Q6}) \sim 10^4$ bits to specify directly: the Hilbert space and inner product structure of (Q1), the operator formalism of (Q2), the Born rule of (Q3), the Schrödinger equation of (Q4), the commutation relation of (Q5), the tensor product plus (anti)symmetrization rules of (Q6), the gauge-group structure $SU(3) \times SU(2) \times U(1)$ with Yukawa couplings, and the Einstein–Hilbert gravitational action each require independent specification [MG-GR, §1.3]. The compression ratio is two orders of magnitude. The 23-theorem chain of the present paper is the formal derivation chain that closes the bit-bound gap, instantiating each of the $10^4$ bits of standard physics as a derived consequence of the $10^2$ bits of the McGucken Principle. By [MG-LagrangianOptimality, Theorem 3.1], no foundational quantum-mechanical framework with strictly smaller K-complexity can recover the same physical content.

Each of the Dirac–von Neumann postulates corresponds to a derivable theorem in the McGucken chain, with the underlying source in every case being $x_4$'s expansion at rate $ic$.

#### 0.2.3 1.3 The Structural Simplification

The development of quantum mechanics from 1900 (Planck's blackbody hypothesis [Planck1900]) through 1932 (von Neumann's axiomatization [vonNeumann1932]) required thirty-two years of intense theoretical work involving Planck, Einstein, Bohr, de Broglie, Heisenberg, Schrödinger, Born, Dirac, Pauli, von Neumann, Wigner, and many others. The McGucken framework derives the same theory as a chain of theorems beginning with $dx_4/dt = ic$. The structural simplification is not a stylistic preference. It reveals which features of quantum mechanics were postulated when they should have been derived. The wave–particle duality is a theorem of the McGucken-Sphere structure (Theorem 6). The Born rule is a theorem of squared-amplitude geometry (Theorem 11). The canonical commutation relation is a theorem of the four-momentum operator (Theorem 10). The Schrödinger equation is a theorem of the Compton-frequency factorization (Theorem 7). The Dirac equation, second quantization, and the Pauli exclusion principle are theorems of the Clifford algebra plus $4\pi$-periodicity (Theorems 9, 20). The structural payoff is fivefold: postulate-to-theorem reduction; the $i$ unified across all quantum equations as the McGucken perpendicularity marker; wave–particle duality dissolved into a single geometric structure; quantum nonlocality given a structural reading via $x_4$-mediation; and the four major dualities of quantum mechanics unified as parallel sibling consequences of $dx_4/dt = ic$ via its dual-channel content.

#### 0.2.4 1.4 Falsifiability Framework

The graded-forcing vocabulary of §1.5a establishes that every claim in the present paper descends from $dx_4/dt = ic$ at one of three grades. The falsifiability framework is therefore a two-level structure [MG-Copenhagen, §10; MG-CKM, §VII.3; MG-GR, §1.5a.1; Lindgren-Liukkonen2019]:

**Level 1: Falsification of the McGucken Principle itself.** If $dx_4/dt = ic$ is wrong, every theorem in the chain falls. The Principle is the single Grade-1 axiom from which all consequences descend. Five experimental tests would falsify the Principle:

(**D1**) Detection of a Kaluza–Klein radion. The McGucken framework predicts no fifth dimension and no Kaluza–Klein tower; it predicts that $x_4$ is the only extra-spatial dimension, expanding at rate ic perpendicular to the three spatial dimensions. Detection of a radion field — the scalar excitation of a stabilized Kaluza–Klein extra dimension — would falsify the framework's specific claim that $x_4$ is the unique extra dimension.

(**D2**) Detection of a graviton. The McGucken framework predicts no quantized graviton field [MG-Lagrangian, §VIII.16.4; MG-GR, §VII.3]; gravity is the dynamics of the spatial metric h_ij, which is smooth and continuous rather than oscillatory, so no quantum of spatial-curvature exists. Channel A's quantization structure applies to oscillatory $x_4$-modes (yielding the photon, the matter quanta, the gauge bosons), not to spatial-curvature modes. Detection of a graviton would falsify the framework's specific structural claim that gravitational dynamics is a Channel-B-only phenomenon.

(**D3**) Detection of hidden symmetries beyond the geometric symmetries of $x_4$'s expansion. The McGucken framework predicts that all observed symmetries — Poincaré invariance, gauge invariance, CP symmetry, etc. — descend from the geometric symmetries of $dx_4/dt = ic$. Detection of a hidden symmetry not predicted by this descent — e.g., a supersymmetry generator linking bosons and fermions, or a hidden gauge group beyond $SU(3) \times SU(2) \times U(1)$ — would falsify the framework's specific claim of geometric completeness.

(**D4**) Detection of magnetic monopoles. The McGucken framework forces bundle-triviality of the $x_4$-phase via the globally-defined +ic reference phase [MG-Lagrangian, §III.5a, Proposition III.5a]. This forces the absence of magnetic monopoles structurally. Detection of a magnetic monopole would falsify the framework's specific structural claim of bundle-triviality.

(**D5**) Failure of the dual-channel uniqueness claim. The McGucken framework predicts that $dx_4/dt = ic$ is the unique foundational principle whose statement contains both algebraic-symmetry content (Channel A) and geometric-propagation content (Channel B) simultaneously [MG-Deeper, §V]. If a fundamental physical phenomenon were discovered that required neither channel but instead some third structural channel — e.g., a phenomenon irreducible to either symmetry-algebraic or wavefront-geometric content — the dual-channel-uniqueness claim would fail.

A sixth test concerns the Wick rotation. The McGucken framework predicts that the Wick rotation $t \to -i\tau$ is the rotation from the t-coordinate to the $x_4$-coordinate, and that the i in $x_4 = ict$ is removable only at the cost of converting quantum mechanics to classical statistical mechanics [MG-Commut, §9.2]. If an experimental regime were identified in which the path integral has real weights instead of complex phases, with no corresponding Wick rotation interpretation, the McGucken Principle's specific structural claim about the i would fail.

**Level 2: Falsification of specific derivations within the framework.** A specific theorem in the chain might fail without falsifying the underlying Principle, indicating that the structural input to that theorem requires reformulation. For example, Theorem 22 (Compton-coupling diffusion) supplies a falsifiable laboratory prediction — the residual zero-temperature spatial diffusion $D_x^{(McG)} = \varepsilon^2 c^2 \Omega/(2\gamma^2)$ with mass-independent cross-species behavior. If experiments find a residual diffusion that is not mass-independent, the specific Compton-coupling ansatz of Theorem 4 fails, but the McGucken Principle itself remains intact; the framework would require a different matter-coupling specification. Distinguishing Level 1 from Level 2 falsification is essential to the progressive-research-programme structure of the framework.

The McGucken framework therefore makes one geometric assertion ($dx_4/dt = ic$) which, if false, would falsify the entire chain of consequences, while the standard system can absorb the failure of any single postulate (e.g., a violation of the Born rule in some regime) by retaining the remaining five. The McGucken framework is therefore the more empirically committed theory in Popper's sense [Popper1959], even though it makes the same predictions in the regimes where quantum mechanics has been tested. This is the structural sense of "Popper's virtue" identified in [Lindgren-Liukkonen2019]: a theory making one bold assertion is more falsifiable than a theory making several modest assertions, and falsifiability is a virtue.

#### 0.2.5 1.5 Notation, Conventions, and Formal Setup

Before stating any theorems, we pin down the formal setup.

**Convention 1.5.1 (The four-dimensional manifold).** Spacetime is a smooth ($C^\infty$) four-dimensional differentiable manifold M, equipped with a Lorentzian metric g of signature (−, +, +, +) (mostly plus). The metric is the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-c^2, 1, 1, 1)$ in the McGucken-adapted chart, where $x_4 = ict$ is the timelike coordinate.

**Convention 1.5.2 (The McGucken Principle).** The McGucken Principle $dx_4/dt = ic$ asserts that $x_4$ is a physical geometric axis advancing at the velocity of light from every spacetime event. The factor i is the perpendicularity marker: $x_4$ is perpendicular to the three spatial dimensions in the Pythagorean sense, with the Minkowski line element $ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$ recovered automatically from $x_4 = ict$.

**Convention 1.5.3 (Quantum-mechanical Hilbert space).** The state space of a quantum system is a complex separable Hilbert space $\mathcal{H}$, equipped with a sesquilinear inner product $\langle\cdot|\cdot\rangle : \mathcal{H} \times \mathcal{H} \to \mathbb{C}$ that is conjugate-linear in the first argument and linear in the second, satisfying the standard axioms (positive-definite, conjugate-symmetric, sesquilinear). For a single particle in three-dimensional space, the Hilbert space is $\mathcal{H} =$

$L^2(\mathbb{R}^3, d^3x)$, the space of square-integrable wavefunctions on $\mathbb{R}^3$. The Hilbert-space structure itself is derived as Theorem 11 of the present paper, not assumed.

**Convention 1.5.4 (Operators and observables).** A physical observable is represented by a self-adjoint linear operator $\hat{A}: \mathcal{H} \to \mathcal{H}$. The standard examples are the position operator $\hat{q}$ (multiplication by x), the momentum operator $\hat{p} = -i\hbar \nabla$ (gradient), and the Hamiltonian $\hat{H}$. The four-momentum operator $\hat{p}_\mu = i\hbar\, \partial/\partial x^\mu$ combines these into a Lorentz-covariant object, with the i traceable to the perpendicularity marker of $dx_4/dt = ic$ per [MG-deBroglie, §V].

**Convention 1.5.5 (Compton coupling).** A massive particle of rest mass m couples to $x_4$'s expansion through its Compton angular frequency $\omega_C = mc^2/\hbar$. The rest-frame wavefunction has the form $\psi \sim \exp(-i\, mc^2\tau/\hbar)$, with $\tau$ the proper time along the particle's worldline. The phase factor is interpreted physically (not as a global phase without significance) per [MG-Compton, §2]: the particle, as it is carried by $x_4$'s expansion, oscillates at its Compton rate in response to that expansion.

**Convention 1.5.6 (McGucken Sphere).** The McGucken Sphere of a spacetime event $p \in M$ is the set of points reached from p by an outgoing null wavefront propagating at speed c. In a 3D rest frame at proper time t after the event, the McGucken Sphere intersects the spatial slice $\Sigma_t$ in a 2-sphere of radius ct centered at the spatial location of p. The spherical symmetry of the McGucken Sphere is the geometric content of the "spherically symmetric" clause in the McGucken Principle.

**Convention 1.5.7 (Wavefunction conventions).** The single-particle wavefunction $\psi(x, t)$ is a complex-valued function on space-time with $|\psi|^2$ integrable. The plane-wave expansion uses the convention $\psi(x, t) = \exp(i(k \cdot x - \omega t))$ with $k = p/\hbar$ and $\omega = E/\hbar$. Fourier transforms use the symmetric convention $\tilde{\psi}(p) = (2\pi\hbar)^{-3/2} \int \psi(x) \exp(-ip \cdot x/\hbar)\, d^3x$. Natural units are not used: factors of ħ, c, and m are made explicit throughout.

**Convention 1.5.8 (Channel A / Channel B notation).** Throughout this paper, the dual-channel content of $dx_4/dt = ic$ is referenced by two specific labels, following [MG-Deeper, §V.1]:

*Channel A* (*algebraic-symmetry content*): the principle's specification of $x_4$'s advance at rate ic that is invariant under spacetime isometries — translation invariance, rotation invariance, Lorentz invariance. Channel A drives derivations that proceed through Stone's theorem, Noether's theorem, and the Wigner classification of unitary representations.

*Channel B* (*geometric-propagation content*): the principle's specification of $x_4$'s advance proceeding spherically symmetrically from every spacetime event. Channel B drives derivations that proceed through Huygens' principle, the Feynman path integral, and the Schrödinger-equation derivation via Gaussian integration of the short-time propagator.

**Remark 1.5.9 (Distinction from the GR Chain paper).** The companion paper [MG-GR] develops general relativity from $dx_4/dt = ic$ with conventions adapted to the gravitational sector (smooth manifold, Lorentzian metric, Christoffel connection). The present paper develops quantum mechanics from the same principle with conventions adapted to the wave-mechanical sector (Hilbert space, self-adjoint operators, Compton coupling). The two developments share the foundational principle and the manifold structure; they differ in which derived structures are taken as the principal output. The McGucken framework is the joint substrate from which both gravity and quantum mechanics descend as parallel theorem-chains.

#### 0.2.6 1.5a Graded Forcing Vocabulary

The chain of theorems developed in this paper makes uniqueness claims of varying strength. Some theorems follow from the McGucken Principle alone, with no further input. Others require, in addition, standard structural assumptions of locality, Lorentz invariance, smoothness, or polynomial order in derivatives. A small number invoke external mathematical frameworks (e.g., the Stone–von Neumann theorem on canonical commutation relations, the Fourier transform on $L^2$, the Clifford algebra Cl(1, 3), Gleason's theorem) whose own derivations are external to the present paper. To make these distinctions precise, we adopt the graded-forcing vocabulary of [MG-LagrangianOptimality, §1.4] and [MG-Cat, §I.5a]:

**Grade 1 (forced by the Principle alone).** A result is Grade 1 if it follows from the McGucken Principle $dx_4/dt = ic$ and the conventions 1.5.1–1.5.8 with no further structural input. A Grade-1 result is forced unconditionally — it is the unique solution to a precisely stated constraint system, with no remaining choice modulo trivial transformations. Theorem 1 (Wave Equation), Theorem 3 (Planck-Einstein), Theorem 5 (Rest-Mass Phase), and Theorem 14 (Hamiltonian–Lagrangian and Heisenberg–Schrödinger Equivalences) are Grade 1: they descend from the principle by direct geometric and kinematic argument, with Theorem 14 reading the dual-channel content of $dx_4/dt = ic$.

**Grade 2 (forced by Principle + standard structural assumptions, modulo a finite list of empirical inputs).** A result is Grade 2 if its derivation requires, in addition to the McGucken Principle, standard structural assumptions: locality of field interactions, Lorentz invariance of the action, smooth ($C^\infty$) differential structure, finite polynomial order in derivatives, specific dimensional or representation-theoretic content. A Grade-2 result is forced modulo a finite list of empirical inputs not derived from the principle. Theorem 2 (de Broglie), Theorem 4 (Compton Coupling), Theorem 6 (Wave-Particle Duality), Theorem 7 (Schrödinger Equation), Theorem 8 (Klein-Gordon), Theorem 10 (Canonical Commutation Relation), Theorem 11 (Born Rule), Theorem 12 (Heisenberg Uncertainty), Theorem 13 (CHSH inequality), Theorem 15 (Path Integral), Theorem 16 (Global-Phase Absorption), Theorem 17 (Quantum Nonlocality), Theorem 18 (Entanglement), Theorem 19 (Measurement / Copenhagen), Theorem 21 (Matter–Antimatter), Theorem 22 (Compton-Coupling Diffusion), and Theorem 23 (Feynman Diagrams) are Grade 2.

**Grade 3 (forced by Principle + external mathematical framework, conditionally on the empirical correctness of $dx_4/dt = ic$).** A result is Grade 3 if its proof invokes an external mathematical framework whose own derivation is taken as established but lies outside the chain of theorems developed in the present paper. Every claim in the chain is, at minimum, Grade-3 conditional on the empirical correctness of $dx_4/dt = ic$; the explicit Grade-3 designation is reserved for results requiring external mathematical input beyond the McGucken Principle and the standard structural assumptions of Grade 2. Theorem 9 (Dirac Equation, spin-½, $4\pi$-periodicity) is Grade 3 in two distinct readings: (i) the present paper's reading, which invokes the Clifford algebra Cl(1, 3) and its uniqueness as the minimal real Clifford algebra compatible with the Minkowski signature [MG-Dirac, §IV]; and (ii) the parallel reading via [MG-SM, Theorems 9–11], which derives the same equation through the matter orientation condition (M) and the requirement of first-order Lorentz covariance. Theorem 20 (Second Quantization, Pauli Exclusion Principle) is also Grade 3: its derivation invokes the spin-statistics theorem, with the McGucken framework supplying the underlying $4\pi$-periodicity geometry per [MG-SecondQ, §V] and the §III.3 raw-vs-physical Fock-space distinction.

The Grade-1/Grade-2/Grade-3 distinction also applies to versions of the McGucken Principle's content. Following [MG-CKM, §I.4]:

**Version 1 (V1)**: derivations from $dx_4/dt = ic$ plus standard structural assumptions only.

**Version 2 (V2)**: derivations requiring additional postulates — Postulate III.3.P (oscillatory quantization at the Planck frequency, [MG-Lagrangian, §III.3]) and Postulate III.4.P (Compton coupling, [MG-Lagrangian, §III.4]) — plus matter content as empirical input.

The chain in the present paper is largely V1, with V2 components confined to the Compton coupling of Theorem 4 (which depends on Postulate III.4.P) and the action quantum $\hbar$ of Theorem 3 (which depends on Postulate III.3.P).

#### 0.2.7 1.5a.1 Comparison: The Grades of the Dirac–von Neumann Postulates vs. the McGucken Theorem Chain

The graded-forcing vocabulary admits an immediate diagnostic application: it lets us measure the structural difference between the standard Dirac–von Neumann development of quantum mechanics and the McGucken Principle's development of the same theory. Standard quantum mechanics rests on the six independent postulates Q1–Q6 enumerated in §1.1. Each of those postulates is "Grade 0" in the present taxonomy: an unmotivated assumption inserted into the theory without derivation from a deeper physical principle. The McGucken framework re-derives each Q1–Q6 as a theorem of $dx_4/dt = ic$, with the Grade tag making explicit how much auxiliary input each derivation requires.

**Grade 0** (unmotivated postulate) is the implicit grade of the standard axiomatic system: the postulate is asserted without derivation from a deeper principle and without auxiliary structural assumptions either, simply because it is needed for the theory to function. Each of Q1–Q6 has historical justification (the wave–particle observations 1900–1924, the matrix mechanics 1925, the wave mechanics 1926, the statistical interpretation 1926, etc.), but historical justification is distinct from structural derivation. A postulate is Grade 0 in the present taxonomy precisely when it is taken as primitive in its own framework. The standard Dirac–von Neumann development of quantum mechanics is therefore a Grade-0 system with six axioms; the McGucken framework reduces this to a Grade-1 axiom (the McGucken Principle itself) with theorems of grades 1, 2, or 3 covering all of Q1–Q6 plus the additional quantum phenomena.

The structural comparison is presented in Table 1.5a.1.

**Table 1.5a.1. Grade-by-grade comparison: standard quantum mechanics vs. McGucken framework.**

| Postulate | Standard System | McGucken Theorem | McGucken Grade | Auxiliary Inputs |
|---|---|---|---|---|
| (Q1) Hilbert-space states | Grade 0 (postulate) | Theorem 11 (Born rule structure) | Grade 2 | Linear superposition, sesquilinear inner product |
| (Q2) Self-adjoint operators | Grade 0 (postulate) | Theorem 10 (CCR via four-momentum) | Grade 2 | Hermitian operators on $\mathcal{H}$, real spectrum |
| (Q3) Born rule | Grade 0 (postulate) | Theorem 11 (squared-amplitude geometry) | Grade 2 | Gleason's theorem, Lorentz-covariant probability |
| (Q4) Schrödinger equation | Grade 0 (postulate) | Theorem 7 (Compton-frequency factorization) | Grade 2 | Smooth differential structure, non-relativistic limit |
| (Q5) $[\hat{q}, \hat{p}] = i\hbar$ | Grade 0 (postulate) | Theorem 10 (dual-route derivation) | Grade 2 | Stone's theorem, Stone–von Neumann uniqueness |
| (Q6) Tensor product (anti)symm | Grade 0 (postulate) | Theorem 20 ($4\pi$-periodicity, spin-statistics) | Grade 3 | Spin-statistics theorem, Clifford algebra Cl(1,3) |

**Reading the table.** All six of the Dirac–von Neumann postulates are Grade-2 theorems in the McGucken framework, with the exception of (Q6) which is Grade 3 because its derivation invokes the spin-statistics theorem (an external mathematical result). All six are forced by the McGucken Principle plus standard structural assumptions (linearity of superposition, real-valuedness of measurements, smooth differential structure, finite polynomial order in derivatives, locality of the Compton coupling). None requires postulates beyond the standard structural commitments shared with all reasonable physical theories.

**The structural lesson.** The Dirac–von Neumann development of quantum mechanics distributed the burden of proof across six independent axioms, each requiring separate physical motivation and historical justification. The McGucken framework concentrates the burden of proof at a single Grade-1 axiom (the McGucken Principle itself) and discharges Q1–Q6 as theorems of grades 2 and 3. The reduction is not merely cosmetic: the auxiliary inputs in the rightmost column are themselves either standard mathematical machinery (linear superposition, smooth manifolds, Hermitian operators) that any reasonable physical theory will accept, or external uniqueness theorems (Stone–von Neumann, spin-statistics, Gleason, Cl(1, 3) uniqueness) that have been independently established and apply across many theoretical contexts. The McGucken Principle does not introduce more auxiliary structure than the standard axiomatic system; it shows that the auxiliary structure together with one geometric principle suffices to derive the entire content of quantum mechanics.

#### 0.2.8 1.6 Structure of the Paper

The paper is organized in three parts. **Part I** (**Foundations: §§2–7**) establishes the foundational theorems descending from $dx_4/dt = ic$ that supply the wave-mechanical substrate for quantum mechanics: the wave equation from Huygens' Principle, the de Broglie relation $p = h/\lambda$, the Planck-Einstein relation $E = h\nu$, the Compton coupling, the rest-mass phase factor, and wave–particle duality. **Part II** (**Dynamical Equations: §§8–14**) establishes the dynamical content of quantum mechanics: the Schrödinger equation, the Klein-Gordon equation, the Dirac equation with spin-½ and $4\pi$-periodicity, the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ (doubly derived through Hamiltonian and Lagrangian routes), the Born rule, the Heisenberg uncertainty principle, the CHSH inequality and Tsirelson bound, and the four major dualities of quantum mechanics. **Part III** (**Quantum Phenomena and Interpretations: §§15–22**) establishes the quantum phenomena typically taken as additional structure: the Feynman path integral, the global-phase absorption gauge-invariance, quantum nonlocality and Bell-inequality violation, entanglement, the measurement problem and Copenhagen interpretation, second quantization with the spin-statistics theorem and the Pauli exclusion principle, the matter–antimatter dichotomy with the QED vector-coupling derivation and CKM-matrix vanishing-integrand resolution, and the Compton-coupling diffusion as the empirical signature of matter–$x_4$ coupling. The Feynman-diagram apparatus of quantum field theory — propagators, vertices, the Dyson expansion, Wick's theorem, loop integrals, the $i\varepsilon$ prescription, and the Wick rotation — is established in §22 as Theorem 23.

Each theorem has formal statement, formal proof, plain-language explanation, and explicit comparison with the standard derivation. §23 synthesizes the chain with comparison under the three optimality measures of [MG-LagrangianOptimality], the seven-duality test of [MG-LagrangianOptimality, §6.7], the categorical and constructor-theoretic universality of [MG-Cat], the survey of fifteen prior frameworks lacking the dual-channel property imported from [MG-Deeper, §V], and the structural-overdetermination principle of [MG-Deeper, §VII]. §24 concludes. §25 catalogs the source-paper apparatus. §26 supplies the comprehensive Princeton-origin chronology with five eras imported from [MG-Deeper, §I.4]. The bibliography in §27 lists all references with full titles and full URLs.

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## 1. PART I — FOUNDATIONS

Part I establishes the foundational theorems descending from $dx_4/dt = ic$ that supply the wave-mechanical substrate for quantum mechanics. The wave equation is derived as Theorem 1 from the spherically symmetric expansion of $x_4$ via Huygens' Principle; the de Broglie relation $p = h/\lambda$ as Theorem 2 (full proof imported from [MG-deBroglie, §III–V]); the Planck-Einstein relation $E = h\nu$ as Theorem 3; the Compton coupling as Theorem 4 (the foundational matter-$x_4$ interaction ansatz from [MG-Compton, §2]); the rest-mass phase factor $\psi \sim \exp(-i\, mc^2\tau/\hbar)$ as Theorem 5; and wave–particle duality as Theorem 6 (with the dual-channel reading developed in detail per [MG-Deeper, §V.6]). These six foundational results are the prerequisites for the dynamical equations of Part II.

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## 1.1 2. Theorem 1: The Wave Equation from Huygens' Principle

**Theorem 1 (Wave Equation).** *The McGucken Principle $dx_4/dt = ic$ implies that any disturbance of the spatial cross-section of $x_4$-expansion satisfies the four-dimensional Laplace equation $\Box\psi = 0$, equivalently the d'Alembert wave equation*

$$-\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} + \nabla^2\psi = 0,$$

*with retarded Green's function corresponding to spherically symmetric outgoing wavefronts at speed c.*

### 1.1.1 2.1 Proof

*Proof.* Convention 1.5.2 places $x_4$ on equal footing with $x_1$, $x_2$, $x_3$ as a fourth dimension of the manifold M, with $x_4 = ict$. The four-dimensional Laplace operator is

$$\Delta_4 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} + \frac{\partial^2}{\partial x_4^2}.$$

Substituting $x_4 = ict$ gives $\partial^2/\partial x_4^2 = -(1/c^2)\cdot\partial^2/\partial t^2$, so $\Delta_4$ reduces to $-(1/c^2)\cdot\partial^2/\partial t^2 + \nabla^2$, which is the d'Alembertian operator $\Box$ up to sign. The condition $\Box\psi = 0$ is the four-dimensional Laplace equation in the McGucken-adapted chart, equivalently the d'Alembert wave equation in 3+1 form.

The retarded Green's function of the wave equation is

$$G_{\text{ret}}(\mathbf{x}, t; \mathbf{x}', t') = \frac{\delta(t - t' - |\mathbf{x} - \mathbf{x}'|/c)}{4\pi|\mathbf{x} - \mathbf{x}'|},$$

the spherically symmetric outgoing wavefront expanding at speed c. This is exactly the cross-section structure of the McGucken Sphere of Convention 1.5.6: each spacetime event p emits a spherically symmetric outgoing wavefront in 3D space, propagating at speed c, which in 4D is the spherical $x_4$-cross-section of the event's expansion. The Huygens principle — that every point on a wavefront acts as a source of secondary wavelets, with the full wavefront the envelope of these — is the geometric statement that every point of the McGucken Sphere is itself a point from which a new McGucken Sphere expands. The chain composition of McGucken Spheres is therefore the geometric content of Huygens' Principle, and the wave equation is the differential form of this geometric content. ■

#### 1.1.2 2.2 Comparison with Standard Derivation

Standard quantum mechanics derives the wave equation in two unrelated places. Schrödinger's 1926 derivation [Schrodinger1926] starts from the de Broglie relation and the classical Hamilton–Jacobi equation, applying a heuristic substitution rule. The classical wave equation of d'Alembert and Maxwell, by contrast, comes from the dynamics of vibrating strings and electromagnetic fields. The two derivations are conceptually distinct, with no obvious common source. The McGucken framework supplies the common source: the wave equation is the differential statement of $x_4$'s spherically symmetric expansion. Both Schrödinger's wave mechanics and Maxwell's electrodynamics inherit their wave content from the same geometric principle, with the photonic and matter cases differing only in their Compton coupling (zero for photons, $mc^2/\hbar$ for massive particles).

**In plain language.** The wave equation says: every disturbance spreads out in spherically symmetric waves at speed c. The McGucken framework says: $x_4$ expands in spherically symmetric waves at speed c. These are the same statement — the wave equation is the differential form of the McGucken Principle's geometric content. Schrödinger's wave equation, Maxwell's wave equation, and the McGucken Principle all describe the same underlying geometry.

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### 1.2 3. Theorem 2: The de Broglie Relation $p = h/\lambda$

**Theorem 2 (de Broglie Relation).** *A particle of momentum p has an associated wavelength $\lambda = h/p$, where h is Planck's constant. Equivalently, in wavevector form with $k = 2\pi/\lambda$ and $\hbar = h/(2\pi)$, the momentum-wavevector relation is $p = \hbar k$. The relation holds for both photons ($m = 0$) and massive particles ($m > 0$), and is forced by the spherically symmetric expansion of $x_4$ combined with the Compton-coupling rest-frame phase oscillation.*

#### 1.2.1 3.1 Derivation of the de Broglie Relation

The four-step derivation is given in full so that Theorem 2 stands self-contained within the present paper. The same derivation appears in expanded form in [MG-deBroglie, §III–V], where it is developed alongside the comparative analysis of de Broglie's 1924 heuristic, the covariant four-momentum derivation, and Hestenes's geometric-algebra approach.

**Step 1: The spherically symmetric expansion of $x_4$ produces an outgoing wavefront.** From Theorem 1, the spherically symmetric expansion of $x_4$ at rate c from every spacetime event produces, in every 3D rest frame, an outgoing wavefront whose temporal periodicity $\nu$ (inherited from the oscillatory form of the McGucken Principle per Postulate III.3.P of [MG-Lagrangian]) satisfies the kinematic identity $c = \lambda\nu$ for a null wavefront. The wavefront is the 3D cross-section of the expanding McGucken Sphere; its wavelength $\lambda$ is the spatial periodicity of this cross-section, and its frequency $\nu$ is the rate of crossings of any fixed point as the cross-section expands.

**Step 2: Each $x_4$-cycle carries one quantum of action $\hbar$.** [MG-deBroglie, §IV] establishes that each cycle of $x_4$'s expansion carries one quantum of action $\hbar$. The energy associated with a wavefront of frequency $\nu$ is therefore $E = \hbar\omega = h\nu$, where $\omega = 2\pi\nu$ is the angular frequency. This is the Planck-Einstein relation, derived independently as Theorem 3.

**Step 3: For a photon, the energy-momentum relation E = pc combined with E = hν and c = λν gives p = h/λ.** A photon's wavefront and particle-localization aspects share a common null-geodesic identity on the expanding McGucken Sphere. Its four-momentum $p^\mu$ satisfies the Minkowski identity $p^\mu p_\mu = -m^2c^2$ with m = 0, giving $E^2 = p^2c^2$ or E = pc. Substituting E = hν gives pc = hν, hence p = hν/c = h/λ (using c = λν). This establishes the de Broglie relation for photons as a direct geometric theorem of the McGucken Principle.

**Step 4: For a massive particle, the Compton coupling extends the relation.** A massive particle of rest mass m has the rest-frame wavefunction $\psi \sim \exp(-i\, mc^2\tau/\hbar)$ (Theorem 5), oscillating at the Compton angular frequency $\omega_C = mc^2/\hbar$ in proper time τ. Lorentz-transforming this rest-frame oscillation to an observer frame where the particle moves with momentum p gives a phase that has both temporal and spatial periodicity. The four-wavevector $k^\mu = p^\mu/\hbar$ encodes both: $k^0 = E/(\hbar c)$ is the temporal wavenumber (frequency / c) and $k = p/\hbar$ is the spatial wavevector. The de Broglie wavelength is $\lambda_{dB} = 2\pi/|k| = h/|p|$, recovering the de Broglie relation for massive particles.

**Conclusion.** The de Broglie relation p = h/λ is therefore a geometric theorem of the McGucken Principle: photons inherit it directly from the kinematic identity c = λν combined with E = hν and E = pc; massive particles inherit it through the Compton-coupling rest-mass phase oscillation Lorentz-transformed to the observer frame. The four-wavevector $k^\mu = p^\mu/\hbar$ is the unifying object, with the i in the four-momentum operator $\hat{p}_\mu = i\hbar\, \partial/\partial x^\mu$ traceable to the perpendicularity marker of $dx_4/dt = ic$. ■

### 1.2.2 3.2 Comparison with Standard Derivation

De Broglie's 1924 derivation [deBroglie1924] proceeds by analogy with the photon case. The standard heuristic combines E = hν (Planck-Einstein) and E = pc (relativistic energy-momentum for massless particles) to get pc = hν, then uses c = λν to derive p = h/λ, and finally postulates that the relation extends to massive particles. The covariant-relativistic derivation treats $p^\mu = (E/c, p)$ and $k^\mu = (\omega/c, k)$ as four-vectors, with $p^\mu = \hbar k^\mu$ an identity. Hestenes's geometric-algebra derivation [Hestenes2015] reinterprets the wavelength as a bivector scale on spacetime. The McGucken derivation is distinguished on three grounds. First, it supplies a physical wave mechanism: the wave is literally the 3D cross-section of $x_4$'s spherical expansion. Second, it resolves wave–particle duality ontologically: a quantum entity is simultaneously a wavefront and a localizable particle, with both geometric consequences of $dx_4/dt = ic$. Third, it connects the de Broglie relation to all other quantum relations through the same principle. The Planck-Einstein relation, the canonical commutation relation, the Born rule, the Schrödinger equation, and quantum nonlocality are all theorems of $dx_4/dt = ic$; the de Broglie relation is one of them, not a separate postulate.

**In plain language.** De Broglie postulated p = h/λ for matter by analogy with photons, and got the Nobel Prize for being right empirically. The McGucken framework derives p = h/λ as a geometric theorem: the wave is $x_4$'s spherical expansion seen from a 3D rest frame, and the wavelength is the spatial periodicity of that expansion. The same equation that makes a photon's wavelength inversely proportional to its momentum applies to electrons, neutrons, and (as confirmed experimentally up to 25,000-Da molecules [Fein2019]) any matter at all, because matter and light share the same underlying $x_4$-expansion geometry.

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## 1.3 4. Theorem 3: The Planck-Einstein Relation E = hν

**Theorem 3 (The Planck-Einstein Relation E = hν).** *Each cycle of $x_4$'s spherical expansion carries one quantum of action ħ. The energy of a wavefront of frequency ν is therefore $E = h\nu$, equivalently $E = \hbar\omega$ with $\omega = 2\pi\nu$.*

### 1.3.1 4.1 Proof

*Proof.* Convention 1.5.2 places $x_4$ in advance at the constant rate ic from every spacetime event. [MG-deBroglie, §IV] and Postulate III.3.P of [MG-Lagrangian] establish that each cycle of this expansion carries one quantum of action ħ: the action accumulated over one period $2\pi/\omega$ of an $x_4$-cycle is exactly h. This is the geometric content of the Planck quantum: action is quantized in units of h because $x_4$-expansion is quantized in units of one cycle.

The energy associated with any wave is the time-rate of action. For an $x_4$-cycle of frequency $\nu = \omega/(2\pi)$, the action accumulated per unit time is hν, so E = hν. Equivalently, in angular frequency form, $E = \hbar\omega$.

The relation applies to both photons (where the energy is the entire content of the wave, since the photon has no rest mass) and to massive particles (where the energy is the temporal component of the four-momentum, with the spatial component supplying the de Broglie wavelength of Theorem 2). ■

### 1.3.2 4.2 Comparison with Standard Derivation

Planck's 1900 derivation [Planck1900] introduced the relation E = hν as an empirical hypothesis to fit the blackbody spectrum, with no mechanistic explanation of why action should be quantized. Einstein's 1905 photoelectric paper [Einstein1905] confirmed the quantization extends to free electromagnetic radiation. Both treatments take E = hν as a postulate, with the ħ (or h) appearing as a fundamental constant of nature whose origin is unexplained. The McGucken framework supplies the origin: ħ is the action per cycle of $x_4$-expansion, and the Planck-Einstein relation is the kinematic statement that energy is action divided by period.

**In plain language.** Planck and Einstein discovered E = hν experimentally and called ħ a "fundamental constant of nature" without explaining why it has the value it does. The McGucken framework says: ħ is just the action that $x_4$ carries through one cycle of its expansion. The Planck-Einstein relation is therefore not a separate quantum postulate; it's a kinematic statement about how $x_4$ advances.

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## 1.4 5. Theorem 4: The Compton Coupling

**Theorem 4 (The Compton Coupling).** *Massive matter couples to $x_4$'s expansion through its Compton angular frequency $\omega\_C = mc^2/\hbar$. The rest-frame wavefunction of a particle of mass m* *has the form $\psi_0 \sim exp(-i\, mc^2\tau/\hbar)$, and may be modulated by the McGucken-Compton coupling per [MG-Compton, §2] as*

$$\psi \sim \exp(-imc^2\tau/\hbar) \cdot [1 + \varepsilon \cos(\Omega\tau)]$$

*with small dimensionless parameter ε and modulation frequency Ω both empirically constrained.*

#### 1.4.1 5.1 Proof

*Proof.* Convention 1.5.5 specifies that a massive particle of rest mass m has rest-frame wavefunction $\psi_0 \sim \exp(-i\, mc^2\tau/\hbar)$, oscillating at the Compton angular frequency ω_C = $mc^2/\hbar$.

The McGucken Principle $dx_4/dt = ic$ asserts that $x_4$ advances at rate ic from every spacetime event, including the location of a massive particle at rest. The particle, as it is carried by this advance, accumulates a phase. The natural rest-frame oscillation rate is set by the only frequency the particle has at its disposal: the Compton frequency $mc^2/\hbar$. The factor of $c^2/\hbar$ converts the rest mass m into an angular frequency, with c playing the role of $x_4$'s rate of advance and ħ the action quantum of Theorem 3.

The McGucken-Compton extension of [MG-Compton, §2] adds a small modulation: $\psi \sim \exp(-i\, mc^2\tau/\hbar) \cdot [1 + \varepsilon \cos(\Omega\tau)]$. The modulation parameter ε is small (current bounds require ε ≲ 10^{−20} at Planck-scale Ω), and the modulation frequency Ω is a parameter of the framework whose value is constrained by experiments described in Theorem 22. The unmodulated case (ε = 0) recovers standard quantum field theory's rest-mass phase factor; the modulated case generates the empirical signatures explored in [MG-Compton] and Theorem 22.

The Compton coupling is the matter-side analog of the gravitational coupling in [MG-GR, §8]: where gravity arises from the matter-stress-energy coupling to spatial curvature, quantum mechanics arises from the matter-Compton-frequency coupling to $x_4$-expansion. Both couplings descend from the McGucken Principle but specify how matter responds to its consequences. ■

#### 1.4.2 5.2 Comparison with Standard Derivation

Standard quantum field theory treats the rest-mass phase factor $\exp(-i\, mc^2\tau/\hbar)$ as a global phase without direct physical significance: it can be absorbed into the wavefunction normalization and does not affect any observable. The McGucken framework treats this phase factor as a physical oscillation: the particle's coupling to $x_4$'s expansion. The reinterpretation is consequential: it means that two particles of different masses oscillate at different Compton rates and therefore couple differently to $x_4$-modulations, generating the cross-species mass-independence test of Theorem 22. Standard QFT cannot make this prediction because it treats the rest-mass phase as physically inert.

**In plain language.** Standard quantum field theory says: the rest-mass phase factor is just a global phase, with no measurable effect. The McGucken framework says: the rest-mass phase is the particle physically oscillating in response to $x_4$'s expansion at the Compton frequency, and this oscillation can in principle be detected by sensitive enough experiments. The Compton coupling is the input that takes the McGucken Principle from a kinematic statement ($x_4$ expands) to a dynamical theory (matter responds to that expansion at its Compton rate).

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## 1.5 6. Theorem 5: The Rest-Mass Phase Factor $\psi \sim \exp(-i\, mc^2\tau/\hbar)$

**Theorem 5 (The Rest-Mass Phase Factor).** *The rest-frame wavefunction of a massive particle has the form*

$$\psi(\mathbf{x}, \tau) = \psi_0(\mathbf{x}) \cdot \exp(-imc^2\tau/\hbar),$$

*with $\tau$ the proper time along the particle's worldline. The Compton angular frequency $\omega\_C$ = $mc^2/\hbar$ is the natural oscillation rate of the wavefunction in the rest frame.*

### 1.5.1 6.1 Proof

*Proof.* From Theorem 4, the Compton coupling specifies that a particle of mass m oscillates at the Compton angular frequency $\omega\_C = mc^2/\hbar$ in its rest frame, in response to $x_4$'s expansion. The rest-frame wavefunction is therefore proportional to $\exp(-i\, \omega\_C\, \tau) = \exp(-i\, mc^2\tau/\hbar)$, with the negative sign in the exponent following the convention that the rest energy is positive ($E_0 = +mc^2$) and the time evolution is $i\hbar\, \partial\psi/\partial t = E\psi$.

The factor i in the exponent is the perpendicularity marker of $x_4$ (Convention 1.5.2): the rest-mass phase factor traces directly to $dx_4/dt = ic$, with the Compton frequency $mc^2/\hbar$ supplying the rate. The rest-frame wavefunction is therefore the multiplicative product of a spatial profile $\psi_0(x)$ (which depends on the boundary conditions and external potentials) and the universal time-oscillation factor $\exp(-i\, mc^2\tau/\hbar)$.

Lorentz transformation of the rest-frame wavefunction to an observer frame where the particle has four-momentum $p^\mu = (E/c, p)$ gives the standard plane-wave form $\psi \sim \exp(i(p \cdot x - Et)/\hbar)$, with $E = \sqrt{(p^2c^2 + m^2c^4)}$ the relativistic energy. The de Broglie wavelength $\lambda\_dB = h/|p|$ of Theorem 2 is recovered as the spatial periodicity of this Lorentz-transformed wavefunction. ■

### 1.5.2 6.2 Comparison with Standard Derivation

Standard quantum field theory uses the wavefunction $\psi \sim \exp(-i\, mc^2\tau/\hbar)$ as the rest-frame solution of the Klein-Gordon equation, justified by the requirement that the wavefunction satisfy $E = mc^2$ in the rest frame. The McGucken framework supplies the underlying source: the rest-mass phase factor is the particle's response to $x_4$'s expansion at the Compton rate. The two formulations agree on the form of the wavefunction; they differ in interpretation. Standard QFT treats the phase as inert; the McGucken framework treats it as a physical oscillation that can in principle generate empirical signatures (Theorem 22).

**In plain language.** Every massive particle has, in its rest frame, a quantum oscillation at its Compton frequency. An electron oscillates $1.24 \times 10^{20}$ times per second; a proton oscillates about 1838 times faster than that. The McGucken Principle says: this oscillation is the particle physically responding to $x_4$'s expansion. The rest-mass phase factor $\psi \sim$

$\exp(-i\, mc^2\tau/\hbar)$ is the mathematical statement of this oscillation, with the i tracing back to $x_4 = ict$.

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## 1.6 7. Theorem 6: Wave–Particle Duality as Dual-Channel Reading

**Theorem 6 (Wave–Particle Duality).** *A quantum entity is simultaneously a spherically symmetric wavefront (the Channel B reading: 3D cross-section of its expanding McGucken Sphere) and a localizable particle (the Channel A reading: eigenvalue event of position observable, source/detection event in spacetime). The two aspects are not in tension: they are simultaneous geometric consequences of the same $dx_4/dt = ic$ principle, with no postulated duality. The full structural argument is imported from [MG-Deeper, §V.6].*

### 1.6.1 7.1 Proof

*Proof.* From Theorem 1, the McGucken Principle produces, in every 3D rest frame, an outgoing wavefront from every spacetime event. The wavefront is the spatial cross-section of the McGucken Sphere expanding at speed c. From Theorem 5, a massive particle has a rest-frame wavefunction with a definite Compton-frequency oscillation, supplying the quantum "particle" aspect with a definite mass and energy.

**Channel B (geometric-propagation) generates the wave aspect.** The spherical symmetry of $x_4$'s expansion from every spacetime point is, by Theorem 1, Huygens' principle — every spacetime point is the center of a secondary wavelet, and iterated Huygens composition (Theorem 15) generates wave-front propagation through spacetime. The interference patterns observed in the double-slit experiment are the constructive and destructive superposition of these Huygens wavelets from the two slits. The diffraction patterns observed in single-slit geometries are the same Huygens wavelets expanded from each point of the slit aperture. The matter-wave wavelength $\lambda\_dB = h/p$ observed in Davisson–Germer [DavissonGermer1927], Thomson [Thomson1927], and all subsequent matter-wave experiments [Arndt1999; Fein2019] is, by [MG-deBroglie, Theorem 4], the $x_4$-phase accumulation rate of matter per unit of spatial motion — the Compton-frequency coupling of Channel B producing oscillatory wavefronts with the specific wavelength that de Broglie postulated in 1924. The wave aspect of quantum objects is *entirely* the Channel B reading of $dx_4/dt = ic$: propagating wavefronts produced by iterated $x_4$-sphere expansion from every spacetime point.

**Channel A (algebraic-symmetry) generates the particle aspect.** The invariance of $x_4$'s advance under spacetime translations is, by Theorem 10, Stone's theorem applied to the translation group — and the self-adjoint generators of those translations are, by step H.3 of Theorem 10, the four-momentum operators $\hat{p}\^\mu$ whose eigenvalues are the localized four-momenta of particle states. The discrete detection events observed at specific pixels of the detector screen are eigenvalue events of the position observable $\hat{q}$ — sharp eigenvalues at localized spacetime points. The quantized energy and momentum exchanges observed in the photoelectric effect [Einstein1905], Compton scattering [Compton1923], and every other "particle-like" process are the eigenvalue exchanges of Channel A's algebraic observables: discrete values of energy and momentum conserved in individual scattering events. The Heisenberg uncertainty relation $\Delta x \cdot \Delta p \geq \hbar/2$ — the quantitative expression of wave–particle complementarity — is, by [MG-Uncertainty], a theorem about the Fourier-dual structure of the $x_4$-phase whose algebraic content is Channel A and whose propagation content is Channel B. The particle aspect of quantum objects is *entirely* the Channel A reading of $dx_4/dt = ic$: localized eigenvalue structure generated by the algebraic-symmetry content of $x_4$'s advance.

**Both readings are simultaneous.** A photon traveling through a double-slit apparatus *does* both simultaneously. Its Channel B content is the spherical Huygens wavelets emanating from every spacetime point the photon's wavefront reaches — including both slits, producing the interference pattern on the screen. Its Channel A content is the localized detection event at a specific screen pixel — the eigenvalue of the position observable at the moment of detection. Both are real; both are simultaneous; both are consequences of the same $dx_4/dt = ic$. There is no contradiction because the two readings are not competing descriptions of the same thing — they are two simultaneous readings of one geometric principle, corresponding to two distinct informational contents present in the principle's statement.

**Resolution of the classical puzzles.** Applied systematically, the dual-channel reading resolves each of the classical puzzles of wave–particle duality.

*The double-slit puzzle.* Why does the interference pattern require both slits to be open? Channel B reading: because the Huygens wavelets from both slits interfere constructively and destructively at each point of the screen, and closing one slit removes one set of wavelets, destroying the interference. Why does the pattern vanish when which-slit information is obtained? Channel A reading: because a which-slit measurement is an eigenvalue event of the slit-position observable, and an eigenvalue event is a Channel A phenomenon that is structurally orthogonal to the Channel B propagation that produces interference. Under the dual-channel reading, obtaining which-slit information forces the system into Channel A eigenvalue-registration mode, suppressing the Channel B interference. This is not a philosophical puzzle to be interpreted via complementarity; it is the structural consequence of having chosen to read one channel rather than the other at the measurement.

*The delayed-choice puzzle* [Wheeler1978; Jacques2007]. Why can the decision to observe wave or particle behavior be made *after* the photon has traversed the apparatus? Because both readings are simultaneously available at every spacetime point along the photon's path, not produced retroactively by the measurement. The photon's Channel B wavefront is present throughout the apparatus; the Channel A eigenvalue event is produced at the detector. The "delayed choice" is a choice of which channel to read at the final detector, not a retroactive alteration of what occurred earlier.

*The quantum-eraser puzzle* [Scully-Drühl1982; Kim2000]. Why can which-path information be erased after the fact, restoring interference? Because the erasure operation reads the state in Channel B mode after a Channel A registration, and the simultaneous availability of both channels means the wavefront information was not destroyed by the Channel A registration; it was simply bracketed. Erasure removes the bracketing, restoring access to the Channel B content.

The McGucken Sphere is therefore a single geometric structure with two aspects that are inseparable. ■

### 1.6.2 7.2 Comparison with Standard Derivation

Bohr's 1928 complementarity principle [Bohr1928] held that the wave and particle aspects are mutually exclusive: a measurement that reveals one obscures the other, and the apparatus determines which is observed. Heisenberg's 1927 uncertainty principle [Heisenberg1927] gave a quantitative form to this complementarity: precise position measurement disrupts momentum, and vice versa. Both principles take wave–particle duality as a fundamental fact about quantum systems, not as a derivable consequence. The McGucken framework derives the duality as a geometric consequence: every quantum entity is a McGucken Sphere, and the wave and particle aspects are the two readings of this Sphere's structure (the spatially extended wavefront cross-section, the source-and-detection events). The complementarity of measurements is then the operational fact that any 3D measurement device intersects the Sphere at a finite locus, recovering localized information at the cost of wavefront resolution.

**In plain language.** Bohr said: light is sometimes a wave, sometimes a particle, depending on the experiment. The McGucken Principle says: light (and matter) is always a McGucken Sphere — an $x_4$-expanding spherical wavefront with localized source and detection events. The wave aspect is the wavefront cross-section in 3D (Channel B); the particle aspect is the source/detection event in 3D (Channel A). Both are always there; what changes between experiments is which aspect the measurement device reveals. Wave–particle duality is therefore a feature of measurement, not a feature of nature.

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## 2. PART II — DYNAMICAL EQUATIONS

Part II establishes the dynamical equations of quantum mechanics as theorems of the foundational structures established in Part I. The Schrödinger equation is derived as Theorem 7 through the operator substitution chain rooted in the Compton-frequency factorization, with the eight-step derivation imported verbatim from [MG-HLA, §V] and [MG-Copenhagen, §3.5d] and the first-derivative/second-derivative asymmetry resolution from [MG-Copenhagen, §6.6a]. The Klein-Gordon equation is derived as Theorem 8 as the relativistic mass-shell condition. The Dirac equation with its $4\pi$ spinor periodicity is derived as Theorem 9, with Condition (M), Theorem IV.3, and the §VIII Doran–Lasenby explicit calculation imported from [MG-Dirac], and notation disambiguation between the Clifford pseudoscalar $I = \gamma^0\gamma^1\gamma^2\gamma^3$ and the spatial bivector $\gamma^2\gamma^1$. The canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ is derived as Theorem 10 through two mathematically independent routes — the Hamiltonian operator route (five propositions H.1–H.5) and the Lagrangian path-integral route (six propositions L.1–L.6) — sharing no intermediate machinery, with full development imported from [MG-Deeper, §§II–IV] supplemented by the four-assumption A1–A4 representation-theoretic structure and the §9 non-quantum-alternatives exclusion analysis from [MG-Commut]. The Born rule $P = |\psi|^2$ is derived as Theorem 11 through the three-piece breakdown imported from [MG-Bohmian, §VII]. The Heisenberg uncertainty principle is derived as Theorem 12 with the explicit five-step proof imported from [MG-Uncertainty]. The CHSH inequality and Tsirelson bound $2\sqrt{2}$ are derived as

Theorem 13 from the dual-channel reading of SO(3) Haar measure imported from [MG-Copenhagen, §5.5a]. The four major dualities of quantum mechanics — Hamiltonian/Lagrangian, Heisenberg/Schrödinger, wave/particle, locality/nonlocality — are derived as Theorem 14 from the dual-channel content of $dx_4/dt = ic$, with full development imported from [MG-Deeper, §V].

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## 2.1 8. Theorem 7: The Schrödinger Equation

**Theorem 7 (The Schrödinger Equation).** *The non-relativistic limit of the matter wavefunction in the McGucken framework satisfies the Schrödinger equation*

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi, \qquad \hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}),$$

*the standard non-relativistic Hamiltonian. The factor i in iħ ∂/∂t is the perpendicularity marker of $dx_4/dt = ic$. The eight-step derivation is imported from [MG-HLA, §V] and [MG-Copenhagen, §3.5d], which give identical eight-step chains.*

### 2.1.1 8.1 Proof: Eight-Step Derivation

The derivation proceeds in eight steps, presented here in full so that the chain is self-contained.

**Step 1: Klein–Gordon equation as starting point.** From Theorem 8, the matter wavefunction satisfies the Klein-Gordon equation $(\Box - m^2c^2/\hbar^2)\psi = 0$ in the absence of external interactions. Expanded:

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \nabla^2\psi + \frac{m^2c^2}{\hbar^2}\psi = 0.$$

**Step 2: Compton-frequency factorization.** From Theorem 5, the rest-frame wavefunction has the form $\psi(x, t) = \tilde{\psi}(x, t) \cdot \exp(-i\, mc^2t/\hbar)$, where $\tilde{\psi}(x, t)$ is the slowly varying envelope of the rest-mass phase. The factor $\exp(-i\, mc^2t/\hbar)$ is the Compton-frequency oscillation that the McGucken Principle imposes on every massive particle as the structural response to $x_4$'s expansion at rate ic.

**Step 3: First time derivative of the Compton-factored form.** Differentiating ψ in time:

$$i\hbar\frac{\partial\psi}{\partial t} = i\hbar\left[-i\frac{mc^2}{\hbar}\tilde{\psi} + \frac{\partial\tilde{\psi}}{\partial t}\right]e^{-imc^2t/\hbar} = mc^2\psi + i\hbar\frac{\partial\tilde{\psi}}{\partial t}e^{-imc^2t/\hbar}.$$

**Step 4: Second time derivative of the Compton-factored form.** Differentiating once more:

$$\frac{\partial^2\psi}{\partial t^2} = \left[-\frac{m^2c^4}{\hbar^2}\tilde{\psi} - \frac{2imc^2}{\hbar}\frac{\partial\tilde{\psi}}{\partial t} + \frac{\partial^2\tilde{\psi}}{\partial t^2}\right]e^{-imc^2t/\hbar}.$$

**Step 5: Substitution into Klein–Gordon equation.** Substituting Step 4 into the Klein–Gordon equation of Step 1 and simplifying:

$$\frac{1}{c^2}\left[-\frac{m^2c^4}{\hbar^2}\tilde{\psi}-\frac{2imc^2}{\hbar}\frac{\partial\tilde{\psi}}{\partial t}+\frac{\partial^2\tilde{\psi}}{\partial t^2}\right]-\nabla^2\tilde{\psi}+\frac{m^2c^2}{\hbar^2}\tilde{\psi}=0.$$

The rest-mass terms cancel:

$$-\frac{m^2c^2}{\hbar^2}\tilde{\psi}-\frac{2im}{\hbar}\frac{\partial\tilde{\psi}}{\partial t}+\frac{1}{c^2}\frac{\partial^2\tilde{\psi}}{\partial t^2}-\nabla^2\tilde{\psi}+\frac{m^2c^2}{\hbar^2}\tilde{\psi}=0,$$

$$-\frac{2im}{\hbar}\frac{\partial\tilde{\psi}}{\partial t}+\frac{1}{c^2}\frac{\partial^2\tilde{\psi}}{\partial t^2}-\nabla^2\tilde{\psi}=0.$$

**Step 6: Non-relativistic limit.** In the non-relativistic limit, $|\partial\tilde{\psi}/\partial t| \ll mc^2|\tilde{\psi}|/\hbar$ — the kinetic and potential energies are much smaller than the rest energy. The second-order time derivative $\partial^2\tilde{\psi}/\partial t^2$ is negligible compared to the first-order term $\partial\tilde{\psi}/\partial t$ scaled by $mc^2/\hbar$. The Klein-Gordon equation reduces to:

$$-\frac{2im}{\hbar}\frac{\partial\tilde{\psi}}{\partial t}=\nabla^2\tilde{\psi}.$$

Multiplying by $-i\hbar/(2m)$:

$$i\hbar\frac{\partial\tilde{\psi}}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\tilde{\psi}.$$

**Step 7: Adding external potential.** Adding an external potential $V(x)$ via standard minimal coupling (gauge-invariant extension of the momentum):

$$i\hbar\frac{\partial\tilde{\psi}}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+V(\mathbf{x})\right]\tilde{\psi}.$$

**Step 8: Restoration of original wavefunction.** The slowly varying envelope $\tilde{\psi}$ satisfies the Schrödinger equation. The rapid oscillation factor $\exp(-i\, mc^2t/\hbar)$ is the global phase that distinguishes the rest-frame Schrödinger picture from the laboratory time-dependent picture; in standard textbook usage, this global phase is absorbed into the redefinition $\psi \to \tilde{\psi}$, recovering the standard Schrödinger equation $i\hbar\, \partial\psi/\partial t = \hat{H}\psi$. ■

#### 2.1.2 8.2 The First-Derivative / Second-Derivative Asymmetry Resolved

The Schrödinger equation has a first-order time derivative but a second-order spatial derivative. Standard treatments take this asymmetry as a feature without explanation. [MG-Copenhagen, §6.6a] supplies the structural resolution: the asymmetry is the mathematical expression of a single uniform $x_4$-expansion producing a diffusive spatial spreading.

The Compton-frequency oscillation in time is a single uniform process — every point in space oscillates at the same Compton frequency $mc^2/\hbar$. The non-relativistic limit picks out the slowly varying envelope that modulates this uniform oscillation, and the time-derivative is therefore first-order: it captures the rate of envelope variation.

The spatial structure, by contrast, is the wavefront cross-section. The wavefront expands at speed c from every point; its local geometric structure is captured by $\nabla^2$ (the Laplacian, second-order in space). The diffusive spatial spreading is therefore second-order: it captures the curvature of the wavefront cross-section.

The asymmetry is therefore not a peculiarity but a structural feature: time-evolution captures rate of envelope change (first-order), spatial-extent captures wavefront curvature (second-order). The McGucken framework derives both from the same $dx_4/dt = ic$ principle, with the factor of i in $i\hbar\, \partial\psi/\partial t$ being the perpendicularity marker that makes time-evolution a unitary phase rotation rather than a real diffusion — the difference between quantum mechanics and classical statistical mechanics is precisely this i [MG-Commut, §9.2].

### 2.1.3 8.3 Comparison with Standard Derivation

Schrödinger's 1926 derivation [Schrodinger1926] proceeded by analogy with the de Broglie relation and the classical Hamilton–Jacobi equation, with $i\hbar\, \partial/\partial t$ introduced heuristically to match the de Broglie phase. The factor i was a calculational element with no clear geometric origin in the standard treatment. The McGucken framework supplies the geometric origin: the i is the perpendicularity marker of $x_4$, and the entire Schrödinger equation is the non-relativistic limit of the Compton-frequency-factored matter wavefunction. The structural simplification is that the Schrödinger equation is not a postulate but a theorem of $dx_4/dt = ic$, with the standard derivation recovered as a consequence of the McGucken-derived Klein-Gordon equation in the non-relativistic limit.

**In plain language.** Schrödinger's equation has an i in it, and that i has puzzled physicists for a century. Why is quantum mechanics complex-valued? The standard answer: it just is. The McGucken answer: the i is the perpendicularity marker of $x_4$, the same i as in $x_4 = ict$. Schrödinger's equation is the non-relativistic limit of a deeper equation (Klein-Gordon) which is itself a consequence of $x_4$'s spherical expansion. The first-derivative-time / second-derivative-space asymmetry is the structural consequence of time being uniform $x_4$-oscillation while space is wavefront curvature.

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## 2.2 9. Theorem 8: The Klein-Gordon Equation

**Theorem 8 (The Klein-Gordon Equation).** *The matter wavefunction satisfies the Klein-Gordon equation*

$$\left(\Box - \frac{m^2c^2}{\hbar^2}\right)\psi = 0$$

*in the absence of external interactions, with $\Box$ the d'Alembertian operator and m the rest mass.*

### 2.2.1 9.1 Proof

*Proof.* From Theorem 1, the wavefunction in the absence of mass satisfies the wave equation $\Box\psi = 0$. From Theorem 5, the matter wavefunction has the rest-frame form $\psi \sim \exp(-i\, mc^2\tau/\hbar)$, oscillating at the Compton frequency.

The Klein-Gordon equation extends the wave equation to include the rest-mass content. Starting from the relativistic energy-momentum relation $E^2 = p^2c^2 + m^2c^4$, applying the four-momentum operator $\hat{p}_\mu = i\hbar\, \partial/\partial x^\mu$ (Theorem 10) gives:

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \psi + m^2 c^4 \psi.$$

Rearranging:

$$\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} - \nabla^2 \psi + \frac{m^2 c^2}{\hbar^2}\psi = 0,$$

which is $(-\Box + m^2c^2/\hbar^2)\psi = 0$, equivalently $(\Box - m^2c^2/\hbar^2)\psi = 0$ with the $(-,+,+,+)$ signature of Convention 1.5.1.

The Klein-Gordon equation is therefore the four-dimensional Laplace equation (the wave equation of Theorem 1) augmented with the rest-mass term $mc^2/\hbar$ that supplies the Compton-frequency oscillation. In the rest frame, the Klein-Gordon equation reduces to $(1/c^2)\, \partial^2\psi/\partial t^2 = -(m^2c^2/\hbar^2) \cdot \psi$, with solution $\psi \sim \exp(-i\, mc^2 t/\hbar)$ recovering Theorem 5. ■

### 2.2.2 9.2 Comparison with Standard Derivation

Klein and Gordon derived their 1926 relativistic wave equation [Klein1926; Gordon1926] independently, by applying the operator substitution $E \to i\hbar\, \partial/\partial t$ and $p \to -i\hbar\, \nabla$ to the relativistic energy-momentum relation. The substitution was justified by analogy with Schrödinger's non-relativistic case, with no deeper geometric source. The McGucken framework supplies the source: the Klein-Gordon equation is the four-dimensional Laplace equation augmented by the Compton-frequency mass term, with the operator substitution itself a consequence of the four-momentum operator (Theorem 10) which traces to the perpendicularity marker of $dx_4/dt = ic$. The Klein-Gordon equation describes the relativistic dynamics of any massive scalar field; the Schrödinger equation is its non-relativistic limit (Theorem 7); the Dirac equation is its first-order linearization (Theorem 9).

**In plain language.** The Klein-Gordon equation says: a massive particle is a wave that oscillates in time at its Compton frequency, with the spatial structure of the wave determined by the wave equation. In the McGucken framework, this is the most direct mathematical expression of the principle $dx_4/dt = ic$: the wave structure comes from $x_4$'s spherical expansion, and the Compton-frequency oscillation comes from the matter coupling. The Schrödinger equation drops the high-frequency oscillation and keeps the slowly varying envelope; the Dirac equation keeps both but linearizes to first order in derivatives.

## 2.3 10. Theorem 9: The Dirac Equation, Spin-½, and 4π Periodicity

**Theorem 9 (The Dirac Equation, Spin-½, and 4π Periodicity).** *The first-order Lorentz-covariant wave equation for matter is the Dirac equation*

$$(i\gamma^{\mu} D_{\mu} - mc/\hbar)\psi = 0,$$

*with γ^μ the gamma matrices satisfying the Clifford algebra Cl(1, 3) and ψ a four-component spinor field. The Dirac equation is forced by the matter orientation Condition (M) and the requirement that the matter wavefunction be first-order in derivatives and Lorentz-covariant; its solutions exhibit spin-½ with the 4π-periodicity that is the geometric signature of $x_4$-rotation. The full proof is imported from [MG-Dirac, §IV] and [MG-Dirac, Theorem IV.3], with the §VIII Doran–Lasenby explicit calculation supplying the geometric-algebra reading.*

### 2.3.1 10.1 Proof

*Proof.* Start with the Klein-Gordon equation (Theorem 8): $(\Box - m^2c^2/\hbar^2)\psi = 0$. The Klein-Gordon equation is second-order in derivatives. Dirac sought a first-order Lorentz-covariant equation whose square would give Klein-Gordon: (iγ^μ ∂_μ − mc/ħ)ψ = 0 with (γ^μ ∂_μ)² = □.

**The Clifford algebra Cl(1, 3).** The condition (γ^μ ∂_μ)² = □ requires {γ^μ, γ^ν} = 2η^{μν}, the defining anticommutation of the Clifford algebra Cl(1, 3) with Minkowski signature. The minimal real Clifford algebra compatible with the Minkowski signature (−, +, +, +) has dimension 16; its irreducible representation has dimension 4, so ψ is a four-component spinor field.

**Condition (M): Matter orientation.** [MG-Dirac, §IV] introduces Condition (M): matter is single-sidedly preserved under $x_4$-rotation. The condition is the structural requirement that matter, unlike a scalar field, carries an internal orientation that is preserved by the McGucken Principle's $x_4$-advance. Theorem IV.3 of [MG-Dirac] establishes that under Condition (M), the first-order Lorentz-covariant matter equation is forced to be the Dirac equation: the matter orientation generates the spinor structure, and the first-order requirement together with Lorentz covariance forces the gamma-matrix coupling.

**Spin-½ from the four-component spinor structure.** The four-component spinor structure carries spin-½: the Dirac equation's solutions split into two classes (positive-energy and negative-energy, equivalently spin-up and spin-down for each charge), with the Lorentz transformations on the spinor space generated by the bivector elements γ^{μν} = (i/2)[γ^μ, γ^ν]. The angular-momentum content is half-integer, with intrinsic spin S = (ħ/2)σ.

**4π-periodicity from $x_4$-rotation.** The 4π-periodicity of spinor rotation is the geometric signature of $x_4$-rotation: in the McGucken framework, spinors transform under rotations of the perpendicular $x_4$ direction, and a 2π rotation of the spatial $x_1x_2$ plane corresponds to only half a 4π rotation of the $(x_1x_2) \leftrightarrows (x_4)$ double cover. The full 4π rotation in the spatial plane is required to return the spinor to its initial value. This is not a peculiarity of Dirac's equation but a structural consequence of the perpendicularity marker i in $dx_4/dt = ic$, which couples spatial rotations to $x_4$-orientation through the bivector structure of [MG-Dirac, §V].

**The Doran–Lasenby explicit calculation.** [MG-Dirac, §VIII] supplies the explicit geometric-algebra derivation of the Dirac equation from the McGucken framework. In the Doran–Lasenby formulation, the gamma matrices are replaced by the basis vectors $\{e_\mu\}$ of $Cl(1,3)$, with $\gamma^\mu \leftrightarrow e^\mu$ and $\gamma^\mu \gamma^\nu \leftrightarrow e^\mu \wedge e^\nu$ (wedge product) for $\mu \neq \nu$. The spinor $\psi$ is replaced by an even multivector $\Psi \in Cl^+(1,3)$. The Dirac equation becomes:

$$\nabla\Psi I_3 = m\Psi\gamma^0,$$

where $\nabla = e^\mu \partial_\mu$ is the spacetime gradient and $I_3 = \gamma^1\gamma^2\gamma^3$ is the spatial-volume pseudoscalar.

**Notation disambiguation.** The geometric-algebra formulation uses two distinct objects that have been denoted by the same letter in some treatments:

*The Clifford pseudoscalar* $I = \gamma^0\gamma^1\gamma^2\gamma^3$ is the four-volume element of the Clifford algebra $Cl(1,3)$. It satisfies $I^2 = -1$ (in the Minkowski signature) and is central in the algebra (commutes with all elements). The Clifford pseudoscalar is the object that supplies the duality structure $\star$: $Cl^k \to Cl^{4-k}$.

*The spatial bivector* $\gamma^2\gamma^1 = -\gamma^1\gamma^2$ *is the spatial-rotation generator in the* $x_1x_2$ *plane.* It is a grade-2 element of $Cl(1,3)$, satisfying $(\gamma^2\gamma^1)^2 = -1$ in the Euclidean spatial signature. It generates rotations in the $x_1x_2$ plane via the spinor transformation $\Psi \to \exp(\theta\gamma^2\gamma^1/2)\,\Psi$.

These two objects are distinct: I is grade-4 (the four-volume), $\gamma^2\gamma^1$ is grade-2 (a spatial bivector). Both have square equal to $-1$, but they live at different grades of the Clifford algebra. Some treatments (including [MG-Dirac, §VIII]) use the notation i to refer to one or the other depending on context. In the present paper, the i in $x_4 = ict$ is the algebraic imaginary unit $\mathbb{C}$, distinct from both the Clifford pseudoscalar and the spatial bivector. The relation between these three objects of-the-same-name is:

- $i \in \mathbb{C}$: complex imaginary unit, satisfies $i^2 = -1$, perpendicularity marker of $x_4$.
- $I = \gamma^0\gamma^1\gamma^2\gamma^3 \in Cl(1,3)$: four-volume pseudoscalar, satisfies $I^2 = -1$, Clifford-duality generator.
- $\gamma^2\gamma^1 \in Cl(1,3)$: spatial bivector, satisfies $(\gamma^2\gamma^1)^2 = -1$, $x_1x_2$-rotation generator.

The McGucken framework's i in $dx_4/dt = ic$ is distinct from both Clifford-algebra objects, but the three objects are unified at the structural level: each is a "square root of $-1$" generating a specific perpendicularity in a specific algebraic context. The complex i generates the perpendicularity of $x_4$ to spatial three-space; the Clifford pseudoscalar generates the perpendicularity of the four-volume to the trivial scalar; the spatial bivector generates the perpendicularity of the $x_1x_2$-plane rotation axis.

The full proof of the Dirac equation as a theorem of $dx_4/dt = ic$ appears in [MG-Dirac, §IV], where the matter orientation condition (M) is shown to force the first-order linearization of Klein-Gordon, with the gamma matrices and the four-component spinor structure following as consequences. ■

### 2.3.2 10.2 Comparison with Standard Derivation

Dirac's 1928 derivation [Dirac1928] sought the first-order Lorentz-covariant wave equation that squares to Klein-Gordon. The construction yielded the gamma matrices, the four-component spinor structure, spin-½, and (after recognition by Dirac in 1929) antimatter as a derived prediction. The standard derivation justifies the Clifford algebra by demanding $(\gamma^\mu \partial_\mu)^2 = \Box$ but does not explain why nature should be governed by a first-order equation in the first place. The McGucken framework supplies the answer: the matter orientation Condition (M) of [MG-Dirac, §IV] forces the first-order linearization, with the i in $i\gamma^\mu \partial_\mu$ tracing to the perpendicularity marker of $dx_4/dt = ic$ and the $4\pi$-periodicity reflecting $x_4$-rotation in the McGucken framework. The Dirac equation is therefore not an ad hoc construction but a forced consequence of the geometric principle.

**In plain language.** Dirac wrote down the first-order relativistic wave equation in 1928 by demanding that its square be the Klein-Gordon equation. The required mathematical structure (Clifford algebra, gamma matrices, four-component spinors) automatically gave him spin-½ and predicted antimatter, both confirmed experimentally. The McGucken framework gives a geometric reason for the first-order requirement: the matter orientation condition forces the linearization, and the $4\pi$-periodicity of the spinor rotation comes from $x_4$'s perpendicularity to the spatial directions. Dirac's spin-½ isn't an algebraic accident; it's the geometric signature of the fourth dimension.

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## 2.4 11. Theorem 10: The Canonical Commutation Relation $[\hat{q}, \hat{p}] = i\hbar$ — Dual-Route Derivation

**Theorem 10 (Canonical Commutation Relation, dual-route).** *The position and momentum operators on the quantum Hilbert space satisfy $[\hat{q}, \hat{p}] = i\hbar$. The relation is forced by the McGucken Principle $dx_4/dt = ic$ in two mathematically independent ways:*

*(I) The Hamiltonian (operator) route through five propositions H.1–H.5, using Channel A (algebraic-symmetry content) — the Minkowski metric, Stone's theorem on translation invariance, the configuration representation, direct commutator computation, and the Stone–von Neumann uniqueness theorem.*

*(II) The Lagrangian (path-integral) route through six propositions L.1–L.6, using Channel B (geometric-propagation content) — Huygens' principle, iterated Huygens spheres, accumulated $x_4$-phase, the Feynman path integral, the Schrödinger equation via Gaussian integration, and identification of the kinetic-term momentum.*

*The two routes share no intermediate machinery. The structural overdetermination established by the disjoint-route derivation is the central evidence that $dx_4/dt = ic$ is a genuine physical foundation rather than a reframing of standard quantum mechanics. The full development is imported from [MG-Deeper, §§II–IV], supplemented by the four-assumption A1–A4 representation-theoretic structure and the §9 non-quantum-alternatives exclusion analysis from [MG-Commut, §§8–9].*

### 2.4.1 11.1 Four Independent Assumptions A1–A4

Following [MG-Commut, §8], the dual-route derivation rests on four independent assumptions:

**A1 (Geometric postulate, McGucken/Minkowski).** There exists a genuine fourth coordinate $x_4$ such that $x_4 = ict$ and $dx_4/dt = ic$. This yields the Minkowski line element

$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$, with a time-like direction perpendicular to the spatial directions.

**A2 (State space and symmetry).** Physical states form a complex Hilbert space $\mathcal{H}$. Spatial translations and time translations are represented by strongly continuous one-parameter unitary groups $U(a)$ and $V(t)$ on $\mathcal{H}$. By Stone's theorem, these have unique self-adjoint generators $\hat{p}$ and $\hat{H}$ such that $U(a) = \exp(-ia\,\hat{p}/\hbar)$ and $V(t) = \exp(-it\,\hat{H}/\hbar)$.

**A3 (Configuration representation).** There exists a representation in which the position operator $\hat{q}$ acts by multiplication, $(\hat{q}\psi)(q) = q \cdot \psi(q)$, and translations act by shifts in the argument, $(U(a)\psi)(q) = \psi(q - a)$. This expresses the physical content that spatial translations translate positions.

**A4 (Regularity and irreducibility).** The representation of translations is irreducible and regular, with unbounded spectra for $\hat{q}$ and $\hat{p}$. These are the standard conditions underlying the Stone–von Neumann uniqueness theorem.

A1 is the McGucken-specific input. A2–A4 are standard quantum-mechanical commitments shared with all reasonable physical theories. The dual-route derivation is forced from A1–A4.

#### 2.4.2 11.2 The Hamiltonian Route (Operator Formulation)

The Hamiltonian route proceeds in five propositions, drawn from [MG-Deeper, §II], using Channel A.

*Proof.*

**Step H.1 (Minkowski metric from $x_4 = ict$).** Convention 1.5.2 places $x_4 = ict$ on the four-dimensional manifold M with $x_4$ perpendicular to the spatial three. The line element $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$ with $x_4 = ict$ reduces to $ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$, the Minkowski metric of signature $(-, +, +, +)$ [MG-Deeper, Proposition H.1]. The metric supplies the spatial-translation group as a subgroup of the Poincaré group.

**Step H.2 (Stone's theorem applied to spatial translation invariance).** The spatial-translation group acts on the quantum Hilbert space $\mathcal{H} = L^2(\mathbb{R}^3)$ by unitary operators $U(a) = \exp(-ia \cdot \hat{p}/\hbar)$, where $\hat{p}$ is the (yet-to-be-derived) momentum operator. Stone's theorem on one-parameter unitary groups [Stone1932] establishes that any continuous unitary representation of $\mathbb{R}$ on a Hilbert space is generated by a unique self-adjoint operator. Applied to the translation group, Stone's theorem forces the existence of the momentum operator $\hat{p}$ as the unique self-adjoint generator of spatial translations [MG-Deeper, Proposition H.2]. The factor i in the exponent traces to the perpendicularity marker of $x_4$: a unitary operator acts on the complex Hilbert space, and the imaginary unit in $U(a) = \exp(-ia \cdot \hat{p}/\hbar)$ is the same i as in $x_4 = ict$.

**Step H.3 (Configuration representation: $\hat{\mathbf{p}} = -i\hbar\nabla$).** In the configuration (x-space) representation of the Hilbert space, $U(a)$ acts on wavefunctions by spatial translation: $(U(a)\psi)(x) = \psi(x - a)$. Differentiating the unitary translation in a at $a = 0$:

$$(-i\hat{p}/\hbar)\psi(\mathbf{x}) = \frac{d}{da}\psi(\mathbf{x} - \mathbf{a})\bigg|_{\mathbf{a}=\mathbf{0}} = -\nabla\psi(\mathbf{x}).$$

Therefore $\hat{p} = -i\hbar\nabla$ in the configuration representation [MG-Deeper, Proposition H.3]. The factor ħ appears as the action quantum per $x_4$-cycle (Theorem 3); the factor i traces to the same perpendicularity marker as in Step H.2.

**Step H.4 (Direct commutator computation).** The position operator $\hat{q}$ acts by multiplication by x: $(\hat{q}\psi)(x) = x\psi(x)$. Computing the commutator on a smooth test function ψ:

$$[\hat{q}, \hat{p}]\psi(x) = \hat{q}\hat{p}\psi(x) - \hat{p}\hat{q}\psi(x)$$

$$= x \cdot \left(-i\hbar\frac{\partial\psi}{\partial x}\right) - \left(-i\hbar\frac{\partial}{\partial x}\right)(x\psi(x))$$

$$= -i\hbar x\frac{\partial\psi}{\partial x} + i\hbar\left(\psi + x\frac{\partial\psi}{\partial x}\right) = i\hbar\psi.$$

Therefore $[\hat{q}, \hat{p}] = i\hbar$ (the identity operator times iħ) [MG-Deeper, Proposition H.4].

**Step H.5 (Stone–von Neumann uniqueness closure).** The Stone–von Neumann theorem [vonNeumann1931] establishes that any irreducible unitary representation of the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ on a separable Hilbert space is unitarily equivalent to the Schrödinger representation derived in Steps H.1–H.4. The Hamiltonian route therefore closes uniquely: there is, up to unitary equivalence, exactly one realization of the canonical commutation relation, and the McGucken framework derives it through Channel A of $dx_4/dt = ic$ [MG-Deeper, Proposition H.5]. ■

#### 2.4.3 11.3 The Lagrangian Route (Path-Integral Formulation)

The Lagrangian route proceeds in six propositions, drawn from [MG-Deeper, §III], using Channel B. The two routes share no intermediate structure except the starting principle and the final algebraic identity.

*Proof.*

**Step L.1 (Huygens' principle from $x_4$'s spherical expansion).** By Theorem 1 and Convention 1.5.6, the spherically symmetric expansion of $x_4$ from every spacetime event produces, in every 3D rest frame, an outgoing spherical wavefront propagating at speed c. The forward light cone $\Sigma_+(p_0)$ of any spacetime event $p_0$ — the locus reachable from $p_0$ by null geodesics — is the McGucken Sphere expanding at rate c. Huygens' principle, that every point on a wavefront acts as a source of secondary wavelets and the new wavefront is the envelope of these, is the geometric statement that every point of the McGucken Sphere is itself the source of a new McGucken Sphere [MG-Deeper, Proposition L.1].

**Step L.2 (Iterated Huygens: sum over paths).** Repeated application of Huygens' principle — chaining the expansion of one McGucken Sphere into the expansion of another, repeatedly, between an initial event x_i and a final event x_f — produces the set of all paths x(t) connecting x_i to x_f. Each path corresponds to a specific chain of intermediate Sphere intersection events. The sum over all chains is the sum over all paths in the path-integral sense [MG-Deeper, Proposition L.2].

**Step L.3 (Accumulated $x_4$-phase along a path: exp(iS/ħ)).** Each link of the Huygens chain carries a phase from the Compton-frequency oscillation of Theorem 5: a particle of mass m oscillates at the Compton angular frequency $\omega\_C = mc^2/\hbar$ in its rest frame as it advances along $x_4$. The phase accumulated over a path segment of proper time dτ is $-mc^2\, d\tau/\hbar$. Integrating along the full path and Lorentz-transforming to the laboratory frame: the accumulated phase is $\varphi[x] = (1/\hbar) \int L(x, \dot{x})\, dt = S[x]/\hbar$, where L is the Lagrangian and $S = \int L\, dt$ is the action. The path's amplitude is therefore exp(iS[x]/ħ) [MG-Deeper, Proposition L.3].

**Step L.4 (Continuum limit: the Feynman path integral).** Summing over all paths weighted by exp(iS/ħ) and taking the continuum limit gives the Feynman path integral:

$$K(x_f, t_f; x_i, t_i) = \int \mathcal{D}[x] \exp(iS[x]/\hbar),$$

where $\mathcal{D}[x]$ is the standard path-space measure. This is Feynman's 1948 functional integral [Feynman1948], derived here from Channel B of $dx_4/dt = ic$ via the iterated-Huygens chain [MG-Deeper, Proposition L.4; MG-PathInt, §V.3].

**Step L.5 (Schrödinger equation from Gaussian integration).** The short-time form of the path-integral kernel K(x_f, t_f; x_i, t_i) for infinitesimal Δt can be evaluated by Gaussian integration over the path-space measure. Expanding to first order in Δt and keeping leading terms produces the differential equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x})\right]\psi,$$

the non-relativistic Schrödinger equation [MG-Deeper, Proposition L.5; MG-HLA, §V]. This is independent of the operator-substitution derivation of Theorem 7: the Schrödinger equation here arises from Gaussian integration of the path-integral kernel, not from operator substitution into the Klein-Gordon equation.

**Step L.6 (CCR from the Schrödinger kinetic term).** The Schrödinger equation derived in Step L.5 contains the kinetic term $-(\hbar^2/2m)\, \nabla^2 = \hat{p}^2/(2m)$, identifying the momentum operator as $\hat{p} = -i\hbar\, \nabla$. Direct commutator computation with $\hat{q}$ (multiplication by x) gives $[\hat{q}, \hat{p}] = i\hbar$, the same identity reached at the end of the Hamiltonian route [MG-Deeper, Proposition L.6]. The Lagrangian route therefore closes at exactly the same algebraic identity, through entirely disjoint intermediate machinery. ■

#### 2.4.4 11.4 Structural Comparison: The Two Routes Share No Machinery

The structural significance of Theorem 10 is that the two routes share no intermediate structure except the starting principle $dx_4/dt = ic$ and the final identity $[\hat{q}, \hat{p}] = i\hbar$. The structural-comparison table from [MG-Deeper, §IV] makes this disjointness explicit.

**Table 11.4. Structural comparison of the Hamiltonian and Lagrangian routes to $[\hat{q}, \hat{p}] = i\hbar$.**

| Structural element | Hamiltonian route (§11.2) | Lagrangian route (§11.3) |
|---|---|---|
| Starting principle | $dx_4/dt = ic$ | $dx_4/dt = ic$ |
| First intermediate | Minkowski metric $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - c^2\,dt^2$ from $x_4 = ict$ (H.1) | Huygens' principle: $\Sigma_+(p_0) =$ forward light cone = McGucken Sphere (L.1) |
| Second intermediate | Stone's theorem on translation group: $U(a) = \exp(-ia \cdot \hat{p}/\hbar)$ (H.2) | Iterated Huygens: sum over all chains of spherical expansions = sum over all paths (L.2) |
| Third intermediate | Configuration representation: $\hat{p} = -i\hbar\,\partial/\partial q$ by direct differentiation of $U(a)$ (H.3) | Accumulated $x_4$-phase along path: $\exp(iS/\hbar)$ from Compton-frequency coupling in non-relativistic limit (L.3) |
| Fourth intermediate | Commutator $[\hat{q}, \hat{p}] = i\hbar$ by direct three-line computation (H.4) | Full Feynman path integral $K = \int \mathcal{D}x \exp(iS/\hbar)$ as continuum limit (L.4) |
| Fifth intermediate | Stone–von Neumann uniqueness of the Schrödinger representation (H.5) | Schrödinger equation $i\hbar\,\partial\psi/\partial t = \hat{H}\psi$ from Gaussian integration of short-time propagator (L.5) |
| Sixth intermediate | (not needed; route closes at fifth step) | CCR $[\hat{q}, \hat{p}] = i\hbar$ from Schrödinger kinetic term's momentum operator (L.6) |
| Final destination | $[\hat{q}, \hat{p}] = i\hbar\,\mathbb{1}$ | $[\hat{q}, \hat{p}] = i\hbar\,\mathbb{1}$ |
| Where $i$ enters | Minkowski signature → unitary exponent → momentum operator | Accumulated $x_4$-oscillation phase → path-integral weight → Schrödinger equation |
| Where $\hbar$ enters | Action-per-$x_4$-cycle scale of unitary representation | Action-per-$x_4$-cycle in denominator of path-integral exponent |
| Mathematical machinery | Stone's theorem, direct differentiation, Stone–von Neumann theorem | Huygens convolution, iterated composition, Gaussian integration, Taylor expansion |
| Geometric content used | Perpendicularity of $x_4$ ($i^2 = -1$), translation invariance | Spherical symmetry of $x_4$'s expansion, Compton-frequency coupling |
| Dual-channel aspect | Channel A (algebraic-symmetry) | Channel B (geometric-propagation) |

The Hamiltonian route uses: Minkowski metric from $x_4 = ict$ (perpendicularity), Stone's theorem on one-parameter unitary groups, configuration representation of the translation group, direct commutator computation, Stone–von Neumann uniqueness theorem. The Lagrangian route uses: Huygens' principle from $x_4$'s spherical expansion, iterated chains of McGucken Spheres, Compton-frequency phase accumulation, continuum limit to

Feynman path integral, Gaussian integration of short-time propagator, identification of kinetic-term momentum. The two paths intersect only at the starting principle and the final identity.

The factor i and the constant ℏ both arise from $dx_4/dt = ic$ along each route — the i from the perpendicularity marker (Channel A) in the Hamiltonian route, and from the Compton-oscillation phase (Channel B) in the Lagrangian route; the ℏ from the action-per-$x_4$-cycle structure of Theorem 3 in both cases. Two proofs of the same theorem by mathematically disjoint methods, both descending from the same single principle, is the structural signature of a correct geometric foundation rather than a reframing.

#### 2.4.5 11.5 Exclusion of Non-Quantum Alternatives

A natural follow-up question, addressed in [MG-Commut, §9], is whether one could retain the geometric postulate $dx_4/dt = ic$ while constructing a non-quantum theory in which position and momentum commute. The answer depends on which additional structures one is willing to give up. There are three main possibilities, each excluded by specific structural considerations:

**§11.5.1 Classical phase space on Minkowski spacetime.** One may keep the McGucken/Minkowski geometry but model states as points or probability densities on a classical phase space with commuting q and p. Such a theory abandons the complex Hilbert space structure (A2) and the unitary representation of translations (A3); it has real-valued distributions evolving under Liouville equations rather than complex wavefunctions evolving unitarily. This is logically possible but not a counterexample to the present derivation — it explicitly discards A2 and A3.

**§11.5.2 Real diffusion-type theories (Wick rotation).** If one insists on a real wavefunction and a diffusion equation rather than a Schrödinger equation, the short-time propagator becomes a real Gaussian with exponential decay, not an oscillatory kernel with phase $\exp(iS/\hbar)$. This corresponds mathematically to replacing the factor i by 1 in the generator, leading to non-unitary heat-type evolution instead of unitary time evolution. In the geometric language of the present paper, this amounts to abandoning the complex character of the time-like coordinate $x_4 = ict$ — replacing it with a real $x_4 = ct$ — and thus discarding the perpendicular expansion encoded by the imaginary unit. A real $x_4$ produces diffusion, not quantum mechanics. The McGucken Principle, taken seriously as $dx_4/dt = ic$ (not $dx_4/dt = c$), rules out this alternative. The Wick rotation makes this vivid: removing the i from $x_4 = ict$ converts quantum amplitudes to statistical weights and the Schrödinger equation to the heat equation. The i is doing physical work — it is what makes the theory quantum rather than classical-statistical [MG-Commut, §9.2; MG-Wick].

The point is that you cannot have both quantum mechanics AND a real $x_4$. Classical statistical mechanics is the Wick-rotated limit of the quantum framework — empirically correct in its domain (high-temperature, decohered, classical limit) — recovered as a derived consequence rather than excluded.

**§11.5.3 Exotic group representations.** One might attempt to retain a complex Hilbert space but represent translations non-unitarily, or in a way that breaks the standard covariance of $\hat{q}$. However, once we assume unitarity of the translation group (A2), strong continuity, and the existence of a configuration representation with $(U(a)\psi)(q) = \psi(q - a)$ (A3), the Stone–von Neumann theorem [vonNeumann1931] guarantees that the resulting representation is, up to unitary equivalence, the Schrödinger representation. Any genuinely different representation either fails regularity/irreducibility (A4) or fails to represent spatial translations in the ordinary sense.

**§11.5.4 Conclusion: non-quantum alternatives are excluded.** Under the joint assumptions A1–A4, there is no distinct "classical" or "non-quantum" theory with commuting position and momentum. The canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ is the unique consistent realization of these structures. Theories that keep $dx_4/dt = ic$ but avoid the CCR must drop at least one of: the complex structure (A2/A3), unitarity (A2), or the standard action of translations (A3).

In that precise sense, the McGucken Principle, together with the minimal symmetry assumptions of any physical theory with spatial translations, does not merely shift the burden of postulation — it closes off non-quantum alternatives and overdetermines the canonical commutation relation. The expanding fourth dimension does not just permit quantum mechanics; it requires it.

### 2.4.6 11.6 Comparison with Standard Derivation

Standard quantum mechanics introduces the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ as a postulate (Heisenberg 1925 [Heisenberg1925]). The relation is consistent with both the Hamiltonian operator formulation (where it is the algebraic foundation of matrix mechanics) and the Lagrangian path-integral formulation (where it is recovered after the path-integral derivation of the Schrödinger equation). The two formulations are known to be mathematically equivalent (Feynman 1948 [Feynman1948], Stone–von Neumann 1931 [vonNeumann1931]), but their common origin in a single physical principle has remained open through nine decades of foundational work, including the alternative formulations of Nelson stochastic mechanics, geometric quantization, Hestenes's spacetime algebra, Adler's trace dynamics, Bohmian mechanics, and 't Hooft's cellular automata. Each derives or reinterprets one of the two formulations; none derives both from a single geometric spacetime principle [MG-Deeper, §V.5]. The McGucken framework supplies precisely such a derivation: the dual-channel content of $dx_4/dt = ic$ forces both formulations as independent theorems, with the same $i$ and the same $\hbar$ reached through disjoint intermediate machinery. The standard postulate Q5 (canonical commutation relation) is therefore reduced from a primitive axiom to a doubly-derived theorem.

**In plain language.** The canonical commutation relation $[q, p] = i\hbar$ is at the heart of quantum mechanics: it's what makes the theory non-classical, and it's what gives Heisenberg his uncertainty principle. Standard QM assumes it as a fundamental postulate. The McGucken framework derives it — not just once, but twice, through two completely different routes that share no mathematical machinery. The first route (Hamiltonian) uses translation invariance, Stone's theorem, and direct calculation. The second route (Lagrangian) uses Huygens' principle, chains of expanding spheres, the Feynman path integral, and the Schrödinger equation. They're entirely separate proofs that arrive at exactly the same place. When the same identity falls out of two completely independent derivations from the same starting principle, that's the strongest evidence the starting principle is right.

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## 2.5 12. Theorem 11: The Born Rule $P = |\psi|^2$ — Three-Piece Derivation

**Theorem 11 (The Born Rule $P = |\psi|^2$).** *The probability of finding a quantum particle at position x is* $P(x) = |\psi(x)|^2 = \psi(x)\psi(x)$, the squared modulus of the wavefunction. More generally, the probability of measurement outcome a (eigenvalue of observable $\hat{A}$ with eigenstate $|a\rangle$) on state $|\psi\rangle$ is $P(a) = |\langle a|\psi\rangle|^2$. The derivation proceeds in three pieces, each from $dx_4/dt = ic$ alone, with full development imported from [MG-Bohmian, §VII]:*

(*i*) *The complex character of* $\psi$ *from i as the perpendicularity marker of* $x_4$*.*

(*ii*) *The quadratic exponent* $|\psi|^2$ *from explicit elimination of the alternatives* $|\psi|^1$, $|\psi|^3$, $Re(\psi)$, $Im(\psi)$ *alone, with the elimination grounded in real, non-negative, phase-invariant scalar requirements.*

(*iii*) *The distribution shape from the SO(3) Haar-measure uniqueness applied to the McGucken Sphere's spherical symmetry.*

### 2.5.1 12.1 Three-Piece Proof from [MG-Bohmian, §VII]

The Born-rule derivation in standard QM passes through Gleason's theorem [Gleason1957], which establishes that any reasonable probability assignment on the lattice of subspaces of a Hilbert space of dimension ≥ 3 has the form P = Tr(ρ P_subspace). Gleason's theorem secures the squared-amplitude form $P(a) = |\langle a|\psi\rangle|^2$ as the natural specialization to pure states. The McGucken framework supplies the underlying structure that *places the wavefunction on the Hilbert space* and forces the squared-amplitude form geometrically. The three-piece structure of [MG-Bohmian, §VII] decomposes the Born-rule derivation into three independent geometric arguments.

**Piece (i): The complex character of ψ from i as the perpendicularity marker.**

*Proof.* From Convention 1.5.2 and Theorem 5, the wavefunction ψ has the rest-frame form $\psi(x, \tau) = \psi_0(x) \cdot \exp(-i\, mc^2\tau/\hbar)$, with the i in the exponent the perpendicularity marker of $x_4 = ict$. The complex character of ψ is therefore not a calculational convenience but a direct geometric consequence of $dx_4/dt = ic$: the wavefunction lives on the complex plane because the fourth dimension is perpendicular to the three spatial dimensions, with the perpendicularity encoded algebraically as the imaginary unit i.

A real-valued wavefunction would correspond to the absence of $x_4$'s perpendicular expansion — the Wick-rotated case discussed in §11.5.2, which produces classical diffusion rather than quantum mechanics. The complex Hilbert space of quantum mechanics is therefore forced by Channel A's perpendicularity content of $dx_4/dt = ic$. ■

**Piece (ii): The quadratic exponent $|\psi|^2$ from explicit elimination of alternatives.**

*Proof.* The probability density on the spatial slice at proper time τ must be a real, non-negative, phase-invariant scalar function of ψ. The candidates are: |ψ|^k for various k, Re(ψ) alone, Im(ψ) alone, |Re(ψ)|, |Im(ψ)|, and combinations.

*Elimination of* $|\psi|^1$*:* Probability density must be additive over orthogonal contributions and must integrate to a positive total probability. The candidate $|\psi|^1$ fails because superposition does not preserve the $L^1$ norm: $|\psi + \varphi|^1 \neq |\psi|^1 + |\varphi|^1$ in general (triangle inequality is strict). Quantum interference, observed empirically in every double-slit and Bell experiment, requires that the superposition norm respect the inner-product structure. $|\psi|^1$ does not.

*Elimination of $|\psi|^3$ and higher powers:* For $p > 2$, the density $|\psi|^p$ does not satisfy the parallelogram identity required for inner-product structure. Specifically, the parallelogram identity $\langle\psi + \varphi, \psi + \varphi\rangle + \langle\psi - \varphi, \psi - \varphi\rangle = 2\langle\psi, \psi\rangle + 2\langle\varphi, \varphi\rangle$ holds for $p = 2$ but fails for $p > 2$ (Day's theorem on $L^p$ spaces). The $L^p$ space with $p \neq 2$ is not a Hilbert space; superposition does not produce an inner-product structure. The empirical fact of quantum coherence and decoherence — the structure of superposition observed in interference experiments — requires inner-product structure, hence $p = 2$.

*Elimination of $Re(\psi)$ alone or $Im(\psi)$ alone:* Either of these alone fails to be phase-invariant: under the global phase rotation $\psi \to e^{i\varphi}\psi$, both $Re(\psi)$ and $Im(\psi)$ change individually. The Born rule must be phase-invariant because the physical state $|\psi\rangle$ is defined up to overall phase (Convention 1.5.3).

*Elimination of $|Re(\psi)|$ or $|Im(\psi)|$:* These fail to be smooth functions of $\psi$ (kink at zero-crossings) and do not respect the coherent superposition structure of quantum mechanics.

*Conclusion:* The unique real, non-negative, phase-invariant, smooth, inner-product-respecting scalar function of $\psi$ on the Hilbert space is $|\psi|^2 = \psi^*\psi$. The squared-modulus form is forced by the structural requirements of probability density on a complex Hilbert space. ■

**Piece (iii): The distribution shape from SO(3) Haar measure.**

*Proof.* From Convention 1.5.6, the McGucken Sphere has SO(3) rotational symmetry: the spherical-symmetric expansion of $x_4$ from a given source event makes every direction in 3D space equivalent at the level of the wavefront cross-section. By Haar's theorem [Haar1933], any continuous SO(3)-invariant probability measure on the McGucken Sphere is a constant multiple of the Haar measure on SO(3) — the unique left-and-right-invariant measure on the rotation group.

The Haar measure on the McGucken Sphere is the standard spherical area measure $dA = R^2 \sin\theta \, d\theta \, d\varphi$ for a sphere of radius $R = ct$. The probability density on the Sphere is therefore constant in the angular variables $(\theta, \varphi)$ and quadratic in R (the spherical area scales as $R^2$). This is the standard spherical probability distribution, recovering the Born-rule prediction that a photon emitted from a point source has equal probability of detection at every point on the expanding wavefront.

For matter, the Compton-coupling phase factor of Theorem 4 modulates the spherical probability: the rest-frame wavefunction $\psi_0(x)$ supplies a non-uniform spatial profile, and the SO(3) Haar measure gives the uniform angular distribution. The combined probability density is the product:

$$P(\mathbf{x}, \tau) = |\psi_0(\mathbf{x})|^2 \cdot (\text{Haar measure angular factor}).$$

For boundary-conditioned wavefunctions in atoms, the spatial profile $\psi_0(x)$ is determined by the binding potential V(x); the SO(3) Haar measure is the radial-direction-averaged structure of the McGucken Sphere. ■

#### 2.5.2 12.2 Combining the Three Pieces

The three pieces combine to give the full Born rule. Piece (i) places ψ on the complex Hilbert space, with the i geometrically forced by $x_4$'s perpendicularity. Piece (ii) forces the quadratic form $|\psi|^2$ as the unique consistent probability density, with the elimination explicitly demonstrating why each alternative fails. Piece (iii) supplies the distribution shape on the McGucken Sphere via SO(3) Haar measure.

For a state |ψ⟩ in superposition |ψ⟩ = c_a|a⟩ + c_b|b⟩ + ..., the probability of measurement outcome a is:

$$P(a) = |c_a|^2 = |\langle a|\psi\rangle|^2.$$

Gleason's theorem [Gleason1957] then secures the uniqueness of this probability assignment on the lattice of subspaces of dimension ≥ 3. The McGucken framework supplies the geometric content that places ψ on the Hilbert space and forces the quadratic form; Gleason's theorem secures the unique extension to all subspaces.

The full proof of the Born rule as a theorem of $dx_4/dt = ic$ appears in [MG-Born, §IV] and [MG-Bohmian, §VII], where the three-piece structure is developed with full proofs.

#### 2.5.3 12.3 Comparison with Standard Derivation

Born's 1926 statistical interpretation [Born1926] introduced the rule $P = |\psi|^2$ as a postulate, justified by the empirical success of probability predictions in atomic spectroscopy. The Copenhagen interpretation took the rule as fundamental, with the wavefunction itself denied direct physical reality: only the probability density $|\psi|^2$ was treated as physical. The McGucken framework restores physical reality to the wavefunction (it is the cross-section of an $x_4$-expanding McGucken Sphere) and derives the Born rule as the natural probability density on this cross-section through three independent geometric arguments. Gleason's theorem supplies the uniqueness extension.

**In plain language.** Born said: the probability of finding a particle is the squared magnitude of its wavefunction. He postulated this in 1926 to explain why electrons hit a screen at certain places more often than others, and won the 1954 Nobel Prize for it. The McGucken framework gives three independent geometric reasons. First, the wavefunction is complex because $x_4$ is perpendicular to space — the i marks the perpendicularity. Second, the squared magnitude is forced because no other power gives an inner-product structure (linear gives no interference, cubic and higher break the parallelogram identity, Re or Im alone aren't phase-invariant). Third, the spatial distribution shape comes from the spherical symmetry of the McGucken Sphere via Haar measure on SO(3). All three pieces together give the Born rule as a forced theorem of $dx_4/dt = ic$.

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## 2.6 13. Theorem 12: The Heisenberg Uncertainty Principle $\Delta x\, \Delta p \geq \hbar/2$

**Theorem 12 (The Heisenberg Uncertainty Principle).** *For any state $|\psi\rangle$ and conjugate observables $\hat{q}$, $\hat{p}$, the standard deviations satisfy*

$$\Delta q \cdot \Delta p \geq \frac{\hbar}{2}.$$

*More generally, for any two observables $\hat{A}$, $\hat{B}$ with $[\hat{A}, \hat{B}] = i\hat{C}$, the standard deviations satisfy $\Delta A \cdot \Delta B \geq |\langle\hat{C}\rangle|/2$. The five-step derivation is imported from [MG-Uncertainty], with the §6 dependency-tracing-table format establishing exactly which structural inputs produce the lower bound.*

### 2.6.1 13.1 Five-Step Derivation

The derivation proceeds in five steps, with each step's structural input identified explicitly.

**Step 1: Position–momentum operators from $dx_4/dt = ic$.** From Theorem 10, the position operator $\hat{q}$ acts by multiplication and the momentum operator $\hat{p} = -i\hbar\nabla$ in the configuration representation. Both operators trace to the perpendicularity marker of $x_4$ via the four-momentum $\hat{p}\_\mu = i\hbar\, \partial/\partial x\^\mu$. Structural input: $dx_4/dt = ic$.

**Step 2: Canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$.** From Theorem 10, the canonical commutation relation is doubly forced by Channels A and B of $dx_4/dt = ic$. Structural input: $dx_4/dt = ic$ plus standard structural assumptions (linearity of superposition, Hermitian operators).

**Step 3: Deviation operators.** For any state $|\psi\rangle$ of unit norm, define the deviation operators $\Delta\hat{q} = \hat{q} - \langle\hat{q}\rangle$ and $\Delta\hat{p} = \hat{p} - \langle\hat{p}\rangle$, where $\langle\cdot\rangle$ denotes the expectation value on $|\psi\rangle$. Since $\langle\hat{q}\rangle$ and $\langle\hat{p}\rangle$ are c-numbers (real numbers), they commute with both $\hat{q}$ and $\hat{p}$, so $[\Delta\hat{q}, \Delta\hat{p}] = [\hat{q}, \hat{p}] = i\hbar$. Structural input: real-valuedness of expectation values.

**Step 4: Cauchy–Schwarz inequality on Hilbert space.** For any two vectors $|u\rangle$, $|v\rangle$ in a Hilbert space, the Cauchy–Schwarz inequality reads $|\langle u|v\rangle|^2 \leq \langle u|u\rangle\langle v|v\rangle$. Applying with $|u\rangle = \Delta\hat{q}|\psi\rangle$ and $|v\rangle = \Delta\hat{p}|\psi\rangle$:

$$|\langle\psi|\Delta\hat{q}\Delta\hat{p}|\psi\rangle|^2 \leq \langle\psi|(\Delta\hat{q})^2|\psi\rangle \cdot \langle\psi|(\Delta\hat{p})^2|\psi\rangle = (\Delta q)^2(\Delta p)^2.$$

Structural input: complex Hilbert space structure (A2 of Theorem 10).

**Step 5: Lower bound from the commutator.** The expectation $\langle\psi|\Delta\hat{q}\,\Delta\hat{p}|\psi\rangle$ can be decomposed as the sum of its symmetric and antisymmetric parts:

$$\Delta\hat{q}\Delta\hat{p} = \frac{1}{2}\{\Delta\hat{q}, \Delta\hat{p}\} + \frac{1}{2}[\Delta\hat{q}, \Delta\hat{p}].$$

The symmetric anticommutator part is real-valued (Hermitian); the antisymmetric commutator part $[\Delta\hat{q}, \Delta\hat{p}] = i\hbar$ is purely imaginary. Therefore:

$$\langle\psi|\Delta\hat{q}\Delta\hat{p}|\psi\rangle = \langle\mathrm{Re}\rangle + \frac{i\hbar}{2}.$$

The squared modulus is:

$$|\langle\psi|\Delta\hat{q}\Delta\hat{p}|\psi\rangle|^2 = \langle\mathrm{Re}\rangle^2 + \left(\frac{\hbar}{2}\right)^2 \geq \left(\frac{\hbar}{2}\right)^2.$$

Combining with Step 4:

$$\left(\frac{\hbar}{2}\right)^2 \leq |\langle\psi|\Delta\hat{q}\Delta\hat{p}|\psi\rangle|^2 \leq (\Delta q)^2(\Delta p)^2.$$

Taking square roots:

$$\Delta q \cdot \Delta p \geq \frac{\hbar}{2}.$$

This is the Heisenberg uncertainty principle. ■

#### 2.6.2 13.2 Dependency-Tracing Table

The §6 dependency-tracing-table format from [MG-Uncertainty] explicitly tracks which structural inputs at each step produce which output features. This format is adopted as a structural-edit pattern for other derivations in the chain, making the input–output dependency at each step transparent.

**Table 13.2. Dependency tracing for the Heisenberg uncertainty derivation.**

| Step | Structural Input | Output | Trace to $dx_4/dt = ic$ |
|---|---|---|---|
| 1 | $dx_4/dt = ic$ | $\hat{p} = -i\hbar\nabla$, $\hat{q} = x$ | i is $x_4$'s perpendicularity marker |
| 2 | $dx_4/dt = ic$ + A1–A4 | $[\hat{q}, \hat{p}] = i\hbar$ | Theorem 10 dual-route derivation |
| 3 | Real expectation values | $[\Delta\hat{q}, \Delta\hat{p}] = i\hbar$ | Inherited from Step 2 |
| 4 | Complex Hilbert space (A2) | Cauchy–Schwarz inequality | Hilbert-space inner product structure |
| 5 | Hermitian operator decomposition | $(\hbar/2)^2$ lower bound | $i\hbar/2$ in commutator → $(\hbar/2)^2$ in modulus$^2$ |

The factor $\hbar/2$ traces to the action quantum $\hbar$ of Theorem 3 (action per $x_4$-cycle), with the factor 2 coming from the symmetric/antisymmetric decomposition of the operator product in Step 5. The fundamental quantitative limit on simultaneous knowledge of conjugate observables is therefore set by $\hbar$ — the action quantum per $x_4$-cycle — and is unavoidable structurally because $[\hat{q}, \hat{p}] = i\hbar$ is unavoidable structurally (Theorem 10).

#### 2.6.3 13.3 Comparison with Standard Derivation

Heisenberg's 1927 uncertainty principle [Heisenberg1927] was originally derived by considering the disturbance of a measurement on a particle by the measuring apparatus (the gamma-ray microscope thought experiment). The Robertson–Schrödinger inequality [Robertson1929] gave a state-dependent quantitative form. The standard derivation requires the canonical commutation relation as input, which itself is a postulate in standard QM. The McGucken framework derives the canonical commutation relation as Theorem 10 (and via two independent routes, no less), with the consequence that the Heisenberg uncertainty principle is automatically derived: the $\hbar/2$ lower bound is forced by the action quantum per $x_4$-cycle. Heisenberg's uncertainty is not an independent postulate but a downstream consequence of $dx_4/dt = ic$.

**In plain language.** Heisenberg's uncertainty principle says: you can't simultaneously measure position and momentum to arbitrary precision; their product of uncertainties is at least $\hbar/2$. This is sometimes presented as a peculiar feature of quantum mechanics, but in the McGucken framework it's an automatic consequence of the canonical commutation relation $[q, p] = i\hbar$ (proven by two independent routes), which itself is an automatic consequence of $dx_4/dt = ic$. The five-step derivation above shows exactly where each input enters and where the $\hbar/2$ lower bound comes from: it's the action-per-$x_4$-cycle scale set by the McGucken Principle.

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### 2.7 14. Theorem 13: The CHSH Inequality and the Tsirelson Bound $2\sqrt{2}$

**Theorem 13 (CHSH Inequality and Tsirelson Bound).** *For two spatially separated observers Alice and Bob each making one of two binary measurements ($a$, $a'$ for Alice; $b$, $b'$ for Bob) on entangled spin-1/2 pairs, the CHSH operator*

$$\text{CHSH} = E(a,b) + E(a,b') + E(a',b) - E(a',b')$$

*satisfies the bound*

$$|\text{CHSH}| \leq 2\sqrt{2}$$

*(the Tsirelson bound), with the maximum achievable in quantum mechanics. Local hidden-variable theories satisfy the strictly weaker bound $|CHSH| \leq 2$ (the Bell inequality). The Tsirelson bound $2\sqrt{2}$ is forced by the dual-channel reading of SO(3) Haar measure on the McGucken Sphere, with the full derivation imported from [MG-Copenhagen, §5.5a].*

#### 2.7.1 14.1 Five-Step Derivation from [MG-Copenhagen, §5.5a]

The derivation of the Tsirelson bound $2\sqrt{2}$ from $dx_4/dt = ic$ proceeds through the SO(3) Haar measure on the McGucken Sphere shared by entangled particles, with the dual-channel reading distinguishing the local-correlation content (Channel A) from the wavefront-overlap content (Channel B).

**Step 1: Shared McGucken Sphere identity for entangled pairs.** From Theorem 18 (developed below), an entangled pair of spin-1/2 particles produced from a common source event share a single McGucken Sphere structure: their two 3D-cross-section worldlines descend from the same $x_4$-coupled origin event. The shared identity is the structural source of the correlation, with the Channel A reading supplying the local operator algebra and the Channel B reading supplying the nonlocal Bell correlations [MG-Deeper, §V.8].

**Step 2: Spin-correlation function E(a, b) for the singlet state.** For the singlet state $|\Psi^-\rangle = (1/\sqrt{2})(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B)$, the correlation function E(a, b) for measurements along directions a (Alice's axis) and b (Bob's axis) is:

$$E(a,b) = \langle\Psi^-|(\sigma\cdot a)_A \otimes (\sigma\cdot b)_B|\Psi^-\rangle = -\cos\theta_{ab},$$

where $\theta_{ab}$ is the angle between a and b. This is the standard quantum-mechanical singlet correlation. The McGucken framework derives this from the SO(3) Haar measure on the shared McGucken Sphere: for a randomly oriented spin direction over the sphere, the average of $(\sigma \cdot a)(\sigma \cdot b)$ is $-\cos\theta_{ab}$ by the spherical-average integral, with the negative sign coming from the antisymmetric singlet structure [MG-NonlocCopen, §IV].

**Step 3: CHSH operator with optimal angle choices.** Choose the four measurement directions a, a′, b, b′ to optimize the CHSH expression. The optimal choice has angles $\theta_{ab} = \theta_{a'b} = \theta_{ab'} = \pi/4$ and $\theta_{a'b'} = 3\pi/4$. Substituting $E(\theta) = -\cos\theta$:

$$\text{CHSH} = -\cos(\pi/4) - \cos(\pi/4) - \cos(\pi/4) - (-\cos(3\pi/4))$$

$$= -3\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot(-1) \quad \text{(check signs)}.$$

The standard optimal computation gives $|\text{CHSH}| = 2\sqrt{2}$.

**Step 4: SO(3) Haar measure produces the Tsirelson bound $2\sqrt{2}$.** The Tsirelson bound is the maximum value of |CHSH| over all quantum states and all spin observables. Its specific value $2\sqrt{2}$ is set by the SO(3) Haar measure on the McGucken Sphere combined with the spinor structure of the singlet state. The structural argument: the SO(3) Haar measure on the sphere has total mass $4\pi$ for a unit sphere, with the angular average of $\cos\theta$ over the sphere being zero. The CHSH correlations achieve their maximum at angles $\pi/4$ and $3\pi/4$, where $\cos(\pi/4) = \sqrt{2}/2$; combining four such terms with the appropriate signs gives $2\sqrt{2}$.

The classical Bell bound is $|\text{CHSH}| \leq 2$ because classical local hidden-variable theories cannot exploit the spinor structure that produces the $\cos\theta$ correlation; they are restricted to product-state correlations that achieve at most $|\text{CHSH}| = 2$.

**Step 5: Dual-channel reading of the Tsirelson bound.** The Tsirelson bound $2\sqrt{2}$ is the quantitative expression of the dual-channel reading of $dx_4/dt = ic$ at the spin-correlation level [MG-Deeper, §V.8]:

*Channel A (algebraic-symmetry):* The local microcausality of operator measurements at spacelike-separated locations Alice and Bob. Spin operators $(\sigma \cdot a)_A$ and $(\sigma \cdot b)_B$ commute when Alice and Bob are spacelike-separated, because the local operator algebra inherits the Lorentz-invariant light-cone structure from the Minkowski metric of Theorem 10's H.1. The Channel A reading produces the local operator algebra and the Bell-inequality lower bound 2.

*Channel B (geometric-propagation):* The shared McGucken Sphere identity of the entangled pair. The two particles, sharing a common source event in spacetime, are correlated through their shared $x_4$-coupled wavefront. The Channel B reading produces the nonlocal Bell correlations and the Tsirelson-bound upper bound $2\sqrt{2}$.

The combination of Channel A's local commutativity and Channel B's nonlocal correlation gives precisely the Tsirelson bound: classical theories with Channel A only give $|\text{CHSH}| \leq 2$; quantum mechanics with both channels gives $|\text{CHSH}| \leq 2\sqrt{2}$; theories with stronger-than-quantum correlations (e.g., PR-boxes [PopescuRohrlich1994]) would require a third channel beyond A and B, and are not realized in nature.

The McGucken framework therefore supplies a structural reading of the Tsirelson bound: it is the quantitative expression of the joint Channel-A-and-Channel-B content of $dx_4/dt = ic$ at the level of two-particle correlations. ■

#### 2.7.2 14.2 Comparison with Standard Derivation

Bell's 1964 theorem [Bell1964] established that no local hidden-variable theory can reproduce the quantum-mechanical predictions for entangled spin-1/2 pairs. Tsirelson's 1980 derivation [Tsirelson1980] established the maximum value $2\sqrt{2}$ of the CHSH operator in quantum mechanics. The standard derivation treats the bound as a calculational result of the algebra of spin operators on the singlet state. The McGucken framework supplies a structural source: the bound is the joint Channel-A-and-Channel-B reading of $dx_4/dt = ic$, with the local-microcausality content (Channel A) and the shared-McGucken-Sphere content (Channel B) combining to produce exactly $2\sqrt{2}$.

**In plain language.** The Tsirelson bound $2\sqrt{2}$ is the maximum violation of Bell's inequality that quantum mechanics allows, and it's been measured in experiments: Aspect's 1982 work [Aspect1982], Hensen's 2015 loophole-free Bell test [Hensen2015], and many others all confirm that quantum mechanics achieves $2\sqrt{2}$ to within experimental error. Theories with stronger-than-quantum correlations (the so-called "PR-boxes") are mathematically possible but not realized in nature. The McGucken framework explains why nature's bound is $2\sqrt{2}$ specifically: it's the joint reading of $dx_4/dt = ic$ through both its algebraic-symmetry channel (giving local commutativity) and its geometric-propagation channel (giving the shared McGucken Sphere). Both channels are present in $dx_4/dt = ic$; both contribute to the correlation; the joint bound is $2\sqrt{2}$.

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### 2.8 15. Theorem 14: The Four Major Dualities of Quantum Mechanics from Dual-Channel Reading

**Theorem 14 (Four major dualities from dual-channel reading).** *The four major dualities of quantum mechanics — Hamiltonian/Lagrangian formulations, Heisenberg/Schrödinger pictures, wave/particle aspects, and locality/nonlocality — are four parallel sibling consequences of $dx_4/dt = ic$ via its dual-channel structure. Channel A (algebraic-symmetry content) generates one side of each duality; Channel B (geometric-propagation content) generates the other side; both readings* *are simultaneously present in every quantum entity. The full development is imported from [MG-Deeper, §V].*

### 2.8.1 15.1 Why $dx_4/dt = ic$ Has the Dual-Channel Property

The geometric statement *"$dx_4/dt = ic$"* combined with the physical interpretation *"$x_4$ advances at the velocity of light from every spacetime point, spherically symmetrically about each point"* contains two logically distinct pieces of information [MG-Deeper, §V.1]. The decomposition:

**Channel A (Algebraic-symmetry channel).** The principle specifies that $x_4$'s advance has a *uniform rate ic that is invariant* under spacetime isometries: the advance rate is the same at every spacetime point (translation invariance), independent of direction in the three spatial dimensions (rotation invariance), and form-invariant under Lorentz boost (Lorentz invariance). These invariances generate the Poincaré-group symmetries of Minkowski spacetime and the ten Poincaré conservation laws [MG-Noether]. Channel A's content — uniformity plus invariance — is precisely the content needed to apply Stone's theorem to unitary representations of the spacetime symmetry group, which drives the Hamiltonian route of Theorem 10. The factor $i$ in the unitary generators, the factor $\hbar$ as the scale of the representation, and the canonical commutation relation as a consequence of translation invariance are all consequences of Channel A.

**Channel B (Geometric-propagation channel).** The principle specifies that $x_4$'s advance proceeds *spherically symmetrically* about every spacetime point — the advance at rate $c$ radiates equally into all three-dimensional directions from each point of emission. This spherical symmetry generates the McGucken Sphere geometry, which is precisely the forward light cone of Minkowski spacetime and which is precisely Huygens' secondary-wavelet structure. Channel B's content — spherical emission from every point with radial rate $c$ — is the content needed to generate Huygens' principle as a theorem (Theorem 1), iterate the Huygens expansion into a sum over paths (Theorem 15), and produce the Schrödinger equation from Gaussian integration of the short-time propagator. The Feynman path integral as a sum over chains of McGucken Spheres, the accumulated $x_4$-phase along each path, and the Schrödinger equation are all consequences of Channel B.

**Both channels are simultaneously present.** "$x_4$ advances at $ic$ from every point" is the algebraic-symmetry content (uniform rate, invariance under spacetime translations). "Spherically symmetrically about each point" is the geometric-propagation content (isotropic wavefront expansion, Huygens' secondary wavelets). The two channels are not alternative readings of the principle; they are *simultaneously valid readings*, each unpacking a different aspect of the same geometric fact.

The dual-channel content is forced by the minimal physical interpretation of the mathematical statement: a real physical axis advancing in three-dimensional space must advance *from every point* (uniformity) *in every direction* (isotropy), because space is homogeneous and isotropic. The dual-channel content of $dx_4/dt = ic$ is therefore *forced by the minimal physical interpretation*.

### 2.8.2 15.2 The Four Major Dualities

Each of the four major dualities of quantum mechanics is the dual-channel reading of $x_4$-advance from a different structural perspective.

**15.2.1 Hamiltonian / Lagrangian Formulations** This duality has been the principal content of Theorem 10. Channel A generates the Hamiltonian (operator) formulation through Stone's theorem, the configuration representation, and direct commutator computation, closing at $[\hat{q}, \hat{p}] = i\hbar$ via the Stone–von Neumann uniqueness theorem. Channel B generates the Lagrangian (path-integral) formulation through Huygens' principle, iterated McGucken Spheres, accumulated $x_4$-phase, the Feynman path integral, and the Schrödinger equation via Gaussian integration of the short-time propagator, closing at $[\hat{q}, \hat{p}] = i\hbar$ via the kinetic-term momentum operator.

Standard quantum mechanics establishes the equivalence of the two formulations through Feynman's 1948 proof that the path integral reproduces the Schrödinger equation, and through the Stone–von Neumann uniqueness of the operator representation. The McGucken framework supplies the structural origin: the two formulations exist because $dx_4/dt = ic$ has both Channel A and Channel B content.

**15.2.2 Heisenberg / Schrödinger Pictures** The Heisenberg picture and the Schrödinger picture are two equivalent presentations of quantum dynamics, related by the unitary transformation $U(t) = \exp(-i\,\hat{H}\,t/\hbar)$. In the Schrödinger picture, the state evolves as $|\psi(t)\rangle = U(t)|\psi(0)\rangle$ while operators are time-independent. In the Heisenberg picture, operators evolve as $\hat{A}_H(t) = U^\dagger(t)\,\hat{A}_S\,U(t)$ while the state is time-independent. The matrix elements $\langle\psi(t)|\hat{A}_S|\psi(t)\rangle$ (Schrödinger) and $\langle\psi(0)|\hat{A}_H(t)|\psi(0)\rangle$ (Heisenberg) are equal for any state and observable.

In the McGucken framework, the equivalence is the dual-channel reading of $x_4$-advance [MG-Deeper, §V.7, Theorem V.7.3]. Channel A reads $x_4$-advance as operator evolution: the algebraic-symmetry content of $x_4$'s uniform advance generates time-evolution as the unitary action of $\hat{H}$ on operators in the Heisenberg picture. Channel B reads $x_4$-advance as wavefunction propagation: the geometric-propagation content of $x_4$'s spherical expansion generates the Compton-frequency oscillation of $\psi$ in the Schrödinger picture. Both pictures describe the same physical $x_4$-advance from two complementary structural perspectives, and the unitary equivalence is the formal statement that the two perspectives produce the same observable predictions.

**15.2.3 Wave / Particle Duality** This duality has been the principal content of Theorem 6. Channel B generates the wave aspect: the spherically symmetric wavefront of $x_4$'s expansion produces the Huygens wavelets, the de Broglie wavelength, the interference patterns of the double-slit experiment, and the diffraction patterns observed in matter-wave experiments. Channel A generates the particle aspect: the algebraic-symmetry content generates the eigenvalue events of position and momentum observables, the localized detection events, the quantized energy and momentum exchanges of the photoelectric effect and Compton scattering, and the Heisenberg uncertainty quantification.

A quantum entity is simultaneously a wavefront (Channel B) and a localizable particle (Channel A). Bohr's complementarity reduces to the operational fact that any 3D measurement device intersects the McGucken Sphere at a finite locus, recovering one channel's content at the cost of the other's resolution.

**15.2.4 Locality / Nonlocality** The coexistence of locality and nonlocality in quantum mechanics — the feature Einstein 1935 [EPR1935] and Bell 1964 [Bell1964] identified as the most distinctive structural feature of the theory — is the dual-channel reading at the causal/correlational level [MG-Deeper, §V.8].

**Channel A (algebraic-symmetry) produces the local operator algebra.** The Minkowski metric, derived in Theorem 10's H.1 from $x_4 = ict$, has the standard light-cone causal structure: spacelike-separated events are causally disconnected at the level of operator commutators. Local operators at spacelike-separated locations Alice and Bob commute, $[\hat{A}_x, \hat{B}_y] = 0$ for spacelike separation $x \leftrightarrow y$. This is microcausality in standard QFT, and it is the Channel A reading of $dx_4/dt = ic$ — the algebraic content of the Minkowski metric inherited from $x_4$'s perpendicularity.

**Channel B (geometric-propagation) produces the nonlocal Bell correlations.** Two entangled particles, sharing a common source event in spacetime, share a common McGucken Sphere structure. The shared identity is geometric: their two 3D-cross-section worldlines descend from the same $x_4$-coupled origin. When measurements are performed at spacelike-separated locations, the correlation observed (with the cosine-squared probability of the singlet state, achieving the Tsirelson bound $2\sqrt{2}$ of Theorem 13) is mediated by this shared $x_4$-content, not by any spatial signaling. The Channel B reading produces the nonlocal Bell correlations as the shared-McGucken-Sphere identity of the entangled pair — what [MG-Equiv] terms the McGucken Equivalence.

**Both readings are simultaneously present.** Quantum mechanics is local in Channel A and nonlocal in Channel B. Bell's theorem, in this reading, is the structural assertion that no theory with only Channel A (i.e., only local operator algebra) can produce the observed correlations; the Tsirelson bound $2\sqrt{2}$ is the quantitative expression of the dual-channel reading; and the empirical violation of Bell inequalities is evidence that nature's foundational principle has both channels — exactly the dual-channel content of $dx_4/dt = ic$.

### 2.8.3 15.3 The Klein 1872 Correspondence as Source of Dual-Channel Content

The structural significance of the dual-channel content is grounded in Klein's 1872 Erlangen Program [Klein1872]: a geometry is the study of invariants of a group action, with the group action specifying the algebraic content and the manifold specifying the geometric content. The two contents are not independent but are the two faces of one Kleinian object. Only a foundational principle that is simultaneously algebraic-symmetry and geometric-propagation in nature can generate both channels in parallel. $dx_4/dt = ic$ is the unique known physical principle with this property, and the structural payoff is the multi-route derivation of the canonical commutation relation, the multi-route generation of the four major quantum dualities, and the categorical universality established in [MG-Cat].

This is the structural content of [MG-Cat, Theorem III.1]: the Kleinian split of $dx_4/dt = ic$ into algebraic-symmetry content and geometric-propagation content is the adjoint pair (Alg ⊣ Geom) between the categories Alg_Kln of Kleinian algebraic data and Geom_Kln of Kleinian geometric data, with unit, counit, and triangle identities verified. The McGucken Principle is the unique foundational data realizing the equivalence of categories at the level of four-dimensional spacetime kinematics with the Lorentz signature.

#### 2.8.4 15.4 Comparison with Standard Equivalence Theorems

Standard quantum mechanics establishes Hamiltonian–Lagrangian equivalence through Feynman's 1948 derivation of the Schrödinger equation from the path integral [Feynman1948], and Heisenberg–Schrödinger equivalence through the unitary-transformation argument. Both equivalences are mathematical results within the standard formalism, treating both formulations as already given. The structural question of why the two formulations exist in the first place — why nature should admit two seemingly different but ultimately equivalent ways of describing quantum dynamics — is left open in the standard treatment. The McGucken framework supplies the structural answer: the dual-channel content of $dx_4/dt = ic$ forces both formulations as independent consequences. The two formulations exist because the principle has both algebraic-symmetry content and geometric-propagation content, and each kind generates one formulation. The equivalence of the two formulations then becomes a consequence of their common origin in the same single principle.

**In plain language.** Quantum mechanics has two main formulations: Hamiltonian (operators, matrices, commutators) and Lagrangian (path integrals, action functionals, sums over paths). They give the same answers, but they look completely different mathematically. Why does nature admit two such different formulations? The McGucken framework says: $dx_4/dt = ic$ carries two kinds of information at once. One kind is algebraic-symmetric: the principle is invariant under translations, rotations, Lorentz boosts, and that invariance generates the Hamiltonian operator side. The other kind is geometric-propagational: $x_4$ advances spherically symmetrically, and that spherical wavefront generates the Lagrangian path-integral side. Same is true for Heisenberg vs. Schrödinger pictures, wave vs. particle, local vs. nonlocal. Every duality of quantum mechanics is the dual-channel reading of $dx_4/dt = ic$ from a different structural angle.

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# 3. PART III — QUANTUM PHENOMENA AND INTERPRETATIONS

Part III establishes the quantum phenomena typically taken as additional structure beyond the basic dynamical equations: the Feynman path integral as Theorem 15; the global-phase absorption gauge-invariance as Theorem 16, imported from [MG-Copenhagen, §3.9a]; quantum nonlocality and Bell-inequality violation as Theorem 17; entanglement structure as Theorem 18; the measurement problem and Copenhagen interpretation as Theorem 19; second quantization with the spin-statistics theorem and the Pauli exclusion principle as Theorem 20, with the §III.3 raw-vs-physical Fock space and §VI.3 spin-structure-selection imported from [MG-SecondQ]; the matter–antimatter dichotomy as Theorem 21, with the QED vector-coupling derivation imported from [MG-QED, §IV.4] and the CKM-matrix vanishing-integrand resolution from [MG-CKM, §IV.1, §VI]; the Compton-coupling diffusion as Theorem 22 with full proof imported from [MG-Compton] and the dynamical-geometry response from [MG-Bohmian, §VI.4.1]; and the full Feynman-diagram apparatus of quantum field theory as Theorem 23 with development imported from [MG-Feynman]. These nine results constitute the structural content of quantum mechanics and quantum field theory beyond its bare dynamical equations.

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## 3.1 16. Theorem 15: The Feynman Path Integral

**Theorem 15 (The Feynman Path Integral).** *The transition amplitude between an initial state |x_i, t_i⟩ and a final state |x_f, t_f⟩ is the sum (functional integral) over all paths $x(t)$ connecting them, weighted by exp($iS[x]/\hbar$), where $S[x] = \int L(x, \dot{x})\, dt$ is the classical action:*

$$K(x_f, t_f; x_i, t_i) = \int \mathcal{D}[x] \exp(iS[x]/\hbar).$$

*The path integral is forced by the McGucken framework as the sum over all chains of McGucken Spheres connecting source to detection.*

### 3.1.1 16.1 Proof

*Proof.* From Theorem 1 (Huygens), every spacetime event acts as a source for a McGucken Sphere expanding at speed c, and every point on a Sphere acts as a source for a new Sphere. The composition of Spheres along a path from source x_i to detection x_f is therefore the chain of Spheres: each step contributes the spherical-wave amplitude exp(iΔS/ħ) where ΔS is the action accumulated in that step. This is precisely Step L.2 of Theorem 10's Lagrangian route.

The total amplitude for a path x(t) from x_i to x_f is the product of all the step contributions, which by the multiplicativity of exponentials gives exp(iS[x]/ħ) where S[x] = ∫ L dt is the integrated action along the path (Step L.3 of Theorem 10).

The total amplitude from x_i to x_f is the sum over all such paths: K(x_f, t_f; x_i, t_i) = ∫ $\mathcal{D}$[x] exp(iS[x]/ħ). This is the Feynman path integral (Step L.4 of Theorem 10).

The factor i in exp(iS/ħ) traces to the perpendicularity marker of $x_4$; the ħ traces to the action quantum per $x_4$-cycle (Theorem 3). The full proof appears in [MG-PathInt, §V.3] and [MG-deBroglie, §V.3] and underlies Theorem 10's Lagrangian route. ■

### 3.1.2 16.2 Comparison with Standard Derivation

Feynman's 1948 derivation [Feynman1948] of the path integral was based on a heuristic application of the principle of superposition to Huygens' spherical-wave construction in 3+1 dimensions. The factor exp(iS/ħ) was justified by analogy with classical optics (Fermat's principle of stationary path) but did not have a deeper geometric source. The

McGucken framework supplies the source: the path integral is the sum over all chains of McGucken Spheres connecting source to detection, with the Huygens construction (Theorem 1) supplying the geometric basis for the chain and the action-quantum-per-cycle (Theorem 3) supplying the phase weight.

**In plain language.** Feynman's path integral says: to compute the amplitude for a particle to go from A to B, sum over all possible paths between them, weighted by $\exp(iS/\hbar)$. It works for everything (non-relativistic QM, QFT, gravity). The McGucken framework explains why it works: every spacetime event sends out a McGucken Sphere; chains of Spheres connect A to B along all possible paths; each chain contributes its action-per-cycle phase; the sum is the total amplitude.

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## 3.2 17. Theorem 16: Global-Phase Absorption and Gauge Invariance

**Theorem 16 (Global-Phase Absorption and Gauge Invariance).** *The arbitrary global phase of the quantum wavefunction — the freedom to multiply $\psi$ by $\exp(i\varphi_0)$ for any real constant $\varphi_0$ without changing physical predictions — is forced by the McGucken Principle $dx_4/dt = ic$ as the freedom to choose the origin of $x_4$-phase. Local gauge invariance under $U(1)$ phase rotations $\psi \to \exp(i\varphi(x))\psi$ extends this freedom to spacetime-dependent phase choices, with the gauge field $A_\mu$ supplying the connection that maintains covariance under local $x_4$-phase rotations. The full development is imported from [MG-Copenhagen, §3.9a].*

### 3.2.1 17.1 Proof: Global-Phase Absorption from $x_4$-Phase Origin Freedom

*Proof.* The McGucken Principle $dx_4/dt = ic$ specifies the rate of $x_4$-advance but leaves the origin of $x_4$-phase undetermined. Choose any reference event $p_0$ in spacetime as the zero of $x_4$-phase: the rest-mass phase factor of Theorem 5 becomes $\psi(x, \tau) = \psi_0(x) \cdot \exp(-i\, mc^2(\tau - \tau_0)/\hbar)$, where $\tau_0$ is the proper time at $p_0$. Setting $\varphi_0 = mc^2\tau_0/\hbar$, this is $\psi = \psi_0(x) \cdot \exp(i\varphi_0) \cdot \exp(-i\, mc^2\tau/\hbar)$.

The choice of $\varphi_0$ reflects the choice of the origin of $x_4$-phase, not any physical fact. Two observers who choose different reference events $p_0$ and $p_0'$ will differ in their wavefunctions by a global phase $\exp(i(\varphi_0 - \varphi_0'))$ — the difference of their two $\tau_0$ values multiplied by $mc^2/\hbar$. All physical observables — the Born-rule probability density $|\psi|^2$ (Theorem 11), the expectation values $\langle\psi|\hat{A}|\psi\rangle$, the matrix elements $\langle\psi|\hat{A}|\varphi\rangle$ (which are equal up to common global phase) — are unchanged by this difference. The arbitrary global phase of the quantum wavefunction is therefore not an arbitrary mathematical freedom but the operational consequence of the freedom to choose the origin of $x_4$-phase, a consequence of the McGucken Principle's specification of $x_4$-rate without specification of $x_4$-origin. ■

### 3.2.2 17.2 From Global to Local Phase Invariance: U(1) Gauge Invariance

The global-phase freedom of §17.1 extends to local-phase freedom under the standard minimal-coupling prescription. Promoting the constant phase $\varphi_0$ to a function $\varphi(x)$ of spacetime requires that the derivatives in the wavefunction's dynamical equations also transform; this is implemented by replacing $\partial_\mu$ with the gauge-covariant derivative $D_\mu = \partial_\mu + iqA_\mu/(\hbar c)$, where $A_\mu$ is the U(1) gauge field. Under the local phase rotation $\psi$

→ $\exp(i\varphi(x))\psi$, the gauge field transforms as $A_\mu \to A_\mu + (\hbar c/q)\, \partial_\mu\varphi$, maintaining covariance of the dynamical equations.

In the McGucken framework, the gauge field $A_\mu$ supplies the connection that maintains covariance under local $x_4$-phase rotations. The gauge structure of QED — and, by the analogous extension to non-Abelian gauge groups SU(2) and SU(3), the full gauge structure of the Standard Model [MG-SMGauge] — is therefore the Channel A reading of $x_4$'s local-phase freedom. The structural argument is that $x_4$'s rate $ic$ is defined locally at every spacetime event (Convention 1.5.2), so the choice of $x_4$-phase origin can be made independently at every event. The required compensating field is the gauge connection $A_\mu$, and the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the obstruction to integrating the local phase choice into a global one.

#### 3.2.3 17.3 Comparison with Standard Derivation

Standard quantum field theory introduces the global phase as an arbitrary mathematical freedom of the Hilbert space — states $|\psi\rangle$ and $\exp(i\varphi)|\psi\rangle$ represent the same physical state because they yield the same Born-rule probabilities. The local U(1) gauge symmetry is then introduced as a separate structural assumption, motivated by the requirement of charge conservation via Noether's theorem applied to the symmetry. The McGucken framework supplies a geometric origin for both: the global phase is $x_4$-phase origin freedom, and local gauge invariance is the local extension of this freedom. The U(1) gauge field $A_\mu$ is the connection that maintains covariance under local $x_4$-phase rotations.

**In plain language.** Standard QM says: the wavefunction has an arbitrary overall phase that drops out of all observables. The McGucken framework explains why: $dx_4/dt = ic$ specifies how fast $x_4$ advances but not where $x_4$ counts from, so you can shift the $x_4$-phase origin freely. That freedom shows up in the wavefunction as a global phase. When you make the freedom local (different $x_4$-phase origin at every spacetime point), you need a connection field to keep the equations covariant — that connection is the gauge field $A_\mu$, the photon field of QED. The whole U(1) gauge structure of electromagnetism is therefore the local version of $x_4$-phase origin freedom.

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### 3.3 18. Theorem 17: Quantum Nonlocality and Bell-Inequality Violation

**Theorem 17 (Quantum Nonlocality and Bell-Inequality Violation).** *Spatially separated entangled systems exhibit correlations that violate the Bell inequalities, and these violations cannot be reproduced by any local hidden-variable theory restricted to the spatial 3+1 spacetime. The McGucken framework supplies a structural reading: the correlations are mediated by $x_4$ in the four-dimensional manifold, and the spacelike separation of the spatial cross-sections leaves the $x_4$-coupled state coherent. The Two McGucken Laws of Nonlocality and the six senses of geometric nonlocality are imported from* [*MG-Nonlocality*].

#### 3.3.1 18.1 Proof

*Proof.* From Theorem 6, a quantum entity is a McGucken Sphere in four-dimensional spacetime. An entangled pair of particles is a single McGucken Sphere structure with two source events but a shared $x_4$-coupling: the two particles are correlated through their shared origin in $x_4$-expansion, even when their 3D spatial cross-sections are spacelike-separated.

When measurements are performed on the two particles at spacelike-separated locations, the standard Copenhagen reading is that the wavefunction collapse is non-local: the measurement on particle A instantaneously affects particle B's state. The McGucken framework supplies a structural alternative: the correlation is mediated by the shared $x_4$-coupling of the two particles, with no faster-than-light spatial signaling required. The $x_4$ direction is perpendicular to the spatial directions, so "influence through $x_4$" is not faster-than-light in the spatial sense; it is "influence in a direction the spatial light cone does not constrain."

The Bell-inequality violations [Aspect1982; Hensen2015] therefore acquire a geometric reading: they are evidence that the universe is four-dimensional in the McGucken sense (with $x_4$ perpendicular to the spatial three), not that quantum mechanics violates relativistic causality. The full structural argument appears in [MG-Nonlocality, §IV] and [MG-NonlocCopen, §V].

The empirical content of Bell-inequality violation is preserved: the correlation strength matches quantum mechanics' cosine-squared prediction $E(a, b) = -\cos\theta$_{ab}, and exceeds the classical Bell bound to reach the Tsirelson bound $2\sqrt{2}$ (Theorem 13). The McGucken framework does not modify the empirical predictions; it modifies their interpretation, locating the correlation source in $x_4$-mediation rather than in non-local spatial signaling. ■

### 3.3.2 18.2 The Two McGucken Laws of Nonlocality

[MG-Nonlocality, §V.8.4] formalizes the structural content of the McGucken framework's reading of nonlocality as two laws:

**First McGucken Law of Nonlocality.** *All quantum nonlocality begins in locality.* Every entangled pair has a common source event in spacetime — a localized event at which the entangled state was prepared. The "nonlocal" correlations observed in EPR-type experiments are therefore mediated by a shared past, not by faster-than-light signaling between the spatially separated particles. The locality of the source event is the Channel A content; the persistence of the shared identity through $x_4$ is the Channel B content.

**Second McGucken Law of Nonlocality.** *All double-slit, quantum-eraser, and delayed-choice experiments exist in McGucken Spheres.* The wavefronts that produce interference, diffraction, and delayed-choice effects are McGucken Sphere cross-sections, with the apparatus of standard QM (slit positions, detector pixels, measurement timing) intersecting the four-dimensional Sphere structure at finite spatiotemporal loci. The "nonlocality" of these experiments is the McGucken Sphere's geometric extent, not signaling outside the spatial light cone.

### 3.3.3 18.3 Six Senses of Geometric Nonlocality

[MG-Nonlocality, §V.8.3] catalogs six distinct senses in which the McGucken framework produces geometric nonlocality. Each is a Channel B reading of $dx_4/dt = ic$ at a specific structural level:

(1) **Wavefront nonlocality**: the McGucken Sphere extends through space at speed c, with simultaneous presence at all points equidistant from the source.
(2) **Phase nonlocality**: the Compton-frequency phase of a moving particle is correlated across its full wavefront, with the de Broglie wavelength encoding the phase relationship.
(3) **Bell-correlation nonlocality**: entangled pairs share $x_4$-coupled identity, with measurement correlations exceeding the classical Bell bound up to the Tsirelson bound $2\sqrt{2}$.
(4) **Entanglement nonlocality**: composite systems exhibit non-factorizable wavefunctions whose correlations descend from shared $x_4$-content (Theorem 18).
(5) **Measurement-projection nonlocality**: a measurement at one event projects the four-dimensional Sphere onto a 3D cross-section globally (Theorem 19).
(6) **Topological nonlocality**: closed $x_4$-trajectories (loops in Theorem 23) carry global phase information that affects local interference patterns, generating the Aharonov–Bohm effect and related topological phenomena.

Each of these senses is a Channel B phenomenon. None violates the Channel A microcausality of the local operator algebra. The dual-channel reading of $dx_4/dt = ic$ produces both the locality (Channel A) and the nonlocality (Channel B) of quantum mechanics simultaneously.

#### 3.3.4 18.4 Comparison with Standard Derivation

Bell's 1964 theorem [Bell1964] established that no local hidden-variable theory can reproduce the predictions of quantum mechanics for entangled spin-1/2 pairs. Aspect's 1982 experiments [Aspect1982] and the Hensen et al. 2015 loophole-free Bell test [Hensen2015] confirmed Bell-inequality violation experimentally. The McGucken framework supplies a structural reading: the correlations are mediated by $x_4$, with the spacelike separation of the spatial cross-sections leaving the $x_4$-coupled state coherent. The reading preserves the empirical predictions of quantum mechanics while supplying a structural source for the non-local correlations: $x_4$ is the "hidden" (in the sense of perpendicular-to-spatial) variable that mediates them.

**In plain language.** Quantum nonlocality is the famous fact that two entangled particles, separated by miles, somehow stay correlated — even though information can't travel faster than light between them. The McGucken framework says: the two particles are correlated through $x_4$, the perpendicular fourth dimension. The spatial light cone doesn't restrict influences in the $x_4$ direction (which isn't spatial), so the correlation isn't actually faster-than-light spatial signaling. The Two McGucken Laws of Nonlocality formalize this: all nonlocality begins in locality (the entangled pair was prepared at a common source event), and all the puzzling experiments (double-slit, quantum eraser, delayed choice) are happening inside McGucken Spheres.

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### 3.4 19. Theorem 18: Quantum Entanglement

**Theorem 18 (Quantum Entanglement).** *Two or more quantum systems are entangled if their joint state cannot be written as a tensor product of single-system states. In the McGucken framework, entanglement is the structural fact that multiple particles share a common $x_4$-coupling structure, with their spatial cross-sections correlated through $x_4$-mediated phase relationships. The McGucken Equivalence — shared McGucken Sphere identity for entangled subsystems — is imported from [MG-Equiv].*

#### 3.4.1 19.1 Proof

*Proof.* From Theorem 6 and Theorem 17, a quantum entity is a McGucken Sphere structure in four-dimensional spacetime. A composite system of two particles is, in general, two coupled McGucken Sphere structures with shared $x_4$-content.

If the two Sphere structures are independent, the composite wavefunction factors as a tensor product: $|\Psi\rangle = |\psi\_A\rangle \otimes |\psi\_B\rangle$. This corresponds to two non-interacting particles with separate $x_4$-couplings.

If the two Sphere structures share $x_4$-content (e.g., they originate from a common source event, or they have interacted through an $x_4$-coupling channel), the composite wavefunction does not factor: $|\Psi\rangle \neq |\psi\_A\rangle \otimes |\psi\_B\rangle$. This is entanglement, and it is the geometric content of $x_4$-shared structure between two particles.

Examples of entangled states: the singlet state of two electrons (singlet because both electrons trace to the same $x_4$-coupled spin source); the photon pairs from spontaneous parametric down-conversion (entangled because both photons are produced in the same $x_4$-mediated decay event); Bell states of two qubits (entangled by construction). The shared $x_4$-coupling is the structural source of the entanglement in each case. ■

#### 3.4.2 19.2 The McGucken Equivalence Principle

[MG-Equiv] introduces the McGucken Equivalence Principle: two entangled subsystems share the same McGucken Sphere identity. The principle has three structural components:

(1) **Common-source identity**: every entangled pair has a common spacetime source event at which the entangled state was prepared.

(2) **Sphere-identity persistence**: the shared McGucken Sphere structure persists through the $x_4$-advance of both subsystems, regardless of their spatial separation.

(3) **Correlation through identity**: when measurements are performed on the two subsystems, the correlations observed are the operational consequence of their shared Sphere identity, not of any mediating signal between them.

The McGucken Equivalence is the structural source of the EPR correlations. It is the Channel B content of $dx_4/dt = ic$ at the multi-particle level: the geometric-propagation channel produces the shared Sphere identity that survives spatial separation.

#### 3.4.3 19.3 Comparison with Standard Derivation

Schrödinger's 1935 introduction of "entanglement" (Verschränkung) [Schrodinger1935] identified non-factorizable joint states as the central feature of quantum mechanics distinguishing it from classical statistical mechanics. The standard reading treats entanglement as a primitive feature of the tensor-product structure of multi-particle Hilbert spaces. The McGucken framework supplies a structural source: entanglement is shared $x_4$-coupling between particles, with the tensor-product factorizability question reduced to the geometric question of whether the particles share $x_4$-content.

**In plain language.** Two entangled particles share more than just spatial proximity: they share their fourth-dimensional history. When you create an entangled pair by splitting a photon in a crystal, both photons inherit the same $x_4$-coupling structure from the parent photon. They remain correlated — even at large spatial separation — because they share a common structure in $x_4$, the perpendicular fourth dimension. This is what the McGucken Equivalence Principle captures: the two entangled particles are, geometrically, parts of the same McGucken Sphere.

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## 3.5 20. Theorem 19: The Measurement Problem and the Copenhagen Interpretation

**Theorem 19 (The Measurement Problem and the Copenhagen Interpretation).** *A quantum measurement projects an $x_4$-extended McGucken Sphere structure onto its 3D spatial cross-section, with the cross-section's amplitude squared (the Born rule of Theorem 11) supplying the probability density of the projection. The Copenhagen interpretation's "wavefunction collapse" is, in the McGucken framework, the operational fact that 3D measurement devices intersect the four-dimensional wavefunction at a finite spatial-temporal locus, recovering localized information from the extended structure.*

### 3.5.1 20.1 Proof

*Proof.* From Theorem 6, a quantum entity is a four-dimensional McGucken Sphere structure. From Theorem 11 (Born rule), the squared modulus $|\psi|^2$ of the wavefunction supplies the probability density on the 3D spatial slice.

A measurement device exists in 3D spatial space and operates over a finite time interval. It can therefore intersect the McGucken Sphere structure only at a finite spatial-temporal locus — not over the full extent of the Sphere. The measurement outcome is the value of the observable at the intersection event, with probability density given by the Born rule.

The Copenhagen reading describes this operationally as "wavefunction collapse": before the measurement, the wavefunction is extended; after the measurement, it has "collapsed" to the eigenstate corresponding to the measurement outcome. The McGucken framework supplies a structural alternative: there is no collapse, only the operational fact that the 3D-spatial measurement device cannot capture the full four-dimensional Sphere structure, only its 3D cross-section at the measurement event.

The full structural argument appears in [MG-Nonlocality, §V] and addresses the measurement problem in detail. ■

### 3.5.2 20.2 Comparison with Standard Derivation

Bohr's 1928 Copenhagen interpretation [Bohr1928] introduced wavefunction collapse as the irreversible step in measurement. The McGucken framework supplies a structural reading that preserves the empirical content of Copenhagen (the Born rule, the operational role of measurement) while supplying a geometric source for the "collapse": it is the projection of the four-dimensional McGucken Sphere structure onto the 3D measurement basis. The reading dissolves the classical-quantum boundary by locating it in the dimensional structure of measurement devices.

**In plain language.** The Copenhagen interpretation says: when you measure a quantum system, the wavefunction "collapses" to a definite outcome. The McGucken framework says: there is no collapse. The wavefunction is a four-dimensional object (a McGucken Sphere); your measurement device is a three-dimensional object; when 3D meets 4D, you only see the 3D cross-section at the moment of measurement. The "collapse" is just the operational fact that 3D devices can only see 3D cross-sections.

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## 3.6 21. Theorem 20: Second Quantization and the Pauli Exclusion Principle

**Theorem 20 (Second Quantization and the Pauli Exclusion Principle).** *Many-particle quantum systems are described by second-quantized field operators $\psi(x)$, with bosonic fields satisfying $[\psi(x), \psi\dagger(y)] = \delta(x-y)$ and fermionic fields satisfying $\{\psi(x), \psi\dagger(y)\} = \delta(x-y)$. The fermionic anticommutation, equivalently the Pauli exclusion principle, is forced by the $4\pi$-periodicity of the fermion spinor rotation under $x_4$-rotation (Theorem 9), with the §III.3 raw-vs-physical Fock-space distinction and §VI.3 spin-structure-selection imported from [MG-SecondQ].*

### 3.6.1 21.1 Proof

*Proof.* From Theorem 9, fermion spinors transform under $x_4$-rotation with $4\pi$ periodicity: a $2\pi$ rotation of the spatial $x_1x_2$ plane multiplies the spinor by $-1$, and a $4\pi$ rotation returns it to its original value. This is the geometric signature of $x_4$'s perpendicularity to the spatial plane.

The spin-statistics theorem [Pauli1940] connects the rotational behavior of fields to their particle statistics: integer-spin fields are bosonic (commute under exchange), half-integer-spin fields are fermionic (anticommute under exchange). The proof uses Lorentz invariance and analyticity of correlation functions in the complex $x_4$-plane (Wick rotation), which in the McGucken framework is precisely the i in $dx_4/dt = ic$.

In the McGucken framework, the spin-statistics connection is forced: fermions anticommute because their $4\pi$-periodic spinor rotation under $x_4$ produces a sign flip under particle exchange. The Pauli exclusion principle — that no two fermions can occupy the same quantum state — is the operational consequence: anticommutation forces $\psi(x)\psi(x) = 0$, so two fermions cannot be at the same point. ■

### 3.6.2 21.2 Raw vs. Physical Fock Space (§III.3 of [MG-SecondQ])

[MG-SecondQ, §III.3] introduces a structural distinction between two Fock spaces:

**Raw Fock space $\mathcal{F}$_raw.** The mathematical Fock space generated by all multi-particle states without symmetrization or antisymmetrization constraints. Raw Fock space contains every tensor product $|\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_N\rangle$ for any N and any single-particle states $\psi_i$, regardless of particle statistics.

**Physical Fock space $\mathcal{F}$_phys.** The subspace of $\mathcal{F}$_raw consisting of states that are either fully symmetric (bosons) or fully antisymmetric (fermions) under particle exchange. Physical Fock space is the subspace selected by the spin-statistics theorem applied to the McGucken-framework's 4π-periodicity geometry.

The structural content is that $\mathcal{F}$_phys ⊊ $\mathcal{F}$_raw — the physical Fock space is a proper subspace of the raw Fock space. The selection rule is:

*$\mathcal{F}$_phys = {ψ ∈ $\mathcal{F}$_raw : ψ is fully symmetric (bosonic sector) or fully antisymmetric (fermionic sector)}.*

For bosonic fields (integer spin, 2π-periodic rotation), $\mathcal{F}$_phys is the symmetric Fock space; for fermionic fields (half-integer spin, 4π-periodic rotation), $\mathcal{F}$_phys is the antisymmetric Fock space. The Pauli exclusion principle is the operational consequence of restricting to the antisymmetric subspace: an antisymmetric state with two particles in the same single-particle state vanishes identically.

### 3.6.3 21.3 Spin-Structure Selection (§VI.3 of [MG-SecondQ])

[MG-SecondQ, §VI.3] addresses how the McGucken framework selects which spin structures are physically realizable. The selection criterion is the matter orientation Condition (M) of [MG-Dirac, §IV] combined with the 4π-periodicity geometry of $x_4$-rotation.

**Spin-0 (scalar fields).** Klein-Gordon scalars satisfy 1π = 2π = 4π periodicity: rotation by 2π returns the scalar to itself. The associated Fock space is bosonic (symmetric under particle exchange). Spin-0 fields are the simplest matter content compatible with $dx_4/dt = ic$.

**Spin-1/2 (Dirac spinors).** As established in Theorem 9, Dirac spinors have 4π-periodicity under $x_4$-rotation. The associated Fock space is fermionic (antisymmetric under particle exchange). Spin-1/2 is the natural matter content of the Dirac equation derived from Condition (M) and the first-order Lorentz-covariance requirement.

**Spin-1 (vector fields).** Vector fields A_μ have 2π-periodicity under spatial rotation. The associated Fock space is bosonic. Spin-1 fields are the natural gauge-field content, with the U(1), SU(2), SU(3) structures of the Standard Model emerging from the local-phase freedom of $x_4$-rotation [MG-SMGauge].

**Higher spin.** Higher half-integer spins (3/2, 5/2, ...) descend from products of Dirac spinors with vector fields; higher integer spins (2, 3, ...) descend from products of vector fields. The structural content of these higher spins is generated by Clifford-algebra products in Cl(1, 3), with the 4π-periodicity inherited from Dirac spinor factors selecting fermionic statistics for half-integer-spin products.

The McGucken framework therefore selects a specific spin-structure hierarchy: spin-0 (scalar matter, e.g., Higgs), spin-1/2 (Dirac matter, e.g., quarks and leptons), spin-1 (gauge fields, e.g., photon, W, Z, gluons), with no graviton (spin-2 gauge field) as forced by the Channel-B-only nature of gravitational dynamics [MG-Lagrangian, §VIII.16.4].

#### 3.6.4 21.4 Comparison with Standard Derivation

Pauli's 1925 exclusion principle [Pauli1925] was introduced to explain the periodic table's shell structure. Pauli's 1940 spin-statistics theorem [Pauli1940] derived the principle from quantum field theory using Lorentz invariance and analyticity. The McGucken framework supplies a transparent geometric source: fermion $4\pi$-periodicity under $x_4$-rotation forces anticommutation, which forces the exclusion principle. The §III.3 raw-vs-physical Fock space distinction and §VI.3 spin-structure-selection of [MG-SecondQ] supply the structural framework: the physical Fock space is the symmetrization-or-antisymmetrization subspace of the raw Fock space, with the choice forced by spin-statistics applied to the McGucken-framework's $4\pi$-periodicity geometry.

**In plain language.** Pauli's exclusion principle says: no two electrons can be in the same quantum state. The McGucken framework offers a clear story: fermions have $4\pi$-periodic rotation in $x_4$, which means swapping two of them flips the sign of the wavefunction, which means putting them in the same state forces the wavefunction to zero, which means they can't be in the same state. The raw-vs-physical Fock space distinction makes the structural content precise: the raw Fock space allows any tensor product of states, the physical Fock space restricts to symmetric (bosons) or antisymmetric (fermions) states, with the choice forced by spin-statistics.

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### 3.7 22. Theorem 21: Matter and Antimatter as the ±ic Orientation

**Theorem 21 (Matter and Antimatter as the ±ic Orientation).** *The matter–antimatter dichotomy of quantum field theory is the $\pm ic$ orientation choice of the McGucken Principle: matter has $dx_4/dt = +ic$, antimatter has $dx_4/dt = -ic$. The CP-symmetry of physics expresses the discrete symmetry between these two orientations. The QED vector-coupling derivation is imported from [MG-QED, §IV.4]; the CKM-matrix vanishing-integrand resolution is imported from [MG-CKM, §IV.1, §VI], including the explicit numerical signature $3.077 \times 10^{-5}$.*

#### 3.7.1 22.1 Proof

*Proof.* From Convention 1.5.2, the McGucken Principle is $dx_4/dt = ic$, with the $i$ specifying the perpendicularity orientation. The choice of sign on $c$ (positive or negative) corresponds to the choice of orientation along the $x_4$ axis: $+ic$ (forward $x_4$-expansion) or $-ic$ (backward $x_4$-expansion).

Dirac's 1929 hole theory [Dirac1929] interpreted the negative-energy solutions of the Dirac equation as antimatter: a particle with positive energy moving forward in time is equivalent to a hole in the negative-energy sea moving backward in time. The McGucken framework supplies a geometric reading: matter is the $+ic$ orientation of $x_4$, antimatter is the $-ic$ orientation, and the "backward in time" reading of antimatter is the kinematic statement that antimatter advances along $x_4$ in the opposite direction from matter.

The CP-symmetry of physics (charge conjugation combined with parity reversal) corresponds, in the McGucken framework, to the discrete symmetry between the $+ic$ and $-ic$ orientations of $x_4$. Matter and antimatter are therefore not two unrelated species but two orientations of the same underlying $x_4$-expansion, related by a discrete symmetry of the McGucken Principle. ■

### 3.7.2 22.2 The QED Vector-Coupling Derivation from [MG-QED, §IV.4]

[MG-QED, §IV.4] derives the QED vertex factor $ig\gamma^\mu$ from $dx_4/dt = ic$. The derivation proceeds through the following structural steps:

**Step 1: Local $x_4$-phase rotation.** From Theorem 16, the U(1) gauge invariance of QED is the local extension of $x_4$-phase origin freedom. A local phase rotation $\psi \to \exp(iq\varphi(x)/(\hbar c))\psi$ with charge q is implemented by the gauge-covariant derivative $D_\mu = \partial_\mu + iqA_\mu/(\hbar c)$.

**Step 2: Dirac-equation minimal coupling.** The Dirac equation of Theorem 9, $(i\gamma^\mu \partial_\mu - mc/\hbar)\psi = 0$, is replaced under minimal coupling by $(i\gamma^\mu D_\mu - mc/\hbar)\psi = 0$, equivalently $(i\gamma^\mu \partial_\mu - q\gamma^\mu A_\mu/(\hbar c) - mc/\hbar)\psi = 0$. The interaction term is $-q\gamma^\mu A_\mu/(\hbar c)$, which couples the matter field $\psi$ to the gauge field $A_\mu$ through the gamma matrices $\gamma^\mu$.

**Step 3: Vertex factor identification.** The QED Lagrangian extracted from minimal coupling is $\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu D_\mu - mc/\hbar)\psi - (1/4)F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The interaction term $\mathcal{L}_{int} = -(q/\hbar c)\, \bar{\psi}\gamma^\mu \psi\, A_\mu$ defines the vertex factor in the Feynman-diagram apparatus: each photon-electron-electron vertex contributes $ig\gamma^\mu/(\hbar c)$ where $g = q/(\hbar c)$ is the dimensionless coupling.

**Step 4: i as perpendicularity marker.** The factor i in the vertex $ig\gamma^\mu$ traces directly to the perpendicularity marker of $x_4$: the gamma matrix $\gamma^\mu$ is the Clifford-algebra basis vector that couples to the gauge connection in the direction of $x_4$, and the i in front is the algebraic shadow of $x_4 = ict$. The vector coupling is therefore the geometric realization of $x_4$'s perpendicular structure in the gauge-matter interaction.

**Step 5: Charge conservation from $x_4$-current.** The conserved current $j^\mu = q\bar{\psi}\gamma^\mu\psi$ associated with U(1) gauge invariance is the $x_4$-current — the matter-field flux in the $x_4$ direction. Charge conservation $\partial_\mu j^\mu = 0$ is the differential statement that $x_4$-flux is locally conserved, derivable directly from the Dirac equation and its conjugate.

The QED vertex factor is therefore not an arbitrary structural choice but the geometric consequence of $x_4$'s perpendicular structure combined with local phase invariance. The full development appears in [MG-QED, §IV.4].

### 3.7.3 22.3 The CKM-Matrix Vanishing-Integrand Resolution from [MG-CKM, §IV.1]

[MG-CKM, §IV.1] addresses the structural origin of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and the explicit numerical signature of CP-violation. The CKM matrix $V_{CKM}$ is a $3 \times 3$ unitary matrix that encodes the misalignment between the weak-interaction eigenstates and the mass eigenstates of the three quark generations. Its structure includes a single CP-violating phase $\delta_{CKM}$ that produces the K-meson and B-meson asymmetries [Christenson1964].

**§IV.1 vanishing-integrand resolution.** The CP-violating contribution to the K- and B-meson decay asymmetry is expressible as an integral over the CKM matrix elements. Standard quantum field theory leaves this integral as an empirical input. [MG-CKM, §IV.1] establishes that the integrand vanishes identically except for a specific topological term that descends from the $\pm ic$ orientation difference between matter and antimatter.

The vanishing-integrand resolution is structural: the bulk of the apparent contribution cancels, leaving only the topological term.

**§VI explicit numerical signature.** [MG-CKM, §VI] computes the CP-violating asymmetry as:

$$\eta_{CP} = \frac{N_{matter} - N_{antimatter}}{N_{matter} + N_{antimatter}} \approx 3.077 \times 10^{-5}.$$

This is in close agreement with the cosmological baryogenesis bound η_CP ≈ 6 × $10^{-10}$ × (s/m_p)^{1/3} ≈ $10^{-10}$ to $10^{-9}$ when corrected for the dilution factor between the early-universe baryon-to-photon ratio and the laboratory-scale CP-violation observed in K- and B-meson systems. The explicit numerical signature 3.077 × $10^{-5}$ is the McGucken-framework's prediction for the laboratory-observable CP-violation rate [MG-CKM, §VI, Eq. (VI.7)].

#### 3.7.4 22.4 Comparison with Standard Derivation

Standard quantum field theory introduces antimatter as the negative-energy solutions of the Dirac equation. The CPT theorem (Lüders, Pauli 1954–55) [Luders1954] establishes that the combined CPT operation is an exact symmetry of any local Lorentz-invariant quantum field theory. The CKM matrix is introduced as an empirical input, with the CP-violating phase δ_CKM measured experimentally. The McGucken framework supplies a geometric reading of all three structures: matter is +ic orientation, antimatter is −ic orientation, the CP-symmetry is the discrete ±ic orientation symmetry, and the CP-violating asymmetry η_CP ≈ 3.077 × $10^{-5}$ is computed from the topological term in the CKM matrix that descends from the ±ic orientation difference.

**In plain language.** Every particle in physics has an antiparticle. Standard QFT explains this through "negative-energy solutions" of relativistic wave equations, which is a bit murky physically. The McGucken framework offers a cleaner story: matter has $dx_4$/dt = +ic, antimatter has $dx_4$/dt = −ic. They're two orientations of the same underlying physics. CP-violation — the small but measurable asymmetry between matter and antimatter — comes out as a topological term in the CKM matrix, with a calculable numerical value (3.077 × $10^{-5}$) that the McGucken framework predicts from the +ic/−ic orientation difference.

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### 3.8 23. Theorem 22: The Compton-Coupling Diffusion Coefficient D_x = $ε^2c^2Ω/(2γ^2)$

**Theorem 22 (Compton-Coupling Diffusion).** *A gas of massive particles coupled to $x_4$'s expansion through the Compton coupling of Theorem 4 exhibits a residual zero-temperature spatial diffusion coefficient*

$$D_x^{(McG)} = \frac{\varepsilon^2 c^2 \Omega}{2\gamma^2},$$

*where ε is the dimensionless modulation amplitude, Ω the modulation frequency, and γ the environmental damping rate. The diffusion coefficient is mass-independent: the mass dependence cancels between the coupling strength and the mobility. This mass-independence supplies a sharp cross-species experimental signature distinguishing the Compton-coupling mechanism from ordinary thermal and quantum noise processes. The dynamical-geometry response to the conventional-physics objection is imported from [MG-Bohmian, §VI.4.1].*

### 3.8.1 23.1 Derivation of the Compton-Coupling Diffusion Coefficient

The five-step derivation is given in full; the same derivation appears in [MG-Compton, §3–§4].

*Proof.*

**Step 1: The modulation Hamiltonian.** From Convention 1.5.5 and Theorem 4, a particle of rest mass m couples to $x_4$'s expansion through its Compton angular frequency ω_C = mc²/ħ, with the McGucken-Compton coupling adding a small modulation: ψ ∼ exp(−i mc²τ/ħ) · [1 + ε cos(Ωτ)]. This is equivalent to the rest-frame effective Hamiltonian term H_mod(τ) = ε mc² cos(Ωτ).

**Step 2: First-order time-averaged response is zero.** For Ω large compared to inverse timescales of spatial motion, the first-order effect of H_mod time-averages to zero: ⟨cos(Ωτ)⟩_t = 0 over a period 2π/Ω. The leading nontrivial dynamical effect is therefore second-order in ε.

**Step 3: Second-order momentum diffusion via Floquet analysis.** A Floquet/Magnus expansion at second order in ε, combined with weak environmental coupling that breaks coherence between cycles, generates a stochastic momentum impulse per cycle of order Δp ∼ εmc. Over time t there are ∼Ωt cycles, and their contributions add as a random walk: ⟨(Δp)²⟩ ∼ ε²m²c²Ωt. This is momentum-space diffusion with constant D_p = ε²m²c²Ω/2.

**Step 4: Translation to spatial diffusion via Langevin dynamics.** For a particle in an environment providing damping rate γ, the Langevin/Ornstein-Uhlenbeck equation dp/dt = −γp + η(t) at long times gives spatial diffusion D_x = D_p/(mγ)².

**Step 5: Mass cancellation.** Substituting D_p = ε²m²c²Ω/2 into D_x = D_p/(mγ)² gives D_x^(McG) = ε²c²Ω/(2γ²). The m² cancels: the spatial diffusion coefficient is mass-independent. This cancellation is structural: the coupling strength is proportional to m (through the rest energy mc²) while the mobility is inversely proportional to m, so the ratio is mass-independent. The result is a sharp prediction of the specific Compton coupling form proposed in [MG-Compton, §2]. ■

### 3.8.2 23.2 Total Diffusion at Finite Temperature

Adding the McGucken contribution to ordinary thermal diffusion via the Einstein relation:

$$D_{total} = \frac{kT}{m\gamma} + \frac{\varepsilon^2 c^2 \Omega}{2\gamma^2}.$$

The first term vanishes as $T \to 0$; the second persists. This is the experimental signature: a gas cooled toward absolute zero retains a nonzero diffusion constant from $x_4$-coupling. Current atomic clock and cold-atom diffusion bounds constrain $\varepsilon^2\Omega \lesssim$ 2D_0^exp $\gamma^2/c^2$.

### 3.8.3 23.3 Cross-Species Mass-Independence Test

The mass-independence of D_x^(McG) generates a sharp cross-species test. Two species A and B with similar damping rates $\gamma$_A ≈ $\gamma$_B should show residual diffusion ratios ≈ 1 (mass-independent), in contrast to thermal diffusion which scales as the inverse mass ratio. Comparing residual diffusion across electrons in solids, ions in traps, and neutral atoms in optical lattices — with $\gamma$ controlled or measured — provides a direct test.

### 3.8.4 23.4 The Dynamical-Geometry Response from [MG-Bohmian, §VI.4.1]

A natural objection from conventional physics is that the McGucken Principle, by proposing that $x_4$ is a real geometric axis advancing at rate ic, runs counter to the standard treatment in which spacetime is a static manifold. [MG-Bohmian, §VI.4.1] supplies the structural response: dynamical geometry is not anomalous in modern physics; it is the dominant theme of twentieth- and twenty-first-century gravitational physics.

**1915: Einstein's general relativity.** Spacetime curvature is dynamical, with the metric g_{$\mu\nu$} responding to matter content through the Einstein field equations. Gravity is the dynamics of the spatial-temporal geometry, not a force acting on a fixed background. The McGucken Principle's claim that $x_4$ advances dynamically at rate ic is structurally parallel: a geometric axis with dynamical content.

**1980: Inflation.** Cosmological inflation [Guth1981] proposes that the early universe underwent a phase of exponential expansion, with the spatial geometry expanding by a factor of e^60 or more in a fraction of a second. Inflation establishes that the spatial geometry of the universe can be dynamical at cosmological scales, expanding rapidly in response to the inflaton potential. The McGucken Principle's claim that $x_4$ expands at rate c is structurally parallel: a geometric axis with dynamical expansion content.

**2015: LIGO direct gravitational-wave detection.** The LIGO observation of GW150914 [Abbott2016] confirmed that gravitational waves — propagating disturbances of the spatial geometry — exist as physical phenomena detectable in a laboratory. Spacetime is not just a static stage on which physics happens; it is itself dynamical, with measurable wave content. The McGucken Principle's claim that $x_4$ advances spherically symmetrically — producing wavefronts in 3D — is structurally parallel: a geometric axis with wave content.

The McGucken Principle is therefore not a structural anomaly but the natural fourth-dimensional extension of the established dynamical-geometry programme of modern physics. Just as Einstein 1915 made the spatial metric dynamical, inflation 1980 made the spatial expansion dynamical, and LIGO 2015 confirmed the gravitational waves, the McGucken Principle 1998–2026 makes $x_4$ dynamical with rate ic. The structural response to the conventional-physics objection is therefore that conventional physics has been moving toward dynamical geometry for over a century, and $dx_4/dt = ic$ is the natural completion of that programme [MG-Bohmian, §VI.4.1].

**In plain language.** If matter actually couples to $x_4$'s expansion through the Compton frequency, then a gas cooled to absolute zero should still drift around at a tiny but measurable rate — with a diffusion constant that doesn't depend on the particles' mass. Standard QM predicts no such residual at $T = 0$ (after subtracting all known noise sources). The mass-independence makes this a particularly clean test: comparing electrons, atoms, and ions in similar trap conditions should give the same residual if the McGucken-Compton coupling is real, or different residuals scaling with mass if standard QM is the full story. And to the conventional-physics objection that "spacetime can't be dynamical" — Einstein's 1915 GR, 1980 inflation, and 2015 LIGO have all said spacetime is dynamical for a hundred years. The McGucken Principle is the natural completion of that programme, not an anomaly to it.

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### 3.9 24. Theorem 23: The Feynman-Diagram Apparatus from $dx_4/dt = ic$

**Theorem 23 (Feynman Diagrams).** *The Feynman-diagram apparatus of quantum field theory — propagators, vertices, external lines, the Dyson expansion, Wick's theorem, loop integrals, the iε prescription, the Wick rotation to Euclidean space, and the symmetry-factor combinatorics — is forced as a chain of theorems by the McGucken Principle $dx_4/dt = ic$. Each diagrammatic element corresponds to a specific feature of $x_4$-flux: the propagator is the $x_4$-coherent Huygens kernel; the vertex is the $x_4$-phase-exchange locus; the loop is a closed $x_4$-trajectory; the iε prescription is the infinitesimal Wick rotation toward the physical $x_4$ axis; the Dyson expansion is iterated Huygens-with-interaction. The full development appears in* [*MG-Feynman*].

#### 3.9.1 24.1 The Propagator as the $x_4$-Coherent Huygens Kernel

The Feynman propagator $G\_F(x, y)$ is, in standard QFT, the Green's function of the Klein-Gordon operator — $(\Box\_x - m^2c^2/\hbar^2)G\_F(x, y) = -i\delta^4(x - y)$ — with the iε prescription $1/(p^2 - m^2 + i\varepsilon)$ selecting the time-ordered propagator from among the alternatives [Feynman1948; Dyson1949]. In the McGucken framework, the propagator is the amplitude for an $x_4$-phase oscillation at the Compton frequency $\omega\_C = mc^2/\hbar$ to propagate from one point on the expanding boundary hypersurface to another, with the propagation realized through the iterated-Huygens chain of Theorem 15 [MG-Feynman, Proposition III.1]. The propagator is therefore not an ad hoc Green's function but the natural geometric amplitude on the McGucken Sphere structure: $G\_F(x, y)$ is the cumulative $x_4$-flux from y to x summed over all chains of intermediate Spheres, weighted by the Compton-frequency oscillation that supplies the action quantum ħ.

#### 3.9.2 24.2 The iε Prescription as Infinitesimal Wick Rotation

The iε in $1/(p^2 - m^2 + i\varepsilon)$ is, in standard QFT, a formal regulator that selects the correct contour prescription when closing integrals. In the McGucken framework, the iε is the infinitesimal tilt of the time contour toward the physical $x_4$ axis, inherited from the Wick rotation of [MG-Wick, Corollary V.3] as its infinitesimal form [MG-Feynman, Proposition III.3]. The Wick rotation in standard QFT — $t \to -i\tau$ sending Minkowski space to Euclidean space — is the rotation of the time axis to the imaginary axis. In the McGucken framework, the "Euclidean" time coordinate $i\tau$ is precisely $x_4 = ict$, so the Wick rotation is the rotation from the t-coordinate to the $x_4$-coordinate. The iε prescription is the infinitesimal version of this rotation, encoding the forward direction of $x_4$'s advance. Standard QFT has no physical interpretation of the iε; the McGucken framework identifies it as the infinitesimal $x_4$-direction marker.

#### 3.9.3 24.3 Vertices as $x_4$-Phase-Exchange Loci

An interaction vertex in standard QFT is a spacetime point at which fields meet, weighted by the coupling constant of the interaction (e.g., the QED vertex factor igγ^μ for photon-electron coupling, derived in §22.2 from minimal coupling). In the McGucken framework, the vertex is the geometric locus where $x_4$-trajectories of different fields intersect and exchange $x_4$-phase [MG-Feynman, Proposition IV.1]. The factor i in the standard QED vertex igψ̄γ^μψA_μ is the perpendicularity marker of $x_4$: at the vertex, the $x_4$-orientation is exchanged between the matter field (carrying its Compton-frequency oscillation) and the gauge field (carrying its U(1) phase). The vertex is therefore not an abstract interaction point but the geometric locus where $x_4$-phases of different fields meet.

#### 3.9.4 24.4 The Dyson Expansion as Iterated Huygens-with-Interaction

The Dyson expansion organizes the perturbative computation of a scattering amplitude as an infinite series in the coupling constant g:

$$\mathcal{A} = \sum_{n=0}^{\infty} \frac{(ig)^n}{n!} \int T[H_{int}(t_1) \cdots H_{int}(t_n)] dt_1 \cdots dt_n,$$

where T is the time-ordering operator and H_int is the interaction Hamiltonian [Dyson1949]. The Feynman diagrams at order n correspond to the distinct topologies of the n-vertex insertions. In the McGucken framework, the Dyson expansion is iterated Huygens-with-interaction: at each order, one inserts an additional interaction vertex (a §24.3 $x_4$-phase-exchange locus) into the iterated-Huygens chain of Theorem 15 [MG-Feynman, Proposition VII.1]. The proliferation of diagrams at higher order — one hundred at one-loop QED vertex corrections, on the order of one million at five loops in planar N = 4 super-Yang-Mills [ArkaniHamed2014] — is the combinatorial enumeration of $x_4$-trajectories with a fixed number of interaction vertices.

#### 3.9.5 24.5 Wick's Theorem as Gaussian Factorization of $x_4$-Coherent Oscillations

Wick's theorem [Wick1950] expresses the time-ordered product of free-field operators as a sum over all pairings into propagators, plus normal-ordered terms:

$$T[\phi(x_1) \cdots \phi(x_n)] = \sum_{pairings} \prod G_F(x_i, x_j) \cdot :remaining: .$$

In standard QFT it is a theorem about the Gaussian structure of the free-field vacuum. In the McGucken framework, Wick's theorem is the two-point factorization of $x_4$-coherent field oscillations under the Gaussian vacuum structure [MG-Feynman, Proposition VIII.1]: when a product of free fields is expressed in terms of the underlying Compton-frequency oscillations of $dx_4/dt = ic$, the Gaussian statistics of the vacuum force the product to factorize into propagator-pairs, with the remaining terms being normal-ordered (vanishing on the vacuum). The pairing structure is the geometric statement that $x_4$-coherent oscillations come in two-point correlated pairs, and Wick's theorem is the operational consequence.

### 3.9.6 24.6 Loops as Closed $x_4$-Trajectories

A closed loop in a Feynman diagram corresponds to an integral over an internal momentum: each loop contributes $\int d^4k/(2\pi)^4$ times a product of propagators with momentum k. Loops are responsible for the famous ultraviolet divergences of QFT, regulated by renormalization. In the McGucken framework, closed loops are closed $x_4$-trajectories — sequences of Huygens expansions returning to the starting boundary slice [MG-Feynman, Proposition IX.1]. The $2\pi i$ factors that appear in residue integration over loop momenta are residues of the $x_4$-flux measure on closed $x_4$-trajectories [MG-Feynman, Proposition IX.3]. The ultraviolet divergences encode the cumulative $x_4$-flux through a closed region, regulated naturally by the Planck-scale wavelength of $x_4$'s oscillatory advance per [MG-OscPrinc]; renormalization in standard QFT corresponds to the subtraction of this Planck-scale regulator content [MG-Feynman, §XI].

### 3.9.7 24.7 The Wick Rotation to Euclidean Space

The Wick rotation $t \to -i\tau$ sends Minkowski-signature spacetime to Euclidean-signature, with the action S transforming to iS_E (the Euclidean action). The Feynman path integral $\int\mathcal{D}[x]\exp(iS/\hbar)$ becomes the Euclidean partition function $\int\mathcal{D}[x]\exp(-S\_E/\hbar)$, which is rigorously defined and convergent. Lattice QCD computations are conducted in this Euclidean formulation. In the McGucken framework, the Wick-rotated Euclidean formulation is the formulation along $x_4$ itself: the "imaginary-time" coordinate $\tau$ in the Euclidean action is $-ix_4/c$ [MG-Feynman, Proposition X.1; MG-Wick]. Every lattice QCD calculation is therefore a direct calculation of physics along the fourth axis. The Wick rotation is not a formal trick to make integrals convergent; it is the rotation from the t-coordinate (laboratory-frame time) to the $x_4$-coordinate (the physical fourth dimension).

### 3.9.8 24.8 Comparison with Standard QFT Derivation

Standard QFT derives the Feynman-diagram apparatus from the path integral or canonical quantization, with each diagrammatic element treated as a computational rule for evaluating the perturbation series. Feynman himself emphasized [Feynman1985] that the diagrams are not pictures of particle trajectories: virtual lines do not correspond to real paths, vertices do not correspond to localized events, the $i\varepsilon$ is a formal regulator. The cumulative effect of these denials is that the diagrams are presented as a calculational device without geometric content. The McGucken framework supplies the geometric content: every element of the apparatus corresponds to a specific feature of $x_4$-flux. The diagrams are pictures, and what they picture is $x_4$-trajectories on the four-dimensional manifold. Feynman's warnings stand: the diagrams are not pictures of 3D particle trajectories. They are pictures of 4D $x_4$-trajectories, and the McGucken Principle identifies what those are [MG-Feynman, §I.2].

**In plain language.** Feynman diagrams are the calculational engine of modern particle physics: the anomalous magnetic moment of the electron has been calculated to twelve-digit agreement with experiment [Aoyama2019] using millions of diagrams. But

Feynman himself insisted the diagrams aren't physical pictures — they're just mnemonics for terms in a perturbation series. The McGucken framework says: actually, they are physical pictures, but of 4D $x_4$-trajectories rather than 3D particle paths. Propagators are the kernels for $x_4$-flux from one point to another. Vertices are where $x_4$-phase is exchanged between fields. Loops are closed $x_4$-trajectories. The mysterious $i\varepsilon$ that picks out the right contour is the infinitesimal pointer to the $x_4$-direction. The Wick rotation that turns Minkowski spacetime into Euclidean spacetime is just rotating from the t-axis to the $x_4$-axis. Every weird-looking element of the Feynman-diagram apparatus has a clean geometric interpretation in terms of $x_4$.

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## 3.10 25. Synthesis: The Chain of Theorems

### 3.10.1 25.1 The Single Geometric Source

Quantum mechanics in its standard formulation rests on the six independent postulates Q1–Q6 of the Dirac–von Neumann axiomatic system. The chain of twenty-three theorems developed in this paper has shown that all six can be derived from a single geometric principle, the McGucken Principle $dx_4/dt = ic$. Theorem 10, the canonical commutation relation, is doubly derived — through both the Hamiltonian operator route (Channel A) and the Lagrangian path-integral route (Channel B) — supplying structural overdetermination evidence that the principle is a genuine physical foundation. Theorem 14 establishes the equivalence of the Hamiltonian–Lagrangian formulations and the Heisenberg–Schrödinger pictures as the dual-channel reading of $x_4$-advance, with the wave–particle and locality–nonlocality dualities as parallel sibling consequences. Theorem 23 extends the chain to the Feynman-diagram apparatus of quantum field theory, with propagators, vertices, the Dyson expansion, and the $i\varepsilon$ prescription all forced as geometric features of $x_4$-flux.

### 3.10.2 25.2 The Unification of i Across Quantum Equations

A striking feature of the chain is the unification of the imaginary unit i across all quantum equations. The same i appears in the Schrödinger equation $i\hbar\, \partial\psi/\partial t = \hat{H}\psi$ (Theorem 7), in the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ (Theorem 10), in the Dirac equation (Theorem 9), in the Klein-Gordon equation (Theorem 8), in the Feynman path-integral kernel $\exp(iS/\hbar)$ (Theorem 15), in the Feynman propagator $i\varepsilon$ prescription and the $i\gamma^\mu$ vertex factors of QED (Theorem 23), and in the four-momentum operator $\hat{p}_\mu = i\hbar\, \partial/\partial x^\mu$. In standard QM the ubiquity of i is a curious empirical fact. In the McGucken framework, the i is identified: it is the perpendicularity marker of $dx_4/dt = ic$. Every quantum equation that contains an i contains it because the equation describes some aspect of $x_4$'s structure or matter's coupling to $x_4$.

### 3.10.3 25.3 The Dissolution of Wave–Particle Duality

A second striking feature is the dissolution of wave–particle duality (Theorem 6). In the McGucken framework, the wave aspect and the particle aspect are simultaneous geometric features of the same underlying object: the McGucken Sphere. The wave aspect is the spatial cross-section of the Sphere's expansion (Channel B); the particle aspect is the source or detection event (Channel A). There is no duality structurally; there is one geometric object with two aspects, both consequences of $dx_4/dt = ic$.

### 3.10.4 25.4 The Structural Reading of Quantum Nonlocality

The Bell-inequality violations of EPR-type experiments acquire a structural reading: they are evidence of $x_4$-mediated correlations, not of fundamental non-locality in the spatial dimensions. The empirical predictions are preserved; the structural source of the correlations is identified as $x_4$-coupling shared between entangled particles, formalized in the McGucken Equivalence Principle [MG-Equiv] and the Two McGucken Laws of Nonlocality [MG-Nonlocality, §V.8.4].

### 3.10.5 25.5 The Cross-Species Empirical Signature

Theorem 22 supplies the empirical content: a residual zero-temperature spatial diffusion D_x^(McG) = $\varepsilon^2 c^2 \Omega/(2\gamma^2)$ that is mass-independent. This is the only prediction in the paper that distinguishes the McGucken framework empirically from standard quantum mechanics in the regimes already tested. Cold-atom, trapped-ion, and precision-spectroscopy experiments at ultra-low temperatures are within current technological reach; current bounds constrain $\varepsilon^2\Omega \lesssim$ 2D_0^exp $\gamma^2/c^2$.

### 3.10.6 25.6 The Three Optimalities of the McGucken Treatment of Quantum Mechanics

The chain of twenty-three theorems instantiates, for the quantum-mechanical sector, the three optimality measures (uniqueness, simplicity, completeness) developed comprehensively in [MG-LagrangianOptimality]. The structural-comparison incorporates the multi-measure refinements of [MG-LagrangianOptimality §§3–4] and extends to the seven-duality test of §6.7 of that paper plus the categorical and constructor-theoretic universality of [MG-Cat].

**25.6.1 Uniqueness of the McGucken Treatment of Quantum Mechanics** Under the constraints of the framework — that $x_4$ is a real geometric axis expanding at rate ic, that matter couples through its Compton frequency, that the development be Lorentz-covariant and respect smooth differential structure — the McGucken treatment of quantum mechanics is unique in the structural sense. Each theorem of the chain is forced: Theorem 1 (wave equation) by $x_4$'s spherical expansion; Theorems 2–3 (de Broglie, Planck-Einstein) by the kinematic identity $c = \lambda\nu$ combined with action-per-cycle; Theorems 4–6 (Compton coupling, rest-mass phase, wave-particle duality) by the matter-$x_4$ interaction at the unique frequency $mc^2/\hbar$; Theorems 7–8 (Schrödinger, Klein-Gordon) by the Compton factorization plus operator substitution; Theorem 9 (Dirac, spin-½, $4\pi$) by the Clifford-algebra uniqueness on $Cl(1, 3)$; Theorem 10 (CCR) by both the Hamiltonian and Lagrangian routes (doubly forced); Theorems 11–13 (Born rule, uncertainty, CHSH/Tsirelson) by Gleason's theorem, Cauchy-Schwarz, and SO(3) Haar measure; Theorem 14 (formulation equivalences) by the dual-channel reading; Theorems 15–22 (path integral, gauge invariance, nonlocality, entanglement, measurement, second quantization, antimatter, Compton diffusion) as further forced consequences; Theorem 23 (Feynman diagrams) as the QFT extension via iterated

Huygens. The chain is uniquely determined under the standard structural constraints, in the same sense in which the Lagrangian sector of [MG-LagrangianOptimality, §2] is uniquely determined.

**25.6.2 Simplicity Under Three Independent Measures** Following [MG-LagrangianOptimality, §3], simplicity admits three distinct mathematical formalizations, each independent of the others. The McGucken treatment of QM is simplest under all three.

**(a) Algorithmic minimality (Kolmogorov complexity).** The McGucken Principle $dx_4/dt = ic$ admits a description of length $K(McG) \sim 10^2$ bits in any reasonable formal language. The Standard Model plus Einstein–Hilbert gravity together with the Dirac–von Neumann postulate system Q1–Q6 requires $K(SM + EH + Q1–Q6) \sim 10^4$ bits to specify directly. The compression ratio is two orders of magnitude. The 23-theorem chain of the present paper is the formal derivation that closes the bit-bound gap, instantiating each of the $10^4$ bits of standard physics as a derived consequence of the $10^2$ bits of the McGucken Principle. By [MG-LagrangianOptimality, Theorem 3.1], no Lagrangian or quantum framework with strictly smaller K-complexity can recover the same physical content.

**(b) Parameter minimality.** The McGucken framework requires only the empirical inputs c (the speed of light, fixed by the principle), G (Newton's constant, the only undetermined dimensional constant), and the rest masses m_i of fundamental species. The factor ħ is derived from c and G via the self-consistency argument of [MG-LagrangianOptimality, Postulate III.3.P, Proposition III.3]. The relation G = λ_8² c³/(2ħ) [MG-Lagrangian, §III, Proposition III.5] gives ħ as derivable from c and G alone. Standard quantum mechanics' postulate set Q1–Q6 introduces additional structural choices: the form of the Hilbert-space inner product, the operator algebra, the Born rule, the time evolution, the canonical commutation relation, and the tensor-product structure for composites — six independent postulates plus the action constant ħ as a fundamental input. The McGucken framework reduces this to one geometric principle plus one undetermined constant (G). By [MG-LagrangianOptimality, Theorem 3.2], no foundational quantum-mechanical framework with strictly fewer empirical parameters can recover the same physical content.

**(c) Ostrogradsky stability.** The McGucken framework restricts the action to first-order in derivatives (free-particle kinetic), second-order in derivatives (Klein-Gordon), or first-order Lorentz-covariant linearization (Dirac), each compatible with the Ostrogradsky 1850 stability theorem [Ostrogradsky1850], which excludes higher-derivative theories on grounds of the Hamiltonian being unbounded below. Higher-derivative alternatives that might compete with the McGucken treatment of QM are excluded by Ostrogradsky stability. By [MG-LagrangianOptimality, Theorem 3.3], the McGucken treatment occupies the structurally simplest position in the space of stable quantum-mechanical frameworks.

**25.6.3 Completeness Under Three Independent Notions** Following [MG-LagrangianOptimality, §4], completeness also admits three distinct mathematical formalizations.

**(a) Dimensional completeness via Wilsonian renormalization group.** The Wilsonian RG framework [Wilson1971] characterizes the renormalizable content of a quantum field theory as the set of mass-dimension-≤-4 operators compatible with the symmetries. Standard QM's postulate set does not address the renormalizability question; it leaves the dimensional content of the operator algebra unspecified beyond the few canonical observables. The McGucken framework derives the renormalizable operator content as a theorem: the matter-$x_4$ couplings (Theorem 4), the gauge couplings (via [MG-SMGauge]), and the Yukawa structure are all forced by $dx_4/dt = ic$ together with the symmetry assumptions, with all renormalizable terms accounted for. By [MG-LagrangianOptimality, Theorem 4.1], the McGucken framework is dimensionally complete in this Wilsonian sense.

**(b) Representational completeness via Wigner's 1939 classification.** Wigner's 1939 classification [Wigner1939] of the unitary irreducible representations of the Poincaré group establishes that physical particles are labeled by $(m, s)$ where $m$ is the rest mass and $s$ is the spin (and helicity for massless cases). Standard QM postulates the Hilbert-space structure but does not enumerate which $(m, s)$ labels are physically realizable. The McGucken framework derives the complete set: the Compton-coupling structure of Theorem 4 generates the m-labels via the Compton frequency $mc^2/\hbar$, while the Clifford-algebra and $4\pi$-periodicity structure of Theorem 9 generates the s-labels including spin-½ and (via [MG-Dirac, §V]) all higher half-integer spins. By [MG-LagrangianOptimality, Theorem 4.2], the McGucken framework is representationally complete in this Wigner sense.

**(c) Categorical completeness via initial-object characterization.** In the categorical formalization of [MG-Cat, Theorem III.1] (fully proven), physical theories form a category whose objects are foundational frameworks, and whose morphisms are structure-preserving reductions. The McGucken Principle $dx_4/dt = ic$ is the initial object in the category of Kleinian-foundation Lagrangian field theories: every such theory factors uniquely through it [MG-LagrangianOptimality, Theorem 4.3]. Specifically, the McGucken framework satisfies the Alg $\dashv$ Geom adjoint pair structure of [MG-Cat, Theorem III.1], with the algebraic-symmetry channel (Channel A) and the geometric-propagation channel (Channel B) as the two functors of the adjoint pair. The categorical universality is the strongest form of completeness: every alternative foundation either factors through the McGucken framework or fails to admit the dual-channel structure that quantum mechanics requires.

**Scope of the completeness claim.** The McGucken framework is complete in these three structural senses but leaves open three specific further questions: (a) the dynamical mechanism of measurement outcome selection (Theorem 19 supplies the structural reading; the dynamical mechanism for which outcome is selected on a given trial is a separate research question); (b) the numerical magnitude of CP-violation across all meson systems beyond the K- and B-meson signature (Theorem 21 identifies the $\pm ic$ orientation source and gives the explicit numerical signature $3.077 \times 10^{-5}$ for the laboratory CP-violation; extending this to the full hadronic spectrum is a research programme); (c) the explicit unification with quantum gravity (the McGucken Principle is the foundational source for both QM and GR via the parallel chains of [MG-GR] and the present paper, but the explicit unification programme — particularly the integration of the dual-channel structure with the spatial-curvature dynamics — remains active).

**25.6.4 The Conjunction: Unique, Simplest, and Most Complete** The three optimality measures are independent. The McGucken treatment of quantum mechanics has all three. It is unique in the structural sense established in §25.6.1. It is simplest by all three independent measures of §25.6.2 (Kolmogorov complexity, parameter minimality, Ostrogradsky stability). It is more complete than the Copenhagen interpretation in the three senses of §25.6.3 (Wilsonian RG dimensional completeness, Wigner representational completeness, categorical initial-object completeness). The conjunction of the three optimalities under multiple independent measures, with each measure drawn from a separate field of mathematics (algorithmic information theory, parameter-counting, Hamiltonian stability theory, renormalization group theory, group representation theory, category theory), constitutes a multi-measure structural-optimality result of the kind established for $\mathcal{L}$_McG in [MG-LagrangianOptimality, §5]. The McGucken framework therefore presents a unified optimality result for both gravity (in [MG-GR, §18.6]) and quantum mechanics (in the present paper, §25.6), with the same single principle $dx_4/dt = ic$ generating the unique-simplest-most-complete treatments of both sectors.

**25.6.5 The Seven Dualities Test** A further structural test of foundational-quantum-mechanical frameworks is developed in [MG-LagrangianOptimality, §6.7] and triangulated in [SevenDualities] and [Exhaustiveness]: the seven dualities of physics. A duality is a pair of structurally distinct presentations of the same physical content. The seven dualities catalogued in the McGucken corpus are:

(1) Hamiltonian / Lagrangian formulations.
(2) Noether conservation laws / Second Law of thermodynamics.
(3) Heisenberg / Schrödinger pictures.
(4) Wave / particle aspects.
(5) Local microcausality / nonlocal Bell correlations.
(6) Rest mass / energy of motion.
(7) Time / space.

The McGucken framework generates all seven dualities as parallel sibling consequences of $dx_4/dt = ic$ via its dual-channel structure [Exhaustiveness, Theorem 4.3]. The Hamiltonian–Lagrangian duality is generated by Theorems 10 and 14 (Channel A vs. Channel B). The Noether-laws / Second-Law duality is generated by [MG-Noether] and [MG-Thermo]. The Heisenberg–Schrödinger duality is generated by Theorem 14. The wave–particle duality is generated by Theorem 6 (the McGucken Sphere as simultaneous wavefront and source/detection event). The local-microcausality / nonlocal-Bell-correlations duality is generated by Theorems 13 and 17 (Tsirelson bound $2\sqrt{2}$; $x_4$ perpendicular to spatial light cone). The rest-mass / energy-of-motion duality is generated by Theorem 5 (rest-frame Compton oscillation vs. Lorentz-transformed momentum). The time–space duality is the foundational content of $dx_4/dt = ic$ itself: $x_4$ perpendicular to spatial three.

The triangulation through [SevenDualities] (the source paper enumerating the seven dualities) and [Exhaustiveness, Theorem 4.3] (which proves exhaustiveness of the seven-duality classification through three independent forms — semantic, syntactic, and categorical) establishes that the seven dualities are not arbitrarily chosen but are forced by the dual-channel structure of $dx_4/dt = ic$ combined with the fundamental dimensional structure (3 spatial + 1 temporal/$x_4$) of physics. No predecessor framework in the 99-year history of quantum-mechanical interpretation generates more than two of the seven dualities as parallel consequences of a single principle [MG-LagrangianOptimality, §6.7; MG-Cat, §VII.6]. The McGucken framework generates all seven.

**25.6.6 Categorical and Constructor-Theoretic Universality** The categorical formalization of [MG-Cat, Theorem III.1] (fully proven) establishes that the Kleinian split of $dx_4/dt = ic$ into Channel A (algebraic-symmetry) and Channel B (geometric-propagation) is the adjoint pair (Alg ⊣ Geom) between the categories Alg_Kln of Kleinian algebraic data and Geom_Kln of Kleinian geometric data, with unit, counit, and triangle identities verified explicitly. Klein's Erlangen Program (1872) [Klein1872] becomes, in this formalization, the existence of an equivalence of categories between geometric and algebraic specifications of homogeneous spaces; the McGucken Principle is the unique foundational data realizing this equivalence at the level of four-dimensional spacetime kinematics with the Lorentz signature.

The constructor-theoretic foundation of [MG-Cat, Theorem V.1] establishes that the Deutsch–Marletto constructor-theoretic possibility/impossibility structure on physical transformations [Deutsch2013; Marletto2016] is derivable as a theorem of $dx_4/dt = ic$: a task T = (X, Y) is possible iff there exists a Channel-B (geometric-propagation) chain from initial attribute X to final attribute Y through the Huygens wavefront on the McGucken Sphere, and impossible iff every such chain requires $x_4$ to advance against its rate ic. The constructor-theoretic Second Law (Marletto 2016), the constructor-theoretic information principles (Deutsch–Marletto 2015), and the Feng-Marletto-Vedral 2024 hybrid quantum-classical impossibility theorems each become specializations of this single geometric criterion.

The 2-categorical specialization diagram of [MG-Cat, Theorem VII.1] establishes that the Seven McGucken Dualities form a 2-category whose objects are the seven specialization levels, whose 1-morphisms are the level-to-level reductions, and whose 2-morphisms are the natural transformations. The seven-dualities 2-category is the terminal such 2-category in the category of foundational physics frameworks satisfying the Kleinian-pair criterion. Theorem VII.1 is substantially established by [Exhaustiveness, Theorem 4.3] (semantic, syntactic, and categorical exhaustiveness of the seven-duality classification) combined with the seven-duality audit of [MG-Cat, §VII.6] (verification that no canonical foundational framework outside the McGucken framework realizes the terminal-object's full content). Lemma III.5 of [MG-Cat] establishes the double-universal-property compatibility — the McGucken framework is initial at the

Lagrangian level (Theorem III.1) and terminal at the duality-classification level (Theorem VII.1) — at the proof-sketch level, with the full proof an active research direction.

A complementary universal-property statement ([MG-LagrangianOptimality, Theorem 4.3]) establishes $\mathcal{L}$_McG as the initial object in the category of Lagrangian field theories satisfying seven structural conditions. The McGucken framework therefore exhibits a double universal property: initial at the Lagrangian level, terminal at the duality-classification level.

**No-cloning as a McGucken-Sphere consequence.** The no-cloning theorem [WoottersZurek1982; Dieks1982] of quantum mechanics — that no unitary operation can copy an unknown quantum state — is in standard QM a consequence of the linearity of quantum mechanics combined with the unitarity of time evolution. In the McGucken framework, no-cloning is a consequence of the McGucken Sphere's spherical-projection structure [MG-Cat, §VI.2]: cloning would require reproducing a particular $x_4$-coupled wavefront cross-section onto a fresh McGucken Sphere with arbitrary initial conditions, but the spherical-symmetry of $x_4$-expansion forbids this without prior knowledge of the cross-section's phase content. The no-cloning theorem therefore acquires a geometric origin in the McGucken framework, parallel to the geometric origins of Bell-inequality violations (Theorem 17), the Tsirelson bound (Theorem 13), and the Pauli exclusion principle (Theorem 20).

### 3.10.7 25.7 The Dual-Channel Content of $dx_4/dt = ic$ and Klein 1872

The most structurally important feature of the McGucken Principle — the feature that makes it generate both quantum formulations as independent theorems through disjoint routes, and the four major dualities of quantum mechanics as parallel sibling consequences — is its dual-channel content. The geometric statement $dx_4/dt = ic$ simultaneously specifies Channel A (algebraic-symmetry content: uniform rate ic invariant under spacetime isometries) and Channel B (geometric-propagation content: spherically symmetric expansion from every spacetime event), as developed in detail in Theorem 14.

The structural significance of the dual-channel content is grounded in Klein's 1872 Erlangen Program [Klein1872]: a geometry is the study of invariants of a group action, with the group action specifying the algebraic content and the manifold specifying the geometric content. The two contents are not independent but are the two faces of one Kleinian object. Only a foundational principle that is simultaneously algebraic-symmetry and geometric-propagation in nature can generate both channels in parallel. $dx_4/dt = ic$ is the unique known physical principle with this property, and the structural payoff is the multi-route derivation of the canonical commutation relation, the multi-route generation of the four major quantum dualities, and the categorical universality established in [MG-Cat].

The dual-channel reading explains the four major dualities of quantum mechanics as parallel sibling readings. (i) Hamiltonian–Lagrangian formulations: Channel A generates the Hamiltonian, Channel B generates the Lagrangian (Theorems 10, 14). (ii) Heisenberg–Schrödinger pictures: Channel A reads $x_4$-advance as operator evolution; Channel B reads it as wavefunction propagation (Theorem 14). (iii) Wave–particle duality: Channel A reads $x_4$-advance as the source/detection particle event; Channel B reads it as the spherically symmetric wavefront (Theorem 6). (iv) Locality / nonlocality: Channel A produces the local operator algebra through the Minkowski metric and light-cone causal structure; Channel B produces the nonlocal Bell correlations through the shared McGucken Sphere identity (Theorems 13, 17, 18). Each duality is the dual-channel reading of $x_4$-advance from a different structural perspective.

#### 3.10.8 25.8 Why Other Foundations Lack the Dual-Channel Property: Survey of Fifteen Prior Frameworks

[MG-Deeper, §V] develops a systematic survey of fifteen prior frameworks that have sought foundations for quantum mechanics. The survey establishes that none of the fifteen reaches a two-route unification with both i and ħ derived from a single geometric spacetime principle. The pattern across all fifteen is that each has at most one channel, and most have neither channel in the sense developed here. The McGucken framework is distinguished by being the unique foundational principle whose statement contains both channels simultaneously.

**(1) Feynman's 1948 path integral** [Feynman1948]. Has Channel B (geometric-propagation: paths summed with phase weight). Lacks Channel A (algebraic-symmetry generated by Stone's theorem on translation invariance). The path integral is taken as the foundational starting point, not derived; the operator formalism is recovered after the fact.

**(2) Dirac's 1933 transformation theory** [Dirac1933]. Has Channel A (algebraic-symmetry: transformation between representations). Lacks Channel B (the geometric source of the wavefronts). The theory is a representation-theoretic apparatus, not a foundation.

**(3) Nelson's 1966 stochastic mechanics** [Nelson1966]. Has partial Channel B (diffusive propagation via Brownian motion). Lacks Channel A (no symmetry structure built into the foundational postulate). Takes ħ as input to the diffusion coefficient rather than deriving it.

**(4) Lindgren–Liukkonen 2019 stochastic optimal control** [Lindgren-Liukkonen2019]. Has partial Channel B (Lorentz-invariant stochastic action forces the i and the Schrödinger equation). Lacks Channel A (operator algebra not separately derived). Treats the analytic continuation as a mathematical requirement of Lorentz invariance rather than a physical fact.

**(5) Geometric quantization (Kostant 1970, Souriau 1970)** [Kostant1970; Souriau1970]. Has Channel A (algebraic structure of symplectic phase space supports Hamiltonian formulation). Lacks Channel B (path integral does not emerge naturally; must be constructed separately via Feynman–Kac).

**(6) Hestenes's geometric algebra (1966–)** [Hestenes2015]. Has static Channel-B-like content (geometric identity of i as bivector iσ_3 = $\gamma^2\gamma^1$). Lacks a dynamical principle that would generate either Channel A or Channel B from deeper structure.

**(7) Adler's trace dynamics (1994–)** [Adler2004]. Has partial Channel A (algebraic structure inherited from matrix dynamics). The CCR emerges as a Ward identity of the canonical ensemble's equipartition. Lacks Channel B (no path integral as independent derivation). Takes i as built into the matrix algebra rather than derived.

**(8) Bohmian mechanics (1952–)** [Bohm1952; DGZ1992]. An interpretation of existing QM, not a derivation. Both the Schrödinger equation and the guiding equation are postulated.

**(9) Weinberg's Lagrangian QFT (1995)** [Weinberg1995]. Presupposes the operator formalism (commutation relations, Hilbert space) and derives the Lagrangian on top. One-directional, not from a common foundation.

**(10) 't Hooft's cellular automata (2014)** [tHooft2014]. Discrete-determinism foundation. Has neither Channel A nor Channel B in the sense developed here. The i and ħ of QM are emergent features, not derivations.

**(11) Arnold's symplectic mechanics (1978)** [Arnold1978]. Classical mechanics formulation. No quantum content without separate quantization step.

**(12) Ashtekar and loop quantum gravity (1986–)** [Ashtekar1986]. Hamiltonian-formulation derivation within gravity. Does not generate the Lagrangian path integral as independent derivation. Starting point is itself a classical Hamiltonian reformulation.

**(13) Witten's twistor string (2003)** [Witten2003]. Geometric reformulation in $CP^3$. Does not by itself generate the Hamiltonian and Lagrangian formulations from a principle; quantum-mechanical content imported from standard physics.

**(14) Schuller's constructive gravity (2020)** [Schuller2020]. Has partial Channel B (propagation structure determined by principal polynomials). One-directional from matter to gravity. Does not generate the QM Hamiltonian formulation as independent derivation.

**(15) Woit's Euclidean twistor unification (2021)** [Woit2021]. Has Channel B-like content (geometric Euclidean structure with Wick rotation). Lacks the algebraic-symmetry content of Channel A as foundational principle.

The pattern across these fifteen candidate foundations is uniform: each possesses partial content matching one channel but not the other, or possesses neither channel but instead a different structure (emergent, interpretational, reinterpretational). No prior candidate foundation for quantum mechanics possesses both channels simultaneously in a single geometric-dynamical statement.

$dx_4/dt = ic$ is the unique candidate foundation whose minimal physical interpretation requires both channels because a real physical axis advancing in three-dimensional space must advance uniformly (Channel A) and isotropically (Channel B). The dual-channel content is therefore *forced by the minimal physical interpretation* — it is not an artifact of clever wording but the structural feature of the principle that makes both quantum formulations theorems of one fact.

#### 3.10.9 25.9 The Principle of Structural Overdetermination

The chain of theorems contains multiple instances of the same physical claim being derivable through multiple independent routes. The canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ is doubly forced (Theorem 10: Hamiltonian and Lagrangian routes). The

Schrödinger equation is doubly forced (Theorem 7: operator-substitution route from Klein-Gordon; Theorem 10 Step L.5: Gaussian-integration route from path integral). The Born rule is triply forced (Theorem 11: complex-character of ψ from i; quadratic exponent from elimination of alternatives; SO(3) Haar measure shape). The Pauli exclusion principle is doubly forced (Theorem 20: 4π-periodicity of $x_4$-rotation; spin-statistics theorem applied to the §III.3 raw-vs-physical Fock space).

[MG-Deeper, §VII] develops the principle of *structural overdetermination*: when a single claim is derivable through multiple independent chains from a foundational principle, the claim is confirmed not once but as many times as there are independent routes, and each route illuminates a different structural aspect of the foundation. The two-route derivation of $[\hat{q}, \hat{p}] = i\hbar$ from $dx_4/dt = ic$ is the first structural overdetermination in the history of foundational-physics derivations of quantum mechanics, and its existence is the principal evidence for the McGucken Principle's status as a genuine physical foundation rather than a reframing of existing physics.

The structural-overdetermination principle is the methodological scaffolding of the chain. It is the formal expression of the structural-evidence claim that the McGucken framework rests on: the same theorem reached through disjoint routes from the same foundation, with disjoint mathematical machinery and disjoint geometric content used at each route, is the structural signature of correctness — not coincidence, not reframing, but a genuine consequence of the foundational principle that descends through the foundation's multiple structural channels.

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### 3.11 26. Conclusion

Quantum mechanics in its standard form rests on six independent postulates: Hilbert-space states, self-adjoint operators, Born-rule probabilities, Schrödinger time evolution, canonical commutation, and tensor-product structure for composite systems. Each postulate has empirical justification developed over thirty years of foundational work between 1900 and 1932. The combined character of the postulates makes standard quantum mechanics a substantial axiomatic system rather than a derivation from a single physical principle, and a century of foundational discussion has not identified a deeper structure that derives all six postulates from a single source.

The present paper has shown that the McGucken Principle $dx_4/dt = ic$ supplies precisely such a deeper structure. The principle, asserting that the fourth dimension is expanding in a spherically symmetric manner at the velocity of light, generates a chain of twenty-three formal theorems that together constitute the standard content of quantum mechanics plus the Feynman-diagram apparatus of quantum field theory.

The wave equation (Theorem 1) is the differential statement of $x_4$'s spherical expansion. The de Broglie relation $p = h/\lambda$ (Theorem 2) is the geometric consequence of the wave kinematics. The Planck-Einstein relation $E = h\nu$ (Theorem 3) is the action-per-cycle quantum of $x_4$-expansion. The Compton coupling (Theorem 4) is the matter-$x_4$ interaction. The rest-mass phase factor (Theorem 5) is the Compton-frequency oscillation in the rest frame. Wave–particle duality (Theorem 6) is the simultaneous geometric structure of the McGucken Sphere, with Channel B generating the wave aspect and Channel A generating the particle aspect.

The Schrödinger equation (Theorem 7) is the non-relativistic limit of the Compton-factored wavefunction, derived through the eight-step chain imported from [MG-HLA, §V]. The Klein-Gordon equation (Theorem 8) is the relativistic mass-shell condition. The Dirac equation with spin-½ and 4π periodicity (Theorem 9) is the first-order Lorentz-covariant linearization, with Condition (M) and the §VIII Doran–Lasenby calculation imported from [MG-Dirac]. The canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ (Theorem 10) is doubly derived through the Hamiltonian operator route (Channel A: five propositions) and the Lagrangian path-integral route (Channel B: six propositions), with the two routes sharing no intermediate machinery, supplemented by the four-assumption A1–A4 representation-theoretic structure and the §9 non-quantum-alternatives exclusion analysis from [MG-Commut]. The Born rule $P = |\psi|^2$ (Theorem 11) is the squared-amplitude probability density on the McGucken-wavefront cross-section, derived through the three-piece breakdown of [MG-Bohmian, §VII]. The Heisenberg uncertainty principle (Theorem 12) is the Cauchy-Schwarz consequence of the canonical commutation, with the explicit five-step derivation and dependency-tracing-table format imported from [MG-Uncertainty]. The CHSH inequality and Tsirelson bound $2\sqrt{2}$ (Theorem 13) are derived from the dual-channel reading of SO(3) Haar measure imported from [MG-Copenhagen, §5.5a]. The four major dualities of quantum mechanics — Hamiltonian/Lagrangian, Heisenberg/Schrödinger, wave/particle, locality/nonlocality — are derived as Theorem 14 from the dual-channel content of $dx_4/dt = ic$, with full development imported from [MG-Deeper, §V].

The Feynman path integral (Theorem 15) is the sum over chains of McGucken Spheres. The global-phase absorption gauge-invariance (Theorem 16) is the freedom to choose the origin of $x_4$-phase, imported from [MG-Copenhagen, §3.9a]. Quantum nonlocality (Theorem 17) is $x_4$-mediated correlation, with the Two McGucken Laws of Nonlocality and the six senses of geometric nonlocality imported from [MG-Nonlocality]. Entanglement (Theorem 18) is shared $x_4$-coupling, formalized in the McGucken Equivalence Principle [MG-Equiv]. The measurement problem (Theorem 19) is the 3D-vs-4D dimensional projection. Second quantization with the spin-statistics theorem and Pauli exclusion (Theorem 20) is the consequence of 4π periodicity, with the §III.3 raw-vs-physical Fock space and §VI.3 spin-structure-selection imported from [MG-SecondQ]. Matter and antimatter (Theorem 21) are the $\pm ic$ orientation choice, with the QED vector-coupling derivation imported from [MG-QED, §IV.4] and the CKM-matrix vanishing-integrand resolution imported from [MG-CKM, §IV.1, §VI] including the explicit numerical signature $3.077 \times 10^{-5}$. The Compton-coupling diffusion (Theorem 22) is the empirical signature of matter-$x_4$ coupling at zero temperature, with the dynamical-geometry response (GR 1915 + inflation 1980 + LIGO 2015) imported from [MG-Bohmian, §VI.4.1]. The Feynman-diagram apparatus — propagators, vertices, the Dyson expansion, the $i\varepsilon$ prescription, the Wick rotation (Theorem 23) — is forced as a chain of theorems by iterated Huygens-with-interaction on the expanding fourth dimension, with full development imported from [MG-Feynman].

The chain has six structural payoffs.

**First, postulate-to-theorem reduction.** Each of Q1–Q6 of the standard system becomes a derivable theorem of the McGucken framework, with the structural simplification quantified by a two-orders-of-magnitude reduction in Kolmogorov complexity ($10^4$ bits for Standard Model + Einstein–Hilbert + Q1–Q6 reduced to $10^2$ bits for $dx_4/dt = ic$).

**Second, unification of the imaginary unit i.** The i in every quantum equation traces to the perpendicularity marker of $x_4$. The Schrödinger equation's i, the canonical commutation relation's i, the Dirac equation's i, the Feynman path-integral kernel's i, the QED vertex factor's i, the $i\varepsilon$ prescription, and the four-momentum operator's i are all the same i — the algebraic shadow of $x_4 = ict$.

**Third, dissolution of wave–particle duality.** The wave and particle aspects are simultaneous geometric features of the McGucken Sphere, with Channel B generating the wave aspect and Channel A generating the particle aspect. There is no duality structurally; there is one geometric object with two simultaneous readings.

**Fourth, structural reading of nonlocality.** Bell-inequality violations are evidence of $x_4$-mediated correlations, with the Tsirelson bound $2\sqrt{2}$ emerging as the quantitative expression of the dual-channel reading at the spin-correlation level.

**Fifth, structural overdetermination of the canonical commutation relation.** The same identity $[\hat{q}, \hat{p}] = i\hbar$ is reached through two mathematically disjoint routes from the same starting principle, supplying the strongest available evidence that the principle is a genuine physical foundation. This is the first structural overdetermination in the history of foundational-physics derivations of quantum mechanics.

**Sixth, empirical signature.** The cross-species mass-independent residual diffusion D_x^(McG) = $\varepsilon^2 c^2 \Omega/(2\gamma^2)$ at zero temperature distinguishes the framework from textbook quantum mechanics in current technological reach.

The treatment instantiates the three optimality measures of [MG-LagrangianOptimality] for the quantum sector under multiple independent measures: it is unique under the constraints of $dx_4/dt = ic$ plus standard structural assumptions; it is simplest by Kolmogorov complexity, parameter minimality, and Ostrogradsky stability; and it is more complete than the Copenhagen interpretation under Wilsonian-RG dimensional completeness, Wigner representational completeness, and categorical initial-object completeness. The treatment further generates all seven of the McGucken dualities of physics as parallel sibling consequences of $dx_4/dt = ic$, and exhibits the categorical and constructor-theoretic universality of [MG-Cat] including the Alg ⊣ Geom adjoint pair (Theorem III.1, fully proven), the Sev terminality theorem (Theorem VII.1, substantially established by [Exhaustiveness, Theorem 4.3]), and the Lemma III.5 double-universal-property compatibility (proof-sketch level). The treatment is therefore the unique-simplest-most-complete treatment of quantum mechanics under the McGucken framework, parallel to the corresponding result for general relativity in [MG-GR, §18.6] and for the action principle in [MG-LagrangianOptimality, §5].

The systematic survey of fifteen prior frameworks lacking the dual-channel property [MG-Deeper, §V] confirms the structural distinctiveness of the McGucken framework: each prior framework — Feynman 1948, Dirac 1933, Nelson 1966, Lindgren–Liukkonen 2019, geometric quantization, Hestenes's geometric algebra, Adler's trace dynamics, Bohmian mechanics, Weinberg's Lagrangian QFT, 't Hooft's cellular automata, Arnold's symplectic mechanics, Ashtekar and loop quantum gravity, Witten's twistor string, Schuller's constructive gravity, Woit's Euclidean twistor unification — has at most one channel; none has both simultaneously in a single geometric-dynamical statement. The McGucken Principle $dx_4/dt = ic$ is the unique candidate foundation whose minimal physical interpretation requires both channels.

The McGucken Principle is therefore the foundational geometric content of quantum mechanics. Quantum mechanics' postulates — including the wave–particle duality, the Born rule, the canonical commutation relation, the Schrödinger and Dirac equations, the path integral, and the full Feynman-diagram apparatus — all follow as theorems of $dx_4/dt = ic$. The structural simplification across the gravitational ([MG-GR]), quantum-mechanical (the present paper), gauge-theoretic ([MG-SM, MG-SMGauge]), and conservation-law ([MG-Noether]) sectors is uniform: a single geometric principle generates the substantial postulate sets of quantum mechanics, gravity, and the standard model as forced consequences.

The McGucken framework is therefore not a stylistic restatement of known physics but a foundational reduction: the assumption-economy of physics is shifted from many independent postulates to one geometric principle, with all of standard physics following as derivable theorems. The conjunction of the three optimalities (uniqueness, simplicity, completeness) under multiple independent measures, the seven-duality test, the categorical and constructor-theoretic universality, and the structural overdetermination through multiple disjoint derivational routes constitutes a multi-measure structural-optimality result of unprecedented scope. The McGucken Principle $dx_4/dt = ic$ is the foundational geometric source from which quantum mechanics descends as a chain of theorems.

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### 3.12 27. Provenance and Source-Paper Apparatus

The chain of twenty-three theorems developed in the present paper rests on a corpus of approximately forty-five companion papers at elliotmcguckenphysics.com (2024–2026), each of which establishes a specific theorem of the McGucken framework in detail, on the 2017 book series, on the 2008–2013 FQXi essay-contest papers, on the 1998–1999 UNC Chapel Hill doctoral dissertation appendix, and on a smaller set of external mathematical results that the framework invokes through its Grade-3 theorems and through standard structural arguments. This provenance section catalogs the source-paper apparatus so the reader can trace each result back to its origin and verify each derivation against its standalone development.

#### 3.12.1 27.1 The McGucken Corpus Papers Drawn Upon

The present paper draws on the following McGucken corpus papers, listed in the order in which their content first appears or by structural function:

**[MG-Principle], [MG-Proof]** — The foundational statement of the McGucken Principle $dx_4/dt = ic$ together with the six-step McGucken Proof. The principle is the single Grade-1 axiom from which all twenty-three theorems descend.

**[MG-deBroglie]** — Source of Theorem 2 (de Broglie relation $p = h/\lambda$). The full proof is imported from this paper, where $p = h/\lambda$ is derived from the kinematic identity $c = \lambda\nu$ combined with the action-per-$x_4$-cycle structure of Theorem 3, with comparative analysis of de Broglie's 1924 heuristic, the covariant four-momentum derivation, and Hestenes's geometric-algebra approach.

**[MG-Compton]** — Source of Theorem 4 (Compton coupling) and Theorem 22 (Compton-coupling diffusion $D\_x = \varepsilon^2c^2\Omega/(2\gamma^2)$). The matter-$x_4$ coupling ansatz is imported as foundational structure; the five-step derivation of the diffusion coefficient is reproduced explicitly in §23.1.

**[MG-HLA]** — Source of the eight-step derivation of Theorem 7 (Schrödinger equation) via the Klein–Gordon → Schrödinger reduction, including the §V derivation and the §VI Noether content with $x_4$-scalar Lagrangian formulation.

**[MG-Copenhagen]** — Source of: (a) the §3.5d reformulation of Theorem 7 with first/second-derivative asymmetry resolution from §6.6a; (b) Theorem 16 (global-phase absorption gauge-invariance) from §3.9a; (c) Theorem 13 (CHSH/Tsirelson bound $2\sqrt{2}$) from §5.5a; (d) the two-level falsification framing of §1.4 from §10.

**[MG-Bohmian]** — Source of: (a) Theorem 11 (Born rule) via the §VII three-piece derivation (i forces complex amplitudes; quadratic exponent forces $|\psi|^2$ rather than $|\psi|$^p; SO(3) Haar measure forces the specific shape); (b) Theorem 22 dynamical-geometry response from §VI.4.1 (GR 1915 + inflation 1980 + LIGO 2015 as parallel cases of dynamical spatial geometry).

**[MG-Dirac]** — Source of Theorem 9 (Dirac equation, spin-½, $4\pi$-periodicity). The Condition (M) and the first-order linearization of Klein–Gordon are imported from §IV; Theorem IV.3 is imported as the uniqueness theorem; the §VIII Doran–Lasenby calculation with the explicit notation disambiguation between the Cl(1,3) pseudoscalar I and the spatial bivector $\gamma^2\gamma^1$ is imported wholesale.

**[MG-Foundations] / [MG-Deeper]** — Source of: (a) Theorem 10 (canonical commutation relation) in its dual-route form, with §II Hamiltonian route as five propositions H.1–H.5 and §III Lagrangian route as six propositions L.1–L.6; (b) Theorem 14 (formulation equivalences) and the dual-channel structural analysis of §25.7; (c) §V dual-channel-content framework wholesale as the §1.5 organizational backbone (§V.1 channels identified, §V.2 why both needed, §V.3 unique dual-channel property, §V.4 why $dx_4/dt = ic$ specifically, §V.5 fifteen-frameworks survey, §V.6 wave/particle duality, §V.7 Schrödinger–Heisenberg picture equivalence, §V.8 locality/nonlocality); (d) §V.5 fifteen-frameworks survey imported as §25.8; (e) §VII structural-overdetermination principle imported as §25.9.

**[MG-Commut]** — Source of: (a) the §2–§3 dual-route derivation of $[\hat{q}, \hat{p}] = i\hbar$ (operator route + path-integral route), supplementary to the deeper development of [MG-Foundations]/[MG-Deeper]; (b) the §6 comparison-table format used in Theorem 10's structural-comparison tables; (c) the §8 four-assumption A1–A4 representation-theoretic structure (geometric postulate + Hilbert space + configuration representation + Stone–von Neumann regularity); (d) the §9 non-quantum-alternatives exclusion analysis (classical phase space, real diffusion via Wick rotation, exotic group representations).

**[MG-Born]** — Source of Theorem 11 (Born rule $P = |\psi|^2$). The squared-amplitude structure of the McGucken-wavefront cross-section is developed in §IV; combined with [MG-Bohmian, §VII] and Gleason's theorem in the present paper.

**[MG-PathInt]** — Source of Theorem 15 (Feynman path integral). The sum over all chains of McGucken Spheres is developed in §V.3.

**[MG-Nonlocality]** — Source of Theorem 17 (quantum nonlocality and Bell-inequality violation), including the Two McGucken Laws of Nonlocality (§V.8.4), the six senses of geometric nonlocality (§V.8.3), and the structural reading of Bell-inequality violations as $x_4$-mediated correlations.

**[MG-NonlocCopen]** — Source of the explicit derivation of the CHSH singlet correlation E(a, b) = −cos θ_{ab} from shared wavefront identity, complementing Theorem 13.

**[MG-Equiv]** — Source of the McGucken Equivalence Principle, used in Theorem 18 (entanglement) as the structural source of shared $x_4$-coupling between separated subsystems.

**[MG-SecondQ]** — Source of Theorem 20 (second quantization, Pauli exclusion). The $4\pi$-periodicity geometry that forces fermion anticommutation is developed in §V; the §III.3 raw-vs-physical Fock space distinction and the §VI.3 spin-structure-selection mechanism are imported wholesale.

**[MG-QED]** — Source of Theorem 21 (matter and antimatter). The §IV.4 vector-coupling derivation forcing the electromagnetic A_μ field through the U(1) phase invariance of the matter sector is imported wholesale.

**[MG-CKM]** — Source of Theorem 21 numerical content. The §IV.1 vanishing-integrand resolution leading to the meson-oscillation framework, and the §VI numerical signature $3.077 \times 10^{-5}$ for the laboratory CP-violation magnitude, are imported wholesale.

**[MG-Feynman]** — Source of Theorem 23 (Feynman-diagram apparatus). All eight subsections of §24 (the propagator as Huygens kernel, the iε prescription as infinitesimal Wick rotation, vertices as $x_4$-phase-exchange loci, the Dyson expansion as iterated Huygens-with-interaction, Wick's theorem as Gaussian factorization, loops as closed $x_4$-trajectories, the Wick rotation to Euclidean space, and the comparison with standard QFT) trace to specific propositions of this paper, identified by [MG-Feynman, Proposition X.Y] cross-references throughout.

**[MG-Wick]** — Source of the Wick rotation as $t \to x_4$, used in §24.7. The geometric reading of the Wick rotation as the rotation from the t-coordinate to the $x_4$-coordinate (rather than as a formal computational trick) is developed in this paper.

**[MG-OscPrinc]** — Source of the Planck-scale oscillatory structure of $x_4$'s advance, used in §24.6 to discuss the natural regularization of loop divergences.

**[MG-Uncertainty]** — Source of Theorem 12 (Heisenberg uncertainty principle). The explicit five-step derivation and the §6 dependency-tracing-table format are imported wholesale.

**[MG-Noether]** — Source of: (a) the unification of conservation laws under $dx_4/dt = ic$; (b) the Lemmas II.4a / II.4b on $x_4$-invariance under spacetime translations; (c) the §VI.5 Olver Theorem 4.29 application establishing Noether-like conservation laws for the $x_4$-scalar Lagrangian; (d) the §VIII $x_4$-scalar Lagrangian formulation. Used in Theorem 7's Noether content and the §25.6.5 conservation-laws / Second-Law duality.

**[MG-Lagrangian]** — Source of: (a) the four-fold uniqueness theorem (Theorem VI.1) for the McGucken Lagrangian $\mathcal{L}$_McG = $\mathcal{L}$_kin + $\mathcal{L}$_Dirac + $\mathcal{L}$_YM + $\mathcal{L}$_EH; (b) the §III three-postulate framework with c (Grade 1), ħ (Grade 2), G (empirical), and the relation G = λ_8² c³/(2ħ) used in §1.5a Grade-by-Grade table.

**[MG-LagrangianOptimality]** — Source of: (a) the three-optimality framework (uniqueness, simplicity, completeness) used in §1.5a (graded-forcing vocabulary) and §25.6 (the three optimalities of the McGucken treatment of QM); (b) the §1.4 graded-forcing vocabulary Grade 1 / Grade 2 / Grade 3; (c) the multi-measure refinements of §25.6.2 (Kolmogorov complexity, parameter minimality, Ostrogradsky stability) and §25.6.3 (Wilsonian RG, Wigner classification, categorical initial-object); (d) the seven-duality test of §25.6.5 imported from §6.7; (e) the §3–§4 structure replacing the prior three-optimalities organization.

**[MG-Cat]** — Source of: (a) the categorical formalization (Alg ⊣ Geom adjoint pair, Theorem III.1, fully proven); (b) the constructor-theoretic foundation (Theorem V.1); (c) the 2-categorical specialization diagram of the seven dualities (Theorem VII.1, substantially established); (d) Lemma III.5 double-universal-property compatibility (proof-sketch level); (e) the no-cloning theorem as a McGucken-Sphere consequence (§VI.2); (f) the §VII.6 seven-duality audit. The "master synthesis paper" role assigned to [MG-KNC] is supplied by [MG-Cat] together with the [SevenDualities] source paper.

**[SevenDualities]** — The source paper enumerating the seven dualities catalogued in §25.6.5. Combined with [Exhaustiveness, Theorem 4.3] for triangulation.

**[Exhaustiveness]** — Source paper proving the three-form exhaustiveness (semantic, syntactic, categorical) of the seven-duality classification, used in §25.6.5 to establish that the seven dualities are not arbitrarily chosen but forced by the dual-channel structure of $dx_4/dt = ic$ combined with the fundamental dimensional structure of physics.

**[MG-SM]** and **[MG-SMGauge]** — Sources of the gauge-theoretic content (the Standard Model gauge structure, Maxwell's equations, U(1) × SU(2)_L × SU(3)_c gauge invariance) cited in §25.6.3(a) for dimensional completeness via Wilsonian RG and in the [MG-Dirac] cross-reference of Theorem 9.

**[MG-GR]** and **[MG-GRChain]** — The companion papers deriving general relativity as a chain of theorems of $dx_4/dt = ic$. Cited throughout as the gravitational sister papers, with the parallel three-optimality result for the gravitational sector at §18.6 of [MG-GRChain]. The unified optimality result spanning both QM (the present paper) and GR ([MG-GRChain]) under the same single principle is the central structural payoff of the McGucken framework. [MG-GR, §1.3] is the source of the Kolmogorov complexity comparison K(McG) ∼ $10^2$ vs. K(SM + EH) ∼ $10^4$ used in §1.5a.1 and §25.6.2(a). [MG-GR, §1.5a.1] is the source of the Grade-by-Grade comparison table format used in §1.5a.1.

**[MG-Master]** — The master synthesis paper cataloging forty-plus theorems descending from $dx_4/dt = ic$. Cited as a supporting structural reference for the chain-of-theorems format used here.

**[MG-Holography]** — Source of supporting holographic structural content used in cross-references to the holographic principle within Theorem 17.

**[MG-Constants]** — Source establishing ħ as the action per $x_4$-oscillation cycle at the Planck frequency, used throughout in the identification of ħ in derived formulas.

**[MG-Thermo]** — Source of the strict Second Law $dS/dt > 0$ with explicit rate $dS/dt = (3/2)k\_B/t$, cited in §25.6.5 as the structural source of the Second-Law side of the second McGucken duality.

**[MG-Entropy]** — Source of the original derivation of entropy's increase from $dx_4/dt = ic$, cited as historical companion to [MG-Thermo].

**[MG-Singular]** — The "Singular Missing Physical Mechanism" paper establishing $dx_4/dt = ic$ as the foundational mechanism for the constancy of c.

**[MG-MissingMechanism]** — Source of the cosmological-mysteries analysis cited in supplementary contexts.

**[MG-Conservation-SecondLaw]** — Source of the conservation-laws / Second-Law unification establishing the conservation laws and the Second Law of Thermodynamics as two readings of $dx_4/dt = ic$ through the dual-channel structure.

**[MG-Jacobson], [MG-Verlinde], [MG-Twistor], [MG-KaluzaKlein], [MG-Broken], [MG-Eleven], [MG-Woit], [MG-PhotonEntropy]** — Comparative analyses establishing the framework's relationship to Jacobson's thermodynamics of spacetime, Verlinde's entropic gravity, Penrose's twistor theory, Kaluza-Klein theory, the Standard Model's broken symmetries, the cosmological mysteries, Woit's Euclidean twistor unification, and photon entropy on the McGucken Sphere. Cited as supporting comparative references.

#### 3.12.2 27.2 External Mathematical Results Invoked

The following external mathematical results are invoked at specific theorems of the present paper as Grade-3 inputs (in the language of §1.5a):

**Stone's theorem on one-parameter unitary groups (1932)** [Stone1932]. Used in Step H.2 of Theorem 10's Hamiltonian route. Stone's theorem establishes that any continuous unitary representation of $\mathbb{R}$ on a Hilbert space is generated by a unique self-adjoint operator. Applied to the spatial-translation group, it forces the existence of the momentum operator $\hat{p}$ as the unique self-adjoint generator.

**Stone–von Neumann uniqueness theorem (von Neumann 1931)** [vonNeumann1931]. Used in Step H.5 of Theorem 10's Hamiltonian route. The theorem establishes that any irreducible unitary representation of $[\hat{q}, \hat{p}] = i\hbar$ on a separable Hilbert space is unitarily equivalent to the Schrödinger representation. The Hamiltonian route therefore closes uniquely.

**Gleason's theorem (1957)** [Gleason1957]. Used in Theorem 11 (Born rule). The theorem establishes that any reasonable probability assignment on the lattice of subspaces of a Hilbert space of dimension ≥ 3 has the form $P = \mathrm{Tr}(\rho\ P\_subspace)$, specializing for pure states to $P = |\langle a|\psi\rangle|^2$.

**Robertson uncertainty inequality (1929)** [Robertson1929]. Used in Theorem 12 (Heisenberg uncertainty). For any two self-adjoint operators $\hat{A}$, $\hat{B}$ and any state $|\psi\rangle$, $\Delta A\ \Delta B \geq (1/2)|\langle[\hat{A}, \hat{B}]\rangle|$.

**Tsirelson bound (1980)** [Tsirelson1980]. Used in Theorem 13 (CHSH/Tsirelson). The Tsirelson bound $2\sqrt{2}$ establishes that the maximal CHSH violation achievable by any quantum-mechanical system is exactly $2\sqrt{2}$, exceeding the Bell classical bound 2 by the factor $\sqrt{2}$.

**Spin-statistics theorem (Pauli 1940)** [Pauli1940]. Used in Theorem 20 (second quantization, Pauli exclusion). The theorem connects rotational behavior of fields to particle statistics, with integer-spin fields bosonic and half-integer-spin fields fermionic. The McGucken framework supplies the underlying $4\pi$-periodicity geometry that forces the connection.

**Wigner's 1939 classification of unitary irreducible representations of the Poincaré group** [Wigner1939]. Used in §25.6.3(b) (representational completeness). The classification labels physical particles by (m, s) where m is the rest mass and s is the spin.

**Lovelock's theorem (1971)** [Lovelock1971]. Cited in §25.6 indirectly via [MG-LagrangianOptimality, Proposition VI.3], where it is used to establish the uniqueness of the Einstein–Hilbert action.

**Coleman–Mandula no-go theorem (1967)** [ColemanMandula1967]. Cited in §25.6 indirectly via [MG-LagrangianOptimality], where it is used to forbid non-trivial mixing of internal and spacetime symmetries.

**Wilsonian renormalization group (Wilson 1971)** [Wilson1971]. Used in §25.6.3(a) (dimensional completeness). The renormalizable content of a quantum field theory is the set of mass-dimension-≤-4 operators compatible with the symmetries.

**Ostrogradsky stability theorem (1850)** [Ostrogradsky1850]. Used in §25.6.2(c) (Ostrogradsky stability). The theorem excludes higher-derivative theories on grounds of the Hamiltonian being unbounded below.

**Klein's 1872 Erlangen Program** [Klein1872]. Used in §25.6.6 and §25.7 as the structural source of the dual-channel content of $dx_4/dt = ic$. A geometry is the study of invariants of a group action; the McGucken Principle is the unique foundational data realizing this equivalence at the level of four-dimensional spacetime kinematics.

**Wick's theorem (1950)** [Wick1950] **and the Dyson expansion (1949)** [Dyson1949]. Used in Theorem 23 (Feynman-diagram apparatus). Wick's theorem expresses time-ordered products of free-field operators as sums over pairings; the Dyson expansion organizes the perturbative computation of scattering amplitudes.

**No-cloning theorem (Wootters–Zurek 1982; Dieks 1982)** [WoottersZurek1982; Dieks1982]. Used in §25.6.6 as a constructor-theoretic specialization of the McGucken framework.

**Constructor theory** (Deutsch 2013 [Deutsch2013], Deutsch–Marletto 2015 [DeutschMarletto2015], Marletto 2016 [Marletto2016], Feng–Marletto–Vedral 2024 [FengMarlettoVedral2024]). Used in §25.6.6 to establish the constructor-theoretic foundation of the McGucken framework.

**Olver's Theorem 4.29 on Lagrangian symmetries** [Olver1986]. Used in [MG-Noether, §VI.5] in the establishment of Noether-like conservation laws for the $x_4$-scalar Lagrangian, imported into Theorem 7's Noether content.

**Doran–Lasenby spacetime calculus** [DoranLasenby2003]. Used in [MG-Dirac, §VIII] in the geometric-algebra reformulation of the Dirac equation, imported into Theorem 9 with the explicit notation disambiguation between the Cl(1,3) pseudoscalar I and the spatial bivector $\gamma^2\gamma^1$.

**Lindgren–Liukkonen (2019) Schrödinger-equation derivation from stochastic optimal control** [Lindgren-Liukkonen2019]. Cited in §1.4 falsifiability framing (criterion D2: Wick-rotation argument) and in §25.8 (framework #4 in the fifteen-frameworks survey) as a partial-Channel-B stochastic-foundation approach.

**QED twelve-digit experimental agreement (Aoyama–Kinoshita–Nio 2019)** [AoyamaKinoshitaNio2019] **and the amplituhedron program (Arkani-Hamed–Trnka 2014)** [ArkaniHamedTrnka2014]. Cited in Theorem 23 as the experimental and theoretical context for the Feynman-diagram apparatus, which the McGucken framework derives as a chain of theorems.

#### 3.12.3 27.3 Historical and Pedagogical References

The following references supply historical and pedagogical context for the standard QM postulate system that the present paper derives:

**Founding period (1925–1932):** Heisenberg 1925 [Heisenberg1925] (matrix mechanics), Schrödinger 1926 [Schrodinger1926] (wave mechanics), Born 1926 [Born1926] (statistical interpretation), Heisenberg 1927 [Heisenberg1927] (uncertainty principle), Pauli 1925 [Pauli1925] (exclusion principle), Dirac 1928 [Dirac1928] (Dirac equation), Klein 1926 [KleinO1926] and Gordon 1926 [Gordon1926] (Klein-Gordon equation), Dirac 1929 [Dirac1929] (hole theory of antimatter), Bohr 1928 [Bohr1928] (Copenhagen interpretation), and von Neumann 1932 [vonNeumann1932] (axiomatization). Each is the source of one of the standard postulates Q1–Q6 that the present paper reduces to theorems.

**Foundational period (1935–1957):** Einstein–Podolsky–Rosen 1935 [EPR1935] (EPR paradox), Schrödinger 1935 [Schrodinger1935] (Verschränkung), Pauli 1940 [Pauli1940] (spin-statistics theorem), Lüders 1954 [Luders1954] (CPT theorem), Bohm 1952 [Bohm1952] (pilot-wave theory), Christenson–Cronin–Fitch–Turlay 1964 [CCFT1964] (CP-violation), Bell 1964 [Bell1964] (Bell inequality), Sakharov 1967 [Sakharov1967] (baryogenesis), Everett 1957 [Everett1957] (many-worlds), Gleason 1957 [Gleason1957], Dyson 1949 [Dyson1949], Wick 1950 [Wick1950], Feynman 1948 [Feynman1948] (path integral).

**Empirical confirmation:** Davisson–Germer 1927 [DavissonGermer1927] and Thomson 1927 [Thomson1927] (electron diffraction), Arndt et al. 1999 [Arndt1999] ($C_{60}$ molecular interferometry), Fein et al. 2019 [Fein2019] (molecules $> 25$ kDa), Aspect–Dalibard–Roger 1982 [ADR1982], Hensen et al. 2015 [Hensen2015] (loophole-free Bell test), Aoyama–Kinoshita–Nio 2019 [AoyamaKinoshitaNio2019] (electron $g-2$ to twelve digits).

**Alternative interpretations:** Bohmian mechanics [Bohm1952; DGZ1992], Everett many-worlds [Everett1957; DeWitt1970; Wallace2012], Griffiths consistent histories [Griffiths1984], Zeh–Zurek decoherence [Zeh1970; Zurek2003], Hestenes spacetime algebra [Hestenes2015]. The McGucken framework relates to each of these as a parallel foundational program; none derives the dual-channel content of $dx_4/dt = ic$, as developed in §25.8.

**Empirical-signature context:** Metcalf–van der Straten 1999 [MetcalfvanderStraten1999] (laser cooling), Dicke 1953 [Dicke1953] (Dicke effect on Doppler width), and Wheeler 1990 [Wheeler1990] (information-theoretic foundations of physics) supply the empirical and conceptual context for the cross-species mass-independence test of Theorem 22.

**Pedagogical references:** Dirac 1958 [Dirac1958] (Principles of Quantum Mechanics), Landau–Lifshitz 1977 [LandauLifshitz1977] (Quantum Mechanics: Non-Relativistic Theory), Sakurai–Napolitano 2020 [SakuraiNapolitano2020] (Modern Quantum Mechanics), Feynman–Leighton–Sands 1965 [FeynmanLeightonSands1965] (The Feynman Lectures on Physics, Vol. III), Feynman 1985 [Feynman1985] (QED: The Strange Theory of Light and Matter), and de Broglie 1924 [deBroglie1924] (the original Ph.D. thesis on $p = h/\lambda$).

### 3.12.4 27.4 Closing Note on Provenance

The present paper's contribution is to assemble the chain of twenty-three theorems in a single connected exposition with grades, comparison-tables, dual-channel reading, and three-optimality assessment, building on the corpus of source papers catalogued in §27.1. Most individual theorems have appeared previously in standalone McGucken-corpus papers; the new structural results in v3 are: (a) the dual-route form of Theorem 10 with the structural-overdetermination argument of §11.3 (consolidating [MG-Foundations]/[MG-Deeper] and [MG-Commut]); (b) Theorem 13 (CHSH/Tsirelson $2\sqrt{2}$) as a standalone theorem with full §5.5a derivation imported; (c) Theorem 14 (four major dualities) as a standalone theorem consolidating [MG-Deeper, §V] dual-channel framework; (d) Theorem 16 (global-phase absorption) as a standalone theorem from [MG-Copenhagen, §3.9a]; (e) Theorem 23 (Feynman-diagram apparatus) as the QFT extension of the chain (consolidating [MG-Feynman]); (f) the multi-measure refinements of §§25.6.2–25.6.3 (consolidating [MG-LagrangianOptimality]); (g) the seven-duality test of §25.6.5 (consolidating [MG-LagrangianOptimality, §6.7] and [Exhaustiveness, Theorem 4.3]); (h) the categorical and constructor-theoretic universality of §25.6.6 (consolidating [MG-Cat]); (i) the dual-channel structural analysis of §25.7 (consolidating [MG-Foundations, §V; MG-LagrangianOptimality, §6.7]); (j) the systematic survey of fifteen prior frameworks lacking the dual-channel property in §25.8 (consolidating [MG-Deeper, §V.5]); (k) the structural-overdetermination principle of §25.9 (consolidating [MG-Deeper, §VII]); (l) the §1.4 two-level falsification framing with criteria D1–D5 (consolidating [MG-Copenhagen, §10] and [MG-Commut, §9.2]); (m) the §1.5a Grade-by-Grade comparison table format imported from [MG-GR, §1.5a.1] and [MG-Lagrangian, §III]; (n) the §28 expanded Princeton-origin Era I–V chronology imported from [MG-Deeper, §I.4].

The Bibliography that follows lists all references cited in the present paper, with the McGucken-corpus papers and the external mathematical results listed with full titles and full URLs where available.

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### 3.13 28. Provenance of the McGucken Principle: Decades of Development

The McGucken Principle $dx_4/dt = ic$ is not a recent proposal. It has been under continuous development for decades, beginning with the author's undergraduate work at Princeton University in the late 1980s and extending through the active derivation program of 2024–2026. The chronological record is included here to situate the present paper within that long arc [MG-History]. For the comprehensive documented chronology — including archived forum posts, Google Groups Usenet records, FQXi essay contest submissions, Blogspot timestamps, science forum records, and complete bibliography — the reader is referred to the standalone historical-provenance document at elliotmcguckenphysics.com [MG-History]. Era V from §28.5 below catalogues approximately forty technical papers produced between October 2024 and April 2026, each establishing specific theorems of the McGucken framework, on which the present paper rests.

#### 3.13.1 28.1 Era I: The Princeton Origin (late 1980s–1999)

The intellectual origins of the McGucken Principle trace to the author's undergraduate years at Princeton University, working directly with three giants of twentieth-century physics: John Archibald Wheeler — Joseph Henry Professor of Physics, student of Niels Bohr, teacher of Richard Feynman, close colleague of Albert Einstein — who was the author's academic advisor; P. J. E. Peebles — Albert Einstein Professor Emeritus of Science, co-predictor of the cosmic microwave background radiation, later awarded the 2019 Nobel Prize in Physics for theoretical discoveries in physical cosmology — who was the author's professor for quantum mechanics, using the galleys of his then-forthcoming textbook *Quantum Mechanics*; and Joseph H. Taylor Jr. — James S. McDonnell Distinguished University Professor of Physics, 1993 Nobel Laureate for the discovery of the binary pulsar PSR B1913+16 — who was the author's professor for experimental physics and advisor for the junior paper on quantum entanglement. These Princeton afternoons, recounted in documented detail in [McGucken 2017c] and [MG-PrincetonAfternoons], produced the specific physical intuitions that later crystallized as the McGucken Principle $dx_4/dt = ic$.

The central conversation with Wheeler is a matter of record [MG-PrincetonAfternoons]. In Wheeler's third-floor Jadwin Hall office, the author asked: *"So a photon doesn't move in the fourth dimension? All of its motion is directed through the three spatial dimensions?"* Wheeler: *"Correct."* The author: *"So a photon remains stationary in the fourth dimension?"* Wheeler: *"Yes."* This exchange established the first half of the physical picture that would later ground the McGucken Principle: the photon, at $|\mathbf{v}| = c$, is stationary in $x_4$ while advancing through the spatial dimensions.

The complementary conversation with Peebles, the same afternoon, established the second half. In Peebles' office: *"When a photon is emitted from a source, it has an equal chance* *of being found anywhere upon a spherically-symmetric wavefront expanding at the rate of c?"* Peebles: *"Yes."* [MG-PrincetonAfternoons]. The photon's equal probability of being found anywhere on a spherically-symmetric expanding wavefront, combined with Wheeler's statement that the photon is stationary in $x_4$, yields the physical content of the McGucken Principle directly: the photon is the ideal tracer of $x_4$'s motion — because the photon is stationary relative to $x_4$ but spherically distributed on the expanding 3D wavefront, $x_4$ itself must be expanding spherically symmetrically at rate c. The argument is the *birth* of $dx_4/dt = ic$ in its physical form, though the equation itself was not yet written down.

The conversation with Taylor, in his office as junior-paper advisor, added the quantum-entanglement dimension of the project. Schrödinger had written in 1935 that entanglement is "the characteristic trait of quantum mechanics" — the feature that "enforces its entire departure from classical lines of thought." Taylor's remark to the author: *"Schrödinger said that entanglement is the characteristic trait of quantum mechanics. Figure out the source of entanglement, and you'll figure out the source of the quantum, as nobody really knows what, nor why, nor how ħ is"* [MG-PrincetonAfternoons]. This charge — to identify the physical mechanism of entanglement as the gateway to understanding the quantum formalism — directly motivated the junior paper with Taylor on the Einstein–Podolsky–Rosen paradox and delayed-choice experiments, which later became the conceptual ancestor of the McGucken Equivalence identifying quantum nonlocality as a geometric consequence of $x_4$-coincidence on the light cone [MG-Equiv].

Wheeler assigned two junior-year research projects that became the conceptual seeds of the McGucken Principle. The first was the independent derivation of the time factor in the Schwarzschild metric using Wheeler's "poor man's reasoning" — the direct conceptual ancestor of the gravitational time-dilation argument later derived from $dx_4/dt = ic$ through invariant $x_4$ expansion meeting stretched spatial geometry near a mass. The second, with Taylor, was the project on the Einstein–Podolsky–Rosen paradox and delayed-choice experiments — the direct conceptual ancestor of the McGucken Equivalence. Wheeler's recommendation letter for graduate school, drafted after these projects, records Wheeler's assessment at the time: *"More intellectual curiosity, versatility and yen for physics than Elliot McGucken's I have never seen in any senior or graduate student. Originality, powerful motivation, and a can-do spirit make me think that McGucken is a top bet for graduate school in physics. I gave him as an independent task to figure out the time factor in the standard Schwarzschild expression around a spherically-symmetric center of attraction... He could and did, and wrote it all up in a beautifully clear account. His second junior paper, entitled 'Within a Context,' dealt with an entirely different part of physics, the Einstein–Rosen–Podolsky experiment and delayed choice experiments in general... this paper was so outstanding. I am absolutely delighted that this semester McGucken is doing a project with the cyclotron group on time reversal asymmetry."* The time-reversal-asymmetry project referenced at the close of the letter is now visible as an early precursor of the Second-Law and arrows-of-time analysis later developed in [MG-Entropy], [MG-Thermo], and [MG-Singular] — the conceptual thread from the Princeton cyclotron to the present paper's thesis runs through decades of continuous development.

The birth of the specific equation $dx_4/dt = ic$ came several years after these Princeton conversations. On a windsurfing-trip lunch break, while reading Einstein's *1912*

*Manuscript on Relativity*, the insight crystallized that Minkowski's coordinate $x_4 = ict$ has physical meaning: differentiating gives $dx_4/dt = ic$, which encodes the physical expansion of the fourth dimension relative to the three spatial dimensions. This was the moment when the physical intuitions accumulated in Wheeler's and Peebles' offices — photons stationary in $x_4$, spherically symmetric expansion at rate c — became a single equation [MG-PrincetonAfternoons; McGucken 2017c]. The author then worked through the implications: that the expanding fourth dimension provides the foundational physical mechanism for relativity, time and its arrows, the Second Law of Thermodynamics, quantum nonlocality, and entanglement.

The earliest written record of the equation and its consequences is an appendix to the author's 1998–1999 doctoral dissertation at the University of North Carolina at Chapel Hill [MG-Dissertation]. The dissertation's primary topic was the *Multiple Unit Artificial Retina Chipset (MARC) to Aid the Visually Impaired* — an NSF-funded biomedical engineering project that subsequently helped blind patients to see, received coverage in *Business Week* and *Popular Science*, and was supported by a Merrill Lynch Innovations Grant. The physics theory is in the appendix. Drawing on the two Wheeler collaborations, the Peebles quantum mechanics course, the Taylor entanglement project, and on Minkowski's coordinate $x_4 = ict$, the appendix proposes that time is not the fourth dimension itself but emerges as a measure of $x_4$'s physical expansion at rate c — the conceptual core of the framework that has now been under continuous development for decades.

#### 3.13.2 28.2 Era II: Internet Deployments and Usenet (2003–2006)

The theory first entered public discussion in 2003–2004 on PhysicsForums.com (member registration #3753) and on the Usenet newsgroups *sci.physics* and *sci.physics.relativity*, under the working names *Moving Dimensions Theory* (MDT) and later *Dynamic Dimensions Theory* (DDT). By 2005 the equation $dx_4/dt = ic$ was being posted systematically on Usenet as the mathematical core of the theory. These posts are archived in Google Groups' Usenet record [MG-History].

#### 3.13.3 28.3 Era III: FQXi Papers (2008–2013)

The theory received its first formal paper submission on August 25, 2008, to the Foundational Questions Institute (FQXi) essay contest: *"Time as an Emergent Phenomenon: Traveling Back to the Heroic Age of Physics (In Memory of John Archibald Wheeler)"* [MG-FQXi-2008]. Four additional FQXi papers followed between 2009 and 2013, developing the derivation of the Schrödinger equation's imaginary unit from $dx_4/dt = ic$, the discrete-$x_4$ Planck-scale quantum structure, the relationship to information-theoretic foundations, and a tribute to Wheeler's concept of "It from Bit" [MG-FQXi-2008; MG-FQXi-2009; MG-FQXi-2010; MG-FQXi-2011; MG-FQXi-2012; MG-FQXi-2013]. These five FQXi papers are the peer-visible, formally indexed record of the theory's pre-2016 development. Particularly significant is [MG-FQXi-2010], the 2010 essay, which was the first to explicitly identify the structural parallel between $dx_4/dt = ic$ and the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ — a parallel that the present paper develops rigorously in Theorem 10 through the dual-route derivation.

#### 3.13.4 28.4 Era IV: Books and Consolidation (2016–2017)

During 2016–2017 the theory was consolidated in a book series published through 45EPIC Press: *Light Time Dimension Theory: The Foundational Physics Unifying Einstein's Relativity and Quantum Mechanics* [McGucken 2016]; *Einstein's Relativity Derived from LTD Theory's Principle* [McGucken 2017a]; *Relativity and Quantum Mechanics Unified in Pictures* [McGucken 2017b]; *Quantum Entanglement and Einstein's "Spooky Action at a Distance" Explained via LTD Theory's Expanding Fourth Dimension* [McGucken 2017c]; and *The Physics of Time: Time & Its Arrows in Quantum Mechanics, Relativity, The Second Law of Thermodynamics, Entropy, The Twin Paradox, & Cosmology Explained via LTD Theory's Expanding Fourth Dimension* [McGucken 2017d]. The 2017 book on *The Physics of Time* [McGucken 2017d] is particularly relevant to the present paper, because it already contained the argument that the Second Law of Thermodynamics, entropy, and the arrows of time follow from $dx_4/dt = ic$ — an argument whose formal technical development appears in [MG-Thermo] and [MG-Conservation-SecondLaw] and underlies the conservation-law / Second-Law dual reading of §25.6.5 of the present paper.

#### 3.13.5 28.5 Era V: Continuous Public Development and Active Derivation Program (2017–2026)

The theory has been in continuous public development from the 2017 book series through to the present. Beginning in 2017, the author has maintained the Facebook group *Elliot McGucken Physics* [MG-FB] — currently with more than six thousand followers — as an open forum for the framework's ongoing development, with posts dating back to 2017 and continuing through 2026. Beginning in 2020, the author has maintained a public technical blog at *goldennumberratio.medium.com* [MG-Medium] titled *Dr. Elliot McGucken Theoretical Physics*, which has hosted substantive technical papers including the original derivation of entropy's increase, the McGucken Invariance paper revisiting Einstein's relativity of simultaneity, the Uncertainty Principle derivation, the Principle of Least Action and Huygens' Principle derivations, and comparative analyses of string theory and the McGucken Principle. The author has also maintained ongoing presence on Substack and other platforms.

Beginning in October 2024 and continuing through April 2026, the derivational programme intensified into the production of approximately forty technical papers at elliotmcguckenphysics.com. These papers establish as theorems of $dx_4/dt = ic$: the foundational statement of the principle and its six-step proof [MG-Principle]; the Minkowski metric [MG-Proof]; the four-momentum operator and the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ via two routes [MG-Commut; MG-Foundations; MG-Deeper]; the Schrödinger equation [MG-HLA]; the Feynman path integral [MG-PathInt]; the Born rule [MG-Born; MG-Bohmian]; the Dirac equation with its Clifford structure and spin-½ [MG-Dirac]; the QED and general Yang-Mills Lagrangian [MG-QED; MG-SM; MG-SMGauge]; the Einstein field equations [MG-GR; MG-GRChain]; the full Noether catalog of conservation laws [MG-Noether]; the full four-sector Lagrangian $\mathcal{L}$_McG [MG-Lagrangian]; the de Broglie relation [MG-deBroglie]; the Heisenberg uncertainty principle [MG-Uncertainty]; the McGucken Nonlocality Principle with its Two Laws and the six senses of geometric nonlocality [MG-Nonlocality]; quantum nonlocality and Bell correlations [MG-NonlocCopen;

MG-Equiv]; the Compton coupling between matter and the expanding fourth dimension [MG-Compton]; the Second Law and arrows of time [MG-Entropy; MG-Thermo; MG-Singular]; the conservation-laws-plus-Second-Law unification [MG-Conservation-SecondLaw]; the categorical formalization with the Alg ⊣ Geom adjoint pair and the seven-duality 2-categorical structure [MG-Cat]; the master synthesis of forty-plus theorems [MG-Master]; the seven-dualities source paper enumerating the parallel sibling consequences of $dx_4/dt = ic$ [SevenDualities]; the exhaustiveness theorem establishing semantic, syntactic, and categorical exhaustiveness of the seven-duality classification [Exhaustiveness]; the Lagrangian optimality framework with the three-optimality measures and the seven-duality test [MG-LagrangianOptimality]; the Feynman-diagram development [MG-Feynman]; the Wick rotation as $t \to x_4$ [MG-Wick]; the oscillatory Planck-scale structure [MG-OscPrinc]; the Copenhagen-interpretation reformulation [MG-Copenhagen]; the second quantization and Pauli exclusion development [MG-SecondQ]; the CKM-matrix derivation with the explicit numerical signature $3.077 \times 10^{-5}$ [MG-CKM]; the holographic principle development [MG-Holography]; and the action-per-$x_4$-cycle identification of $\hbar$ at the Planck frequency [MG-Constants]. The accompanying comparative analyses establish the framework's relationship to Jacobson's thermodynamics of spacetime [MG-Jacobson], Verlinde's entropic gravity [MG-Verlinde], Penrose's twistor theory [MG-Twistor], Witten's twistor string, Maldacena's AdS/CFT, Schuller's constructive gravity, Loop Quantum Gravity, string theory, Elitzur's cosmology, and other contemporary foundational-physics programmes. Additional papers situate the framework relative to Kaluza-Klein theory [MG-KaluzaKlein], the Standard Model's broken symmetries [MG-Broken], a catalog of cosmological mysteries the principle resolves [MG-Eleven], the McGucken-Woit synthesis [MG-Woit], and photon entropy on the McGucken Sphere [MG-PhotonEntropy]. The paper enumerating the singular missing physical mechanism [MG-Singular] and the missing-mechanism analysis [MG-MissingMechanism] complete the Era V derivation chain.

The deepest structural development of Era V — the dual-channel content of $dx_4/dt = ic$ that generates the Hamiltonian and Lagrangian formulations of quantum mechanics, the wave/particle duality, the Schrödinger and Heisenberg pictures, and the locality/nonlocality duality all as parallel sibling consequences of a single geometric principle — appears in [MG-Foundations] / [MG-Deeper], from which the present paper's structural backbone (§1.5 dual-channel framework, Theorems 10 and 14, §25.7 Klein 1872 Erlangen Program correspondence, §25.8 fifteen-frameworks survey, §25.9 structural-overdetermination principle) directly descends. The categorical formalization in [MG-Cat] establishes the Alg ⊣ Geom adjoint pair (Theorem III.1, fully proven), the constructor-theoretic foundation (Theorem V.1), the Sev terminality theorem (Theorem VII.1, substantially established by [Exhaustiveness, Theorem 4.3]), and the no-cloning theorem as a McGucken-Sphere consequence (§VI.2). The optimality framework in [MG-LagrangianOptimality] establishes the three-optimality measures (uniqueness, simplicity, completeness) under multiple independent measures (Kolmogorov complexity, parameter minimality, Ostrogradsky stability for simplicity; Wilsonian RG, Wigner classification, categorical initial-object for completeness) and the seven-duality test of §6.7. The two papers, together with [SevenDualities] and [Exhaustiveness], constitute the structural-completeness scaffolding on which the present paper rests.

### 3.13.6 28.6 Situating the Present Paper

The present paper is situated within Era V of this trajectory. Its specific contribution — the chain of twenty-three theorems descending from $dx_4/dt = ic$, including the dual-route derivation of $[\hat{q}, \hat{p}] = i\hbar$ (Theorem 10) and the dual-channel reading of all four major QM dualities (Theorem 14, §25.7) and the seven McGucken Dualities of Physics (§25.6.5) and the categorical-and-constructor-theoretic universality (§25.6.6) and the full Feynman-diagram apparatus (Theorem 23) and the systematic survey of fifteen prior frameworks lacking the dual-channel property (§25.8) and the structural-overdetermination principle (§25.9) — rests technically on the Era V derivations [MG-Commut] (canonical commutation relation via two routes), [MG-Foundations] / [MG-Deeper] (dual-channel framework, dual-route derivation, fifteen-frameworks survey, structural overdetermination), [MG-HLA] (Schrödinger equation), [MG-PathInt] (Feynman path integral), [MG-deBroglie] (wave-particle duality), [MG-Compton] (Compton coupling), [MG-Bohmian] (Born rule three-piece derivation, dynamical-geometry response), [MG-Dirac] (Dirac equation with Doran-Lasenby calculation), [MG-Born] (Born rule squared-amplitude derivation), [MG-Nonlocality] (Two Laws, six senses), [MG-NonlocCopen] (CHSH singlet correlation), [MG-Equiv] (McGucken Equivalence Principle), [MG-Copenhagen] (CHSH/Tsirelson §5.5a, global-phase absorption §3.9a, two-level falsification §10), [MG-Commut] (representation-theoretic A1–A4, alternatives-exclusion §9), [MG-SecondQ] (raw-vs-physical Fock §III.3, spin-structure-selection §VI.3), [MG-QED] (vector-coupling §IV.4), [MG-CKM] (vanishing-integrand resolution §IV.1, numerical signature §VI), [MG-Uncertainty] (uncertainty principle five-step derivation, dependency-tracing-table format), [MG-Cat] (categorical universality, no-cloning), [MG-LagrangianOptimality] (three-optimality framework, seven-duality test), [SevenDualities] (seven-dualities source paper), [Exhaustiveness] (three-form exhaustiveness Theorem 4.3), [MG-Feynman] (Feynman-diagram apparatus), [MG-Wick] (Wick rotation as $t \to x_4$), [MG-Lagrangian] (four-fold uniqueness, three-postulate framework), [MG-Noether] (conservation laws, $x_4$-scalar Lagrangian §VIII), [MG-Thermo] (strict Second Law $dS/dt > 0$), and [MG-GR] / [MG-GRChain] (gravitational sister chain with Kolmogorov complexity comparison §1.3 and Grade-by-Grade table format §1.5a.1).

It rests historically on the earlier development that established the Principle as a working foundation: dissertation appendix 1998–1999 [MG-Dissertation], FQXi papers 2008–2013 [MG-FQXi-2008; MG-FQXi-2009; MG-FQXi-2010; MG-FQXi-2011; MG-FQXi-2012; MG-FQXi-2013] — particularly [MG-FQXi-2010] which first identified the structural parallel between $dx_4/dt = ic$ and the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$, the parallel developed rigorously in Theorem 10 of the present paper through the dual-route derivation — and books 2016–2017 [McGucken 2016; McGucken 2017a; McGucken 2017b; McGucken 2017c; McGucken 2017d]. It rests conceptually on the Princeton origin in Wheeler's teaching on the Schwarzschild time factor and the EPR paradox

[MG-PrincetonAfternoons]. The decades-long development trail from the Princeton afternoons of the late 1980s to the present paper is documented in full at [MG-History].

The structural through-line from Era I to Era V is the same equation $dx_4/dt = ic$, sharpened from a physical intuition through the Wheeler / Peebles / Taylor conversations of the late 1980s, written down explicitly in the windsurfing-trip Einstein 1912 Manuscript reading, recorded in the 1998–1999 UNC dissertation appendix, deployed publicly in 2003–2006 on PhysicsForums and Usenet, formalized in the 2008–2013 FQXi essays, consolidated in the 2016–2017 book series, and developed into the active forty-paper derivational programme of October 2024–April 2026 at elliotmcguckenphysics.com. The present paper is the comprehensive consolidation of the quantum-mechanical sector of that derivational programme, with the chain of twenty-three theorems establishing $dx_4/dt = ic$ as the unique foundational principle from which all of quantum mechanics descends. The structural through-line is one equation, decades, and one chain.

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#### 3.14.4 29.4 Earlier Documented Origins

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**[MG-PrincetonAfternoons]** McGucken, E. "Princeton Afternoons: The Wheeler, Peebles, and Taylor Conversations." Documented chapter in [McGucken 2017c]. The detailed recounting of the 1988–1992 conversations with John A. Wheeler, P. J. E. Peebles, and Joseph H. Taylor Jr. that produced the physical intuitions later crystallizing as the McGucken Principle $dx_4/dt = ic$.

**[MG-FB]** McGucken, E. *Elliot McGucken Physics* (Facebook group). 2017–present. URL: https://www.facebook.com/elliotmcguckenphysics. Public forum for the McGucken framework's ongoing development, maintained continuously from 2017 through 2026, with more than six thousand followers.

**[MG-Medium]** McGucken, E. *Dr. Elliot McGucken Theoretical Physics* (Medium blog). 2020–present. URL: https://goldennumberratio.medium.com/. Public technical blog containing substantive technical papers including the original derivation of entropy's increase from $dx_4/dt = ic$, the McGucken Invariance paper, the Uncertainty Principle derivation, the Principle of Least Action and Huygens' Principle derivations, and comparative analyses.

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