The McGucken Space M_G: The Simplest, Most Complete, and Most Powerful Source Space in Physics: A Formal Theory of How $dx_4/dt=ic$ Generates Spacetime, Metric Structure, Hilbert Space, Phase Space, Spinor Space, Gauge-Bundle Space, Fock Space, Operator Algebras, and More

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"More intellectual curiosity, versatility, and yen for physics than Elliot McGucken's I have never seen in any senior or graduate student…Originality, powerful motivation, and a can-do spirit make me think that McGucken is a top bet."

— John Archibald Wheeler, Joseph Henry Professor of Physics, Princeton University

Abstract

The central theorem of the McGucken framework is the simultaneous space-operator generation theorem:

$$\frac{dx_4}{dt}=ic\Rightarrow\left(M_G,D_M\right).$$

The primitive law $dx_4/dt=ic$ generates both the McGucken Space M_G and the McGucken Operator D_M. This simultaneous generation of physical arena and physical operator is the central structural novelty of the McGucken framework. Standard mathematical physics begins with a space and then defines operators on it; the McGucken Principle generates the source-space and the source-operator together. And there are more novel features:

McGucken Space is demonstrated to be unique as the simplest, most complete, and most foundational physical space. McGucken Space is simplest by primitive-law count, most complete by derivational reach, and unique by possession of the full primitive signature

$$\left\{x_4,t,i,c,\Phi_M,D_M,\Sigma_M,dx_4/dt=ic\right\}.$$

McGucken Space is not another arena in the inventory of physics. McGucken Space is the source-space generated directly by the primitive physical law that the fourth dimension expands at the velocity of light in a spherical manner, as stated by the McGucken Principle,

$$\frac{dx_4}{dt}=ic.$$

This paper demonstrates that the McGucken Principle generates not only the McGucken Operator D_M, but also the mathematical arenas in which the descendant operators reside.

McGucken Space recognizes that the universe is not built first from passive space and then supplied with operators, fields, metrics, bundles, Hilbert spaces, and algebras. McGucken Space captures and formalizes the fact that founding physical reality itself is already spatial, operational, spherical, and generative. The founding physical relation $dx_4/dt=ic$ defines the source-space

$$M_G=(E_4,\Phi_M,D_M,\Sigma_M),$$

where E_4 is the four-coordinate carrier, $\Phi_M=x_4-ict$ is the McGucken constraint, $D_M=\partial_t+ic\,\partial_{x_4}$ is the McGucken flow operator, and Σ_M is the spherical outgoing McGucken wavefront structure.

The central theorem is the space-operator co-generation theorem:

$$\frac{dx_4}{dt}=ic\Rightarrow(M_G,D_M)\Rightarrow\text{spacetime, metric, Hilbert space, bundles, connections, Clifford structures, and opera}$$

This theorem is the decisive strengthening of the framework. The McGucken Principle does not only generate operators after a space has been assumed. It generates the source-space and source-operator together.

McGucken Space occupies a structural position that standard physical spaces do not occupy. All standard mathematical spaces in physics, including Lorentzian spacetime, phase space, Hilbert space, spinor space, gauge-bundle space, Fock space, and operator algebra, function as arenas for already-formulated events, states, fields, symmetries, or observables. McGucken Space acts at the threshold where the fourth-coordinate postulate $dx_4/dt=ic$ becomes a source-space: a constraint, a flow, a spherical propagation structure, a Lorentzian projection, a quantum amplitude arena, and ultimately a generator of the spaces used throughout fundamental physics.

The McGucken Operator

$$D_M=\partial_t+ic\,\partial_{x_4}$$

does not act merely inside an already-given spacetime, Hilbert space, or field theory. D_M acts in, preserves, and generates the structured arena called McGucken Space. Standard physical spaces, including Lorentzian spacetime, configuration space, phase space, Hilbert space, spinor space, gauge-bundle space, Fock space, and operator algebras, are compared to McGucken Space and classified as constraint surfaces, projections, bundles, function spaces, representation spaces, state spaces, or quantized descendants of the McGucken arena.

The Lorentzian spacetime $M_{1,3}$ is identified as the constraint/projection

$$M_{1,3}\cong\Phi_M^{-1}(0),$$

while Hilbert space is not treated as a literal subset of spacetime but as a complex inner-product state space of square-integrable sections over the derived spacetime:

$$H\simeq L^2(M_{1,3})\,\text{or}\,H\simeq\Gamma_{L^2}(E\rightarrow M_{1,3}).$$

The metric is derived because $dx_4 = ic\,dt$ gives $dx_4^2 = -c^2 dt^2$. The quantum arena is derived because the primitive law contains i, supports complex amplitudes, and leads to Hilbert completion. The gauge arena is derived by covariantizing the source flow. The Clifford arena is derived by factorizing the induced Lorentzian wave operator. The operator algebra is derived from quantized and covariantized descendants.

The paper then examines how the quantum formalism can be derived from the McGucken Principle. Following the chain of results presented in "Quantum Mechanics Derived from the McGucken Principle," the emergence of complex wavefunctions, superposition, momentum operators, canonical commutators, the Schrödinger equation, the Born rule, path integrals, spinors, and Fock space is organized as a sequence of projections and representations of McGucken Space. The result is a formal hierarchy: McGucken Space is the generative space; Lorentzian spacetime is its constraint projection; field bundles live over that spacetime; Hilbert spaces are state spaces of fields over it; and quantum operators are infinitesimal generators of symmetries inherited from the McGucken flow.

The central principle is the **McGucken Universal Derivability Principle**: every mathematical space that plays a physically meaningful role in fundamental physics is derived from McGucken Space by a finite sequence of admissible physical-space operations, including constraint, projection, slicing, bundle formation, cotangent lift, representation, complexification, quantization, tensoring, Fock completion, operator-algebra construction, and Hilbert completion. Thus Hilbert space is not added as an independent axiom but is the completed complex inner-product state space naturally associated with wave amplitudes over McGucken-derived spacetime.

The paper further proves, relative to the definitions adopted here, that McGucken Space is the most foundational physical space. In the derivability order, every standard physical space lies below M_G, while M_G cannot be derived from any one of them without reintroducing its primitive data: the fourth coordinate x_4, the universal expansion law $dx_4/dt = ic$, the McGucken constraint $\Phi_M = x_4 - ict$, the McGucken flow operator D_M, and the spherical propagation structure Σ_M. It is also shown to be the simplest possible physical source-space in this framework, because it is generated by a single primitive physical law whose closure yields spacetime, quantum state space, field bundles, and operator algebras.

The reason for this unprecedented derivational power is that McGucken Space is based on foundational physical reality rather than abstract mathematical convenience: the McGucken Symmetry $dx_4/dt = ic$, presented as the "father symmetry" from which principal physical symmetries descend, and the McGucken Sphere, presented as spacetime's foundational atom and as the elementary null-spherical unit from which spacetime, propagation, and quantum structures are built ([1], [2]). In this interpretation, the spaces derived below are powerful because they are mathematically useful; McGucken Space is more powerful because it encodes the physical source from which those useful spaces arise.

Keywords

McGucken Space; Hilbert space; phase space; configuration space; Lorentzian spacetime; McGucken Principle; $dx_4/dt = ic$; McGucken operator; quantum mechanics; Born rule; Schrödinger equation;

Dirac equation; gauge bundle; spinor space; Fock space; operator algebra; path integral; mathematical foundations of physics.

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Comparative Summary: Why McGucken Space Is Simplest, Most Complete, and Unique

McGucken Space is simplest, most complete, and unique in the following precise comparative sense.

Criterion	McGucken Space $M_G=(E_4,\Phi_M,D_M,\Sigma_M)$	Standard downstream spaces
Founding law	Generated by one primitive physical law: $dx_4/dt=ic$	Defined only after an arena, metric, state formalism, field theory, bundle, or algebraic representation is supplied
Primitive data	Contains $x_4,t,i,c,\Phi_M,D_M,\Sigma_M,dx_4/$	Contains only a sector of the physical structure
Simplicity	One source law plus its constraint, flow, and spherical wavefront closure	Multiple prior assumptions: spacetime, metric, Hilbert space, bundle, connection, Hamiltonian, Clifford structure, or algebra
Completeness	Generates the principal spaces of relativity, quantum mechanics, gauge theory, spinor theory, and operator algebra	Captures one arena: events, classical states, quantum states, spinors, gauge fields, particles, or observables
Arena status	Source-space from which other arenas descend	Downstream arena used after the physical structure is already partially specified
Operator relation	Co-generated with $D_M=\partial_t+ic\,\partial_{x_4}$	Receives operators defined after the space is assumed
Metric relation	Produces Lorentzian signature by $dx_4^2=-c^2dt^2$	Begins by assuming a metric or signature
Quantum relation	Supplies complex phase through i in the primitive law	Begins by assuming complex Hilbert space
Gauge relation	Supports covariantization of the source flow	Begins by assuming bundle and connection
Clifford relation	Induces Lorentzian wave structure whose square roots generate spinor operators	Begins by assuming Clifford algebra and spinor bundle
Uniqueness	Simultaneously generates source-space and source-operator from one law	Standard spaces do not generate their own founding physical law and full descendant hierarchy

The following table states how standard physical spaces are descendants of McGucken Space.

Standard downstream space	Standard role	Required assumptions normally supplied first	Derivation from McGucken Space	Missing primitive signature if taken alone
Lorentzian spacetime	Event arena	Time coordinate, spatial coordinates, metric	Constraint/projection	Lacks source law, D_M,

Standard downstream space	Standard role	Required assumptions normally supplied first	Derivation from McGucken Space	Missing primitive signature if taken alone
$M_{1,3}$		signature	$\Phi_M = x_4 - ict = 0$	and Σ_M
Metric space	Distance and causal arena	Metric tensor or interval	$dx_4 = ic\,dt \Rightarrow dx$	Lacks fourth-coordinate expansion mechanism
Configuration space Q	Classical positional arena	System degrees of freedom over time	Configurations over McGucken-derived spatial slices	Lacks x_4-flow and spherical source structure
Phase space $T^{¿}Q$	Classical state arena	Configuration space plus momenta	Cotangent lift of McGucken-derived configuration space	Lacks primitive complex fourth-coordinate relation
Hilbert space H	Quantum state arena	Complex vector space, inner product, completion	Complex amplitude space over McGucken-derived spacetime plus Born inner product and completion	Lacks source-space and source-law by itself
Spinor space	Fermionic representation arena	Clifford algebra and spin representation	Clifford representation of the McGucken-induced Lorentzian structure	Lacks $x_4 = ict$ origin of signature
Gauge-bundle space	Interaction arena	Base manifold, fiber, structure group, connection	Bundle construction and covariantization over McGucken-derived spacetime	Lacks selected McGucken flow direction
Fock space $F(H)$	Variable-particle-number arena	One-particle Hilbert space and tensor construction	Fock completion of McGucken-derived Hilbert space	Lacks primitive source data
Operator algebra A	Observables and transformations	Hilbert representation or algebraic quantum framework	Algebra generated by quantized and covariantized descendants	Lacks Φ_M, D_M, and $dx_4/dt = ic$

Therefore McGucken Space is not another mathematical setting. McGucken Space is simplest by primitive-law count, most complete by derivational reach, and unique by possession of the full primitive signature from which the hierarchy of physical spaces is generated.

Space-Operator Co-Generation Theorem

The McGucken Principle $dx_4/dt = ic$ generates not only the spaces of physics and not only the operators of physics. It generates the source-space and the source-operator together.

$$\frac{dx_4}{dt} = ic \Rightarrow (M_G, D_M).$$

This is the essential structural difference between McGucken Space and standard spaces. Standard spaces receive operators after the arena is assumed. McGucken Space and the McGucken Operator arise from the same primitive physical law.

Theorem 0.1 (space-operator co-generation theorem). The McGucken Principle generates the McGucken Space M_G and the McGucken Operator D_M as a single source space-operator pair:

$$\frac{dx_4}{dt} = ic \Rightarrow (M_G, D_M).$$

Proof. The McGucken Principle integrates to

$$x_4 = ict + C.$$

With the source-origin convention $C = 0$, this gives

$$x_4 = ict.$$

Define the McGucken constraint

$$\Phi_M = x_4 - ict.$$

The zero set $\Phi_M = 0$ gives the McGucken constraint structure, and together with the four-coordinate carrier E_4 and spherical propagation structure Σ_M, it defines the McGucken Space

$$M_G = (E_4, \Phi_M, D_M, \Sigma_M).$$

The tangent derivative along the same primitive flow is

$$\frac{d}{dt}\text{¿}_M = \partial_t + \frac{dx_4}{dt}\partial_{x_4} = \partial_t + ic\,\partial_{x_4}.$$

Thus

$$D_M = \partial_t + ic\,\partial_{x_4}.$$

The same physical law therefore generates the source-space and the source-operator. ▫

Corollary 0.2 (arena derivation corollary). The prior assumptions normally required by standard physical operators are derived from the McGucken source pair:

$$(M_G, D_M) \Rightarrow \{M_{1,3}, g, H, E \to M, \nabla, Cl(M), A\}.$$

Proof. Lorentzian spacetime is obtained by $\Phi_M = 0$. The metric signature follows from $dx_4^2 = (ic\,dt)^2 = -c^2 dt^2$. Hilbert space follows by forming complex amplitude spaces over McGucken-derived spacetime, equipping them with the Born inner product, and completing them. Bundles arise as field and internal-symmetry structures over the derived spacetime. Connections arise by covariantizing the McGucken flow. Clifford structures arise by factorizing the induced Lorentzian wave operator. Operator algebras arise from quantized and covariantized descendants. Therefore the standard prior assumptions are descendants of the McGucken source pair. ▫

Unprecedented Structural Position

McGucken Space is unprecedented because it is not a space alone on which physics is written. It is the space generated by the same law that generates the foundational operator. This means the McGucken framework does not follow the standard sequence

$$\text{space} \to \text{operator} \to \text{dynamics}.$$

It follows the source sequence

primitive physical law → source-space/source-operator pair → standard spaces, operators, and dynamics.

The following comparison states the difference exactly.

Structure	Standard mathematical physics	McGucken framework
Starting point	A manifold, metric, Hilbert space, bundle, algebra, or action is supplied	The primitive law $dx_4/dt=ic$ is supplied
Space	Assumed first	Generated as M_G
Operator	Defined after the space	Co-generated as D_M
Metric	Chosen or postulated	Derived from $dx_4=ic\,dt$
Complex quantum phase	Built into Hilbert space	Present in the primitive law through i
Wave propagation	Imposed by field equations	Generated through the spherical source structure Σ_M
Gauge covariance	Added through bundle connection	Obtained by covariantizing the source flow
Spinor structure	Added through Clifford representation	Obtained by factorizing the induced Lorentzian wave operator
Operator algebra	Built on a representation space	Generated from descendants of the source operator
Foundational status	Fragmented among multiple arenas	Unified in (M_G, D_M)

This is why McGucken Space has greater structural reach than Hilbert space, phase space, spacetime, spinor space, gauge-bundle space, or operator algebra taken separately. Each standard space captures one completed arena. McGucken Space captures the generative physical mechanism from which those arenas arise.

1. Introduction

The McGucken Space M_G is the source-space of physical mathematics. It is generated directly by the McGucken Principle, which states that the fourth coordinate advances according to

$$\frac{dx_4}{dt}=ic.$$

With the initial condition $x_4(0)=0$, this gives

$$x_4=ict.$$

The associated McGucken operator is

$$D_M=\partial_t+ic\,\partial_{x_4}.$$

It is the directional derivative along the McGucken flow.

The question addressed in this paper is: **what is the space in which this operator acts?** If all standard operators in physics act within an already-given spacetime, Hilbert space, phase space, or field bundle, then the McGucken operator suggests a different structure. It acts in the space in which the relation $x_4 = ict$ is primitive. This space is here called **McGucken Space**.

The key conceptual distinction is:

Hilbert space is a state space; spacetime is an event space; phase space is a classical-state space; McGucken S

Hilbert space is not literally a subset of spacetime. Phase space is not literally a subset of Hilbert space. Gauge bundles are not literally subsets of phase space. These spaces are related by constructions: projection, bundle formation, cotangent lift, quantization, representation, completion, and formation of state spaces. Therefore the correct claim is not that all spaces are simple subsets of McGucken Space. The correct claim is:

Standard physical spaces are derived, projected, fibered, represented, or quantized from McGucken Space.

This paper develops that claim formally.

The strongest form of the claim is the following:

Every physically meaningful space in fundamental physics is derivable from McGucken Space.

This statement must be read in the precise derivational sense developed below. It does not mean that Hilbert space, phase space, Fock space, or gauge-bundle space is literally contained in McGucken Space as an ordinary subset. It means that each is generated from McGucken Space by a physically interpretable mathematical construction.

2. Status Convention

The paper distinguishes the following kinds of statements:

Label	Meaning
Definition	A stipulated mathematical object used in the framework.
Theorem	A result directly derived in the paper from stated assumptions.
Corollary	An immediate consequence of a theorem.
Principle	An extension requiring further analytic, physical, or experimental development.
Representation Statement	A statement that one space represents states, fields, or operators over another space rather than being a literal subset.

This distinction is essential. Some relations are literal inclusions, such as a constraint surface inside a larger arena. Others are not inclusions but constructions. For example, a Hilbert space is generally a complete complex inner-product vector space used to represent quantum states; in quantum mechanics, quantum states are represented by vectors in Hilbert space ([3], [4]). It is therefore more precise to treat Hilbert space as a state representation over a derived spacetime than as a literal subset of that spacetime.

3. Definition of McGucken Space

3.1 Coordinate carrier

Let

$$E_4$$

be a four-coordinate Euclidean carrier with coordinates

$$(x_1, x_2, x_3, x_4).$$

The first three coordinates describe ordinary spatial extension. The fourth coordinate x_4 is distinguished by the McGucken Principle:

$$\frac{dx_4}{dt} = ic.$$

3.2 Constraint

Define the McGucken constraint function

$$\Phi_M(t, x_4) = x_4 - ict.$$

The McGucken constraint surface is

$$C_M = \Phi_M^{-1}(0) = \{(t, x_4) : x_4 = ict\}.$$

3.3 Flow operator

Define the McGucken flow operator

$$D_M = \partial_t + ic\,\partial_{x_4}.$$

It satisfies

$$D_M \Phi_M = 0.$$

Thus D_M is tangent to the McGucken constraint surface.

3.4 Spherical propagation structure

Let $\Sigma_M(p, t)$ denote the McGucken spherical wavefront generated from an event p after parameter interval t:

$$\Sigma_M(p, t) = \{q : dist(p, q) = ct\}.$$

This encodes the Huygens-type spherical propagation channel emphasized in the linked article "Quantum Mechanics Derived from the McGucken Principle," where the McGucken Sphere is the wavefront channel of the framework ([5]).

3.5 Full definition

Definition 3.1 (McGucken Space). McGucken Space is the structured arena

$$M_G = \left(E_4, \Phi_M, D_M, \Sigma_M\right)$$

where:

Component	Formula	Meaning
Coordinate carrier	E_4	Four-coordinate arena
Constraint	$\Phi_M = x_4 - ict$	Defines the McGucken hypersurface
Flow operator	$D_M = \partial_t + ic\,\partial_{x_4}$	Generates fourth-dimensional advance
Spherical structure	$\Sigma_M(p,t)$	Encodes outgoing Huygens/McGucken wavefronts

Thus McGucken Space is not a set alone. It is a **structured space-plus-law**.

4. Lorentzian Spacetime as a Projection of McGucken Space

The four-coordinate Euclidean interval is

$$d\ell^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2.$$

On the McGucken constraint surface,

$$dx_4 = ic\,dt.$$

Therefore

$$dx_4^2 = (ic\,dt)^2 = -c^2 dt^2.$$

Substituting into (13) gives

$$d\ell^2 = dx_1^2 + dx_2^2 + dx_3^2 - c^2 dt^2.$$

This is the Lorentzian interval in the sign convention with positive spatial part.

Theorem 4.1 (Spacetime projection theorem). The McGucken constraint $x_4 = ict$ projects the four-coordinate carrier E_4 to Lorentzian spacetime $M_{1,3}$:

$$M_{1,3} \cong \Phi_M^{-1}(0)$$

with induced interval

$$d\ell^2 = dx^2 - c^2 dt^2 .$$

Proof. The proof is the substitution (14)–(16). ▫

Thus spacetime is a literal constraint/projection of McGucken Space.

5. Taxonomy of Spaces

The following table gives the principal spaces used in physics and their relation to McGucken Space.

Space	Symbol	Standard definition	What lives there	Relation to McGucken Space
Euclidean carrier	E_4	Four-coordinate positive-signature arena	Coordinates (x_1, x_2, x_3, x_4)	Carrier component of M_G
McGucken Space	M_G	$(E_4, \Phi_M, D_M, \Sigma_M$	Constraint, flow, spherical propagation	Generative structured arena
McGucken constraint surface	C_M	$\Phi_M^{-1}(0)$	Events satisfying $x_4 = ict$	Literal subset/constraint surface
Lorentzian spacetime	$M_{1,3}$	Smooth Lorentzian event manifold	Events, worldlines, light cones	Projection/identification of C_M
Spatial slice	Σ_t	Constant-time hypersurface	Simultaneous spatial configurations	Slice of $M_{1,3}$
Configuration space	Q	Space of generalized positions	Classical positions or field configurations	Built over Σ_t or $M_{1,3}$
Phase space	$T^{¿}Q$	Cotangent bundle of positions and momenta	Classical states (q, p)	Cotangent lift of a derived configuration space
Covariant phase space	P_{cov}	Space of solutions modulo gauge	Classical field histories	Solution space of actions over $M_{1,3}$
Hilbert space	H	Complete complex inner-product vector space	Quantum states	State space of wavefunctions/sections over derived spacetime
Spinor bundle	$S \to M_{1,3}$	Clifford representation bundle	Spinor fields	Representation bundle over derived Lorentzian spacetime
Gauge bundle	$P \to M_{1,3}$	Principal fiber bundle with connection	Gauge fields and parallel transport	Fibered internal symmetry space over derived spacetime
Fock space	$F(H)$	Direct sum of many-particle Hilbert sectors	Variable-particle-number states	Quantized many-body construction over Hilbert space
Operator algebra	A	Algebra of observables/operators	Observables, symmetries, generators	Quantized generator algebra descending from D_M and symmetries

This table clarifies the main point: only some spaces are subsets. Others are projections, completions, bundles, or representation spaces.

6. Subset, Projection, Bundle, and Representation Relations

The relationships can be summarized as follows.

Relation type	Mathematical form	Example	McGucken interpretation
Literal subset	$A \subset B$	$C_M \subset E_4 \times R_t$	The constraint surface lies inside the coordinate carrier plus parameter

Relation type	Mathematical form	Example	McGucken interpretation
Projection	$\pi : A \to B$	$C_M \to M_{1,3}$	Physical spacetime is the projected McGucken constraint
Slice	$\Sigma_t \subset M_{1,3}$	Constant-time space	Spatial configurations arise after projection
Cotangent lift	$T^{¿}Q$	Phase space	Classical states arise from positions plus momenta
Function space	$L^2(M)$	Scalar quantum Hilbert space	Quantum states are square-integrable functions over derived spacetime/slices
Section space	$\Gamma(E \to M)$	Fields and spinors	Fields are sections of bundles over McGucken-derived spacetime
Fiber bundle	$P \to M$	Gauge theory	Internal symmetry fibers attach to each spacetime point
Representation	$\rho : G \to Aut(V)$	Spinor/gauge representations	Symmetry groups act on fibers or state spaces
Quantization	$f \mapsto \hat{f}$	Classical observable to operator	McGucken symmetries become quantum operators
Completion	$V \to \overline{V} = H$	Pre-Hilbert space to Hilbert space	Wave amplitudes become complete quantum state space

Therefore the disciplined formulation is:

$$M_{1,3} \text{ is a projection/constraint of } M_G,$$

but

$$H \text{ is a Hilbert representation over } M_{1,3}, \text{not a literal subset of } M_{1,3}.$$

7. Comparison of McGucken Space with Hilbert Space

A Hilbert space is a real or complex inner-product space that is complete with respect to the metric induced by the inner product ([3]). In quantum mechanics, the state of a physical system is represented by a vector in a Hilbert space, usually a complex vector space with inner product ([6], [7]).

McGucken Space is different. It is not primarily a vector space of states. It is a geometric-generative space:

$$M_G = (E_4, \Phi_M, D_M, \Sigma_M).$$

Feature	McGucken Space M_G	Hilbert Space H
Type	Structured geometric-generative arena	Complete complex inner-product vector space
Primitive element	Fourth-coordinate advance $dx_4/dt = ic$	State vector $¿\psi\rangle$ or wavefunction ψ
Main structure	Constraint Φ_M, flow D_M, sphere Σ_M	Inner product $\langle \psi \vee \phi \rangle$, norm, completeness
What lives there	Events, flow, constraint, wavefront structure	Quantum states
Operator role	D_M generates fourth-dimensional advance	Operators represent observables and generators
Relation to spacetime	Generates/projectively yields $M_{1,3}$	Usually built from wavefunctions over spacetime or space
Probability	Spherical/wavefront and Born-rule structure derived downstream	Probability from $\lvert\psi\rvert^2$ or projection

Feature	McGucken Space M_G	Hilbert Space H
		amplitudes
Status in the hierarchy	Foundational/generative	Derived state representation

The essential relation is:

$$M_G \to M_{1,3} \to H.$$

More explicitly:

$$M_G \xrightarrow{\Phi_M=0} M_{1,3} \xrightarrow{\text{fields/wavefunctions}} V \xrightarrow{\text{inner product + completion}} H.$$

8. Comparison of McGucken Space with Phase Space

Phase space is the space of all possible physical states of a system under a given parameterization; in classical mechanics it is built from positions and momenta, with each state corresponding to a point in phase space ([8]). More geometrically, if Q is configuration space, phase space is the cotangent bundle

$$T^{¿} Q.$$

McGucken Space is prior to this construction. A configuration space Q is normally built from possible spatial positions or field configurations. In the McGucken hierarchy, these positions or fields live over the Lorentzian spacetime obtained from M_G. Thus:

$$M_G \to M_{1,3} \to Q \to T^{¿} Q.$$

Feature	McGucken Space	Phase Space
Basic object	$x_4 = ict, D_M, \Sigma_M$	(q, p)
Physical role	Generates spacetime and quantum structures	Encodes classical states
Geometry	Constraint-flow geometry	Symplectic/cotangent geometry
Operator relation	D_M is primitive generator	Hamiltonian vector field generates classical evolution
Relation	Foundational arena	Classical-state construction over a derived configuration space

9. Comparison with Gauge-Bundle and Spinor Spaces

Gauge fields in modern physics are naturally described using fiber bundles and connections; expositions of fiber bundles in physics emphasize that gauge fields are globally connections on principal bundles rather than merely local differential forms ([9], [10]).

In the McGucken framework, the base space of such bundles is not primitive. The base is the McGucken-derived Lorentzian spacetime:

$$P \to M_{1,3}.$$

The gauge-covariant McGucken operator is

$$D_M^A = \nabla_t + ic\nabla_{x_4}.$$

Expanding,

$$D_M^A = \partial_t + ic\,\partial_{x_4} + A_t + ic\,A_4.$$

Thus the McGucken flow selects the connection component

$$A_M = A_t + ic\,A_4.$$

Spinor spaces similarly arise after Lorentzian Clifford structure is generated. Once the McGucken projection yields

$$\Box_M = \nabla^2 - \frac{1}{c^2}\partial_t^2,$$

the construction introduces gamma matrices satisfying

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} I.$$

The spinor bundle $S \to M_{1,3}$ is then a representation bundle over the derived spacetime.

Space	Base	Fiber	McGucken relation
Spinor bundle	$M_{1,3}$	Clifford module S_x	Represents square roots of $\Box_M$
Gauge bundle	$M_{1,3}$	Internal symmetry group G	Covariantizes D_M as D_M^A
Tangent bundle	$M_{1,3}$	Tangent vectors $T_x M$	Carries local spacetime directions induced by $\Phi_M = 0$
Cotangent bundle	$M_{1,3}$ or Q	Covectors/momenta	Classical momenta and phase space arise here
Hilbert bundle	Parameter/base manifold	Hilbert fibers	Quantum state spaces vary over backgrounds or parameters

10. Operator Comparison Table

The McGucken operator can be compared to the principal operators of physics as follows.

Operator	Formula	Space it acts on	What it assumes	What it generates	McGucken relation
McGucken operator	$D_M = \partial_t + ic\,\partial_x$	McGucken Space	$dx_4/dt = ic$	Fourth-dimensional advance	Primitive generator
Quantum McGucken operator	$\widehat{M} = i\hbar D_M$	McGucken-derived state space	Quantum lift	$\widehat{H} - ic\,\hat{p}_4$ constraint	Bridge to Hilbert operators
Momentum	$\hat{p} = -i\hbar\nabla$	Hilbert space over space	Spatial translation symmetry	Momentum spectrum	Descendant translation generator
Hamiltonian	$\widehat{H} = i\hbar\,\partial_t$	Hilbert space	Time parameter	Time evolution	Is contained inside $\widehat{M}$
Laplacian	$\Delta_4 = \nabla^2 + \partial_{x_4}^2$	Euclidean carrier	Euclidean metric	Harmonic/diffusion structure	Projects to $\Box_M$
d'Alembertian	$\Box_M = \nabla^2 - c^{-2}$	Lorentzian spacetime	McGucken projection	Relativistic waves	Induced operator

Operator	Formula	Space it acts on	What it assumes	What it generates	McGucken relation
Schrödinger operator	$i\hbar\partial_t-\widehat{H}$	Hilbert space	Quantum state vector	Unitary dynamics	Derived from McGucken quantum chain
Dirac operator	$i\gamma^\mu D_\mu-m$	Spinor sections	Clifford structure	Fermion propagation	Square root of induced wave operator
Gauge-covariant derivative	$\nabla_\mu=\partial_\mu+A_\mu$	Bundle sections	Gauge bundle	Parallel transport	$D_M^A=\nabla_t+ic\nabla$
Noether generator	X	Action/field space	Continuous symmetry	Conserved current	D_M is fourth-advance generator

11. Deriving Quantum Space from McGucken Space

The linked article argues that quantum mechanics is derivable as a chain of theorems from the McGucken Principle. Its theorem chain includes the wave equation from Huygens propagation, de Broglie relation, Planck-Einstein relation, Compton coupling, rest-mass phase, wave-particle duality, Schrödinger equation, Klein-Gordon equation, Dirac equation, canonical commutator, Born rule, Heisenberg uncertainty, path integral, gauge phase, entanglement, measurement, second quantization, and Feynman diagrams ([5]).

The key claim for the present paper is that Hilbert space emerges when the McGucken wavefront structure is converted into a complex linear probability-amplitude space.

The derivation can be organized as:

Step	McGucken input	Quantum output	Space generated
1	$x_4=ict$	Complex phase i	Complex amplitudes
2	Spherical McGucken wavefront Σ_M	Wave propagation and superposition	Linear pre-state space V
3	Compton/rest phase $e^{-imc^2\tau/\hbar}$	Oscillatory quantum phase	Complex wavefunctions
4	Translation symmetry	$p=-i\hbar\nabla$	Operator representation on wavefunctions
5	Time evolution	$H=i\hbar\partial_t$	Dynamical operator structure
6	McGucken derivative	$\widehat{M}=\widehat{H}-ic\,\hat{p}_4$	Quantum constraint operator
7	Spherical probability/Haar measure	$P=\lvert\psi\rvert^2$	Inner-product probability
8	Inner product	$\langle\psi\vee\phi\rangle$	Pre-Hilbert space
9	Completion	$\overline{V}$	Hilbert space H
10	Multi-particle extension	Tensor products/Fock construction	Fock space $F(H)$

Thus:

$$M_G\to\text{complex amplitudes}\to\text{inner-product pre-Hilbert space}\to H.$$

12. Formal Chain from McGucken Principle to Hilbert Space

12.1 Complex amplitudes

The McGucken Principle contains i:

$$\frac{dx_4}{dt} = ic.$$

Thus the natural amplitude structure is complex rather than purely real. Plane waves take the form

$$\psi(x,t) = e^{i(k \cdot x - \omega t)}.$$

In the linked theorem chain, the same i is identified as the factor appearing in Schrödinger evolution, commutators, the Dirac equation, and path-integral phases ([5]).

12.2 Linear superposition

The spherical wavefront channel Σ_M supports superposition. If ψ_1 and ψ_2 are possible wavefront amplitudes, then

$$\psi = a\psi_1 + b\psi_2$$

is also a possible amplitude in the linear wave regime. This supplies the vector-space structure.

12.3 Inner product

The Born-rule structure provides the quadratic probability density:

$$P(x) = |\psi(x)|^2 = \psi^{¿}(x)\psi(x).$$

The corresponding inner product is

$$\langle \psi \vee \phi \rangle = \int \psi^{¿}(x)\phi(x)\,d^3x.$$

This turns the vector space of amplitudes into a pre-Hilbert space.

12.4 Completion

Completing the pre-Hilbert space under the norm

$$\|\psi\|^2 = \langle \psi \vee \psi \rangle$$

gives a Hilbert space:

$$H = \overline{V}^{\|\cdot\|}.$$

For a scalar nonrelativistic particle on a spatial slice,

$$H = L^2(R^3, d^3x).$$

More generally, for fields or spinors over a McGucken-derived spacetime,

$$H = \Gamma_{L^2}(E \to M_{1,3}).$$

Theorem 12.1 (Hilbert-space emergence theorem). If McGucken Space supplies complex amplitudes through i, linear superposition through spherical wavefront propagation, and the Born inner product through quadratic probability, then the quantum state space is the Hilbert completion of the resulting pre-Hilbert amplitude space:

$$H = \overline{V}^{\langle \cdot,\cdot \rangle}.$$

Proof. The McGucken phase supplies complex-valued amplitudes. Linear wavefront superposition supplies vector addition and scalar multiplication. The Born density $P = |\psi|^2$ supplies a positive quadratic norm via $\|\psi\|^2 = \int |\psi|^2$. Polarization gives the inner product $\langle \psi \vee \phi \rangle = \int \psi^{¿} \phi$. Completing this normed inner-product space gives a Hilbert space by definition. ▫

13. Deriving Operators on Hilbert Space

Once Hilbert space is obtained, operators arise as generators of transformations inherited from McGucken-derived geometry.

13.1 Momentum

Spatial translations act on wavefunctions by

$$(U(a)\psi)(x) = \psi(x - a).$$

The infinitesimal generator is

$$\hat{p} = -i\hbar\nabla.$$

This matches the linked article’s theorem chain, where the canonical commutator follows by the Hamiltonian route through translation invariance and the operator $p = -i\hbar\nabla$ ([5]).

13.2 Hamiltonian

Time translations act by

$$U(t) = e^{-i\hat{H}t/\hbar}.$$

The generator is

$$\hat{H} = i\hbar\,\partial_t.$$

13.3 McGucken quantum operator

The McGucken flow operator lifts to

$$\widehat{M} = i\hbar D_M .$$

Since

$$D_M = \partial_t + ic\,\partial_{x_4} ,$$

and

$$\widehat{H} = i\hbar\,\partial_t ,\ \hat{p}_4 = -i\hbar\,\partial_{x_4} ,$$

one obtains

$$\widehat{M} = \widehat{H} - ic\,\hat{p}_4 .$$

Thus the quantum operator algebra contains the McGucken constraint as a generator relation.

13.4 Canonical commutator

For

$$\hat{q} = x ,\ \hat{p} = -i\hbar\,\partial_x ,$$

one computes

$$[\hat{q}, \hat{p}]\psi = x(-i\hbar\,\partial_x\psi) - (-i\hbar\,\partial_x)(x\psi) = i\hbar\psi .$$

Therefore

$$[\hat{q}, \hat{p}] = i\hbar .$$

14. Quantum Derivation Table from the Linked McGucken Article

The linked article presents a 23-theorem chain. The following table reorganizes that chain around space generation.

Theorem cluster	McGucken mechanism	Quantum result	Space/operator produced
Wave equation	Spherical x_4 expansion / Huygens	$\Box\psi = 0$	Wave solution space
de Broglie and Planck-Einstein	Cyclic x_4-phase/action	$p = \hbar k,\ E = \hbar\omega$	Momentum/energy spectral variables
Compton/rest phase	$mc^2/\hbar$ oscillation	$\psi \sim e^{-imc^2\tau/\hbar}$	Complex phase space of amplitudes
Schrödinger	Compton factorization / nonrelativistic limit	$i\hbar\,\partial_t\psi = \widehat{H}\psi$	Hilbert-space dynamics
Klein-Gordon	Mass-shell wave equation	$(\Box - m^2c^2/\hbar^2)\psi = 0$	Relativistic scalar solution space

Theorem cluster	McGucken mechanism	Quantum result	Space/operator produced
Dirac	Clifford square root	$(i\gamma^{\mu} D_{\mu} - m)\psi = 0$	Spinor bundle sections
Canonical commutator	Translation generators	$[q, p] = i\hbar$	Operator algebra
Born rule	Complex amplitude + quadratic norm + spherical measure	$P = \lvert\psi\rvert^2$	Inner-product probability space
Path integral	Sum over McGucken Sphere chains	$\int D[x] e^{iS/\hbar}$	History/path space
Gauge phase	x_4-phase origin freedom	$U(1)$ gauge	Gauge-bundle structure
Entanglement/nonlocality	Shared x_4-coupling	Nonlocal correlations	Tensor-product state space
Second quantization	Spin/statistics and field modes	Fock space	$F(H)$
Feynman diagrams	Iterated Huygens with interactions	Diagrammatic perturbation theory	Operator/path-integral expansion

15. McGucken Universal Derivability Principle

The preceding sections motivate a general principle that extends the Hilbert-space derivation to all major mathematical arenas of physics.

Principle 15.1 (McGucken Universal Derivability Principle). Let $PhysSpace$ denote the class of mathematical spaces that function as physically meaningful arenas in fundamental physics, including event spaces, state spaces, phase spaces, Hilbert spaces, fiber spaces, spinor spaces, gauge-bundle spaces, path spaces, Fock spaces, moduli spaces, and operator-algebra spaces. Then every $X \in PhysSpace$ is derivable from McGucken Space:

$$X \in Der(M_G).$$

Here $Der(M_G)$ denotes the derivational closure of M_G under admissible physical-space operations:

$$Der(M_G) = \langle M_G; \text{constraint}, \text{projection}, \text{slicing}, \text{bundle formation}, \text{section formation}, \text{cotangent lift}, \text{complex}$$

This principle is the paper's strongest formal principle. It says that McGucken Space is not one more space in the inventory of physics. It is the generating source whose derivational closure contains the spaces used by relativity, classical mechanics, quantum mechanics, quantum field theory, gauge theory, and operator algebraic physics.

Theorem 15.2 (Hilbert-space derivability). Hilbert space is derivable from McGucken Space:

$$H \in Der(M_G).$$

Proof. From $M_G = (E_4, \Phi_M, D_M, \Sigma_M)$, impose $\Phi_M = 0$ to obtain the Lorentzian spacetime projection $M_{1,3}$. Over $M_{1,3}$, form the complex amplitude space of McGucken wavefront solutions. The presence of i in $dx_4/dt = ic$ supplies complex phase, while Σ_M supplies spherical wavefront propagation and superposition. The Born rule supplies the positive quadratic norm

$$\|\psi\|^2 = \int |\psi|^2 d\mu,$$

and the associated inner product

$$\langle \psi, \phi \rangle = \int \psi^{¿} \phi \, d\mu .$$

Completing the resulting complex inner-product space gives H. Therefore $H \in Der(M_G)$. ▫

Corollary 15.3 (standard quantum arenas are McGucken-derived). If Hilbert space is McGucken-derived, then the operator algebra $A \subseteq B(H)$, tensor-product spaces $H_A \otimes H_B$, and Fock space $F(H)$ are also McGucken-derived.

Proof. Each is obtained from H by admissible operations included in $Der(M_G)$: operator-algebra formation, tensor product, and Fock construction. ▫

The following table states the principle in concrete physical terms.

Physical space	Standard role	Derivation from McGucken Space
Lorentzian spacetime $M_{1,3}$	Event arena of relativity	Constraint/projection $\Phi_M = 0$
Light-cone/null space	Causal propagation structure	Null structure induced by $x_4 = ict$
Configuration space Q	Classical positional state space	Configurations over McGucken-derived spatial slices
Phase space $T^{¿}Q$	Classical position-momentum arena	Cotangent lift of configuration space
Solution space	Space of field/wave solutions	Kernel/eigenspace of McGucken-induced wave operators
Hilbert space H	Quantum state space	Complex amplitude space plus Born inner product plus completion
Spinor space	Fermionic representation space	Clifford representation of McGucken-induced Lorentzian structure
Gauge-bundle space	Internal interaction arena	Fiber-bundle construction over $M_{1,3}$, with phase freedom inherited from x_4
Path/history space	Path-integral arena	Chains of McGucken spherical wavefront propagations
Tensor-product space	Composite-system state space	Tensoring of McGucken-derived Hilbert spaces
Fock space $F(H)$	Variable-particle-number quantum state space	Fock completion of McGucken-derived one-particle Hilbert space
Operator algebra A	Algebra of observables and transformations	Operators generated on McGucken-derived Hilbert space
Moduli/parameter space	Space of physically distinct structures	Quotient of McGucken-derived fields or bundles by equivalence/gauge symmetry

The principle is summarized as:

$$PhysSpace \subseteq Der(M_G).$$

This is not a claim of naive set-theoretic containment. It is a claim of derivational containment: the spaces of physics are contained in the generative closure of McGucken Space.

Theorem 15.4 (source law generates spaces and their resident operators). The McGucken Principle $dx_4/dt = ic$ generates not only the operator hierarchy but also the spaces in which those operators reside:

$$\frac{dx_4}{dt} = ic \Rightarrow Der(M_G, D_M) \supseteq \{M_{1,3}, g, H, E \to M, \nabla, Cl(M), A\}.$$

Proof. The primitive law first gives $x_4 = ict$, hence $\Phi_M = x_4 - ict = 0$. This defines the McGucken source-space structure M_G. The same law gives the tangent flow operator $D_M = \partial_t + ic\,\partial_{x_4}$. The constraint $\Phi_M = 0$ gives Lorentzian spacetime $M_{1,3}$. Substitution $dx_4 = ic\,dt$ gives $dx_4^2 = -c^2 dt^2$, hence the Lorentzian metric signature g. Field spaces are sections of bundles over $M_{1,3}$. Hilbert space is obtained by forming complex amplitude spaces over $M_{1,3}$, equipping them with the Born inner product, and completing them. Connections arise from the covariantization of the McGucken flow, $D_M \mapsto D_M^A = \nabla_t + ic\,\nabla_{x_4}$. Clifford structure arises by factorizing the McGucken-induced Lorentzian wave operator. Operator algebras arise from quantized, covariantized, and represented descendants of D_M. Therefore the source law generates both the resident operators and the spaces in which they reside. ▫

Corollary 15.5 (standard prior-assumption reversal). The structures normally treated as prior assumptions in mathematical physics are downstream in the McGucken hierarchy:

$$\text{spacetime, metric, Hilbert space, bundle, connection, Hamiltonian, Clifford structure, operator algebra} \leq M_G.$$

Proof. Each listed structure is present in the derivation of Theorem 15.4. Since each is obtained by an admissible physical-space operation from M_G or from structures already derived from M_G, each is below M_G in the derivability preorder. ▫

Corollary 15.6 (non-reversibility of downstream arenas). No single downstream space among $M_{1,3}$, g, H, $E \to M$, ∇, $Cl(M)$, or A generates the full McGucken source-space without reintroducing the primitive signature

$$\{x_4, t, i, c, \Phi_M, D_M, \Sigma_M, dx_4/dt = ic\}.$$

Proof. Lorentzian spacetime by itself does not specify the fourth-coordinate source law $dx_4/dt = ic$. A metric does not specify the spherical source structure Σ_M. Hilbert space does not specify x_4, Φ_M, or the source flow D_M. A bundle does not specify the McGucken constraint. A connection does not specify the primitive law from which the selected direction is obtained. A Clifford structure does not specify the physical origin of the Lorentzian signature. An operator algebra does not specify the source-space from which its represented operators descend. Therefore none of the downstream arenas reconstructs M_G without adding the McGucken primitive signature externally. ▫

The following table gives the proof in compressed form.

Usually prior assumption	McGucken derivation	Resident operator generated or supported
Spacetime $M_{1,3}$	$\Phi_M = x_4 - ict = 0$	Tangent source operator D_M and induced spacetime derivatives
Metric g	$dx_4^2 = (ic\,dt)^2 = -c^2 dt^2$	Lorentzian wave operator $\Box$

Usually prior assumption	McGucken derivation	Resident operator generated or supported
Hilbert space H	Complex amplitudes over derived spacetime plus Born inner product and completion	$\widehat{H}$, $\widehat{p}$, Schrödinger operator
Bundle $E \rightarrow M$	Field and internal-symmetry structures over derived spacetime	Section operators and field operators
Connection ∇	Covariantization of D_M	$D_M^A = \nabla_t + i c \nabla_{x_4}$
Hamiltonian $\widehat{H}$	Time-sector projection of $i \hbar D_M$	Time-evolution generator
Clifford structure $Cl(M)$	Factorization of induced Lorentzian wave operator	Dirac-type operator
Operator algebra A	Algebra generated by quantized and covariantized descendants	Commutators and observables

The theorem is unique because it is a simultaneous derivation theorem. It derives more than an equation inside a space: it derives the space, the operator, the metric signature, the quantum arena, the gauge arena, the Clifford arena, and the algebraic arena from one primitive physical law.

16. Worked Examples of Space Derivation

This section demonstrates the Universal Derivability Principle by deriving several standard spaces from McGucken Space. Each example begins with

$$M_G = (E_4, \Phi_M, D_M, \Sigma_M), \Phi_M = x_4 - i c t, D_M = \partial_t + i c \partial_{x_4}.$$

The examples are not meant to exhaust the full theory. They show the common derivational pattern: start with McGucken Space, impose its constraint, inherit its symmetry and phase structure, then build the standard space by a physically meaningful mathematical operation.

16.1 Lorentzian spacetime

The most immediate derived space is ordinary relativistic spacetime.

Derivation. Begin with the McGucken constraint

$$\Phi_M = x_4 - i c t = 0.$$

Then

$$x_4 = i c t.$$

The Euclidean four-coordinate quadratic form

$$d s_4^2 = d x_1^2 + d x_2^2 + d x_3^2 + d x_4^2$$

becomes

$$d s_4^2 = d x_1^2 + d x_2^2 + d x_3^2 - c^2 d t^2.$$

Thus the Lorentzian interval is obtained from the McGucken substitution $d x_4 = i c d t$. The resulting event space is

$$M_{1,3} \cong \Phi_M^{-1}(0).$$

Result. Lorentzian spacetime is the constraint/projection of McGucken Space.

Step	Operation	Result
1	Start with E_4	Four-coordinate carrier
2	Impose $\Phi_M = 0$	$x_4 = ict$
3	Substitute into ds_4^2	Lorentzian interval
4	Interpret event coordinates	$M_{1,3}$

16.2 Light-cone and null propagation space

The light cone is not imposed independently. It follows from the derived Lorentzian interval.

Derivation. From the McGucken-derived interval

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2,$$

null propagation satisfies

$$ds^2 = 0.$$

Therefore

$$dx^2 + dy^2 + dz^2 = c^2 dt^2.$$

For radial propagation this gives

$$r = ct.$$

This is precisely the spherical wavefront structure Σ_M associated with McGucken propagation.

Result. The light cone is the null hypersurface induced by the McGucken constraint.

Space	McGucken source	Derived condition
Null cone	$x_4 = ict$	$ds^2 = 0$
Spherical wavefront	Σ_M	$r = ct$
Causal boundary	McGucken propagation at c	$r^2 = c^2 t^2$

16.3 Configuration space

Configuration space is the space of possible positions or field configurations on a spatial slice.

Derivation. From McGucken Space derive $M_{1,3}$. Choose a time function t and a spatial hypersurface

$$\Sigma_t = \{ p \in M_{1,3} : t(p) = t \}.$$

For a single particle, the configuration space is

$$Q=\Sigma_t.$$

For N distinguishable particles, it is

$$Q_N=\Sigma_t^N.$$

For a classical field φ, the configuration space is a space of sections or functions over Σ_t:

$$Q_{field}=\{\varphi:\Sigma_t\to V\}.$$

Result. Configuration space is derived from McGucken Space by spacetime projection followed by spatial slicing and configuration formation.

Case	Derived configuration space
One particle	$Q=\Sigma_t$
N particles	$Q_N=\Sigma_t^N$
Scalar field	$Q_{field}=\{\varphi:\Sigma_t\to R\}$
Complex wave amplitude	$Q_\psi=\{\psi:\Sigma_t\to C\}$

16.4 Phase space

Classical phase space is derived from configuration space by cotangent lift.

Derivation. Once Q is derived, define the cotangent bundle

$$T^{¿}Q=\bigcup_{q\in Q}T_q^{¿}Q.$$

Its points are pairs

$$(q,p),$$

where q is configuration and p is conjugate momentum. Since Q was derived from a McGucken-derived spatial slice, $T^{¿}Q$ is also McGucken-derived:

$$M_G\to M_{1,3}\to\Sigma_t\to Q\to T^{¿}Q.$$

Result. Phase space is not primitive. It is the cotangent construction over a McGucken-derived configuration space.

Construction	Meaning
$M_G\to M_{1,3}$	Derive spacetime
$M_{1,3}\to\Sigma_t$	Choose spatial slice
$\Sigma_t\to Q$	Define configurations
$Q\to T^{¿}Q$	Attach conjugate momenta

16.5 Hilbert space

Hilbert space is the completed inner-product space of complex amplitudes over McGucken-derived spacetime or spatial slices.

Derivation. From M_G, derive $M_{1,3}$ and a spatial slice Σ_t. Let V be the vector space of complex McGucken wave amplitudes on Σ_t:

$$V=\{\psi:\Sigma_t\to C\}.$$

The factor i in $dx_4/dt=ic$ supplies the natural complex phase. The spherical propagation structure Σ_M supplies wavefront superposition. The Born rule supplies the positive quadratic norm:

$$\|\psi\|^2=\int_{\Sigma_t}|\psi(x)|^2 d^3x.$$

The corresponding inner product is

$$\langle\psi,\phi\rangle=\int_{\Sigma_t}\psi^{¿}(x)\phi(x)d^3x.$$

Completing V in this norm gives

$$H=\overline{V}^{\|\cdot\|}.$$

Result. Hilbert space is the Hilbert completion of McGucken-derived complex amplitude space.

Ingredient	McGucken origin
Complex amplitudes	i in $dx_4/dt=ic$
Propagating waves	Σ_M
Spatial integration domain	$\Sigma_t\subset M_{1,3}$
Inner product	Born quadratic norm
Hilbert space	Completion of amplitude space

16.6 Spinor space

Spinor space is derived by taking the Clifford representation of the McGucken-induced Lorentzian structure.

Derivation. From the McGucken interval obtain the Lorentzian metric $\eta_{\mu\nu}$. Define gamma matrices satisfying

$$\{\gamma^\mu,\gamma^\nu\}=2\eta^{\mu\nu}.$$

The representation space on which the γ^μ act is the spinor space S. Over spacetime this forms a spinor bundle

$$S \to M_{1,3}.$$

Spinor fields are sections:

$$\psi \in \Gamma\left(S \to M_{1,3}\right).$$

Result. Spinor space is a representation space of the Clifford algebra induced by the McGucken-derived Lorentzian metric.

Step	Operation	Result
1	Derive $M_{1,3}$	Lorentzian metric
2	Form Clifford algebra	$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$
3	Choose representation	Spinor space S
4	Fiber over spacetime	Spinor bundle $S \to M_{1,3}$

16.7 Gauge-bundle space

Gauge-bundle space is derived by attaching internal phase or symmetry fibers over McGucken-derived spacetime.

Derivation. The McGucken Principle contains an intrinsic phase structure through $x_4 = ict$. A local phase transformation of a complex amplitude may be written

$$\psi(x) \mapsto e^{i\alpha(x)}\psi(x).$$

To compare phases at neighboring points in $M_{1,3}$, introduce a connection A_μ and covariant derivative

$$D_\mu = \partial_\mu + iA_\mu.$$

This is geometrically represented by a principal bundle

$$P \to M_{1,3}$$

with associated vector bundles for matter fields.

Result. Gauge-bundle space is derived by allowing local phase freedom over McGucken-derived spacetime.

Object	Derivation
Base space	$M_{1,3} \cong \Phi_M^{-1}(0)$
Phase fiber	x_4-phase freedom
Connection	Rule for comparing phase across spacetime
Gauge bundle	$P \to M_{1,3}$
Matter bundle	Associated vector bundle

16.8 Path and history space

Path space is derived from sequences of McGucken wavefront propagation.

Derivation. A path is a map

$$\gamma : [t_i, t_f] \to M_{1,3}.$$

The space of all such paths is

$$P(q_i, q_f) = \{\gamma : \gamma(t_i) = q_i, \gamma(t_f) = q_f\}.$$

In the McGucken picture, each infinitesimal propagation step is constrained by the spherical wavefront structure $r = c\,dt$. A path integral then sums over chains of such propagations:

$$K(q_f, t_f; q_i, t_i) = \int_{P(q_i, q_f)} e^{iS[\gamma]/\hbar} D\gamma.$$

Result. Path space is the space of histories assembled from McGucken-derived propagation steps.

Standard path-integral object	McGucken interpretation
Path γ	Chain of allowed propagation events
Measure $D\gamma$	Sum over possible chains
Phase $e^{iS/\hbar}$	Action phase from McGucken complex structure
Propagator K	Accumulated McGucken wavefront amplitude

16.9 Fock space

Fock space is derived from Hilbert space by allowing variable excitation number.

Derivation. Once H is derived, define

$$F(H) = \oplus_{n=0}^{\infty} H^{\otimes n}.$$

For bosons one takes the symmetric tensor powers:

$$F_B(H) = \oplus_{n=0}^{\infty} Sym^n(H).$$

For fermions one takes the antisymmetric tensor powers:

$$F_F(H) = \oplus_{n=0}^{\infty} \wedge^n(H).$$

Result. Fock space is the tensor-completion of McGucken-derived Hilbert space.

Space	Construction from H	Physical role
H	One-particle Hilbert space	Single excitation
$H^{\otimes n}$	n-fold tensor product	n-particle sector

Space	Construction from H	Physical role
$F_B(H)$	Symmetric tensor sum	Bosonic field space
$F_F(H)$	Exterior tensor sum	Fermionic field space

16.10 Operator-algebra space

Operator algebras are derived once the Hilbert space of states has been derived.

Derivation. From the McGucken-derived Hilbert space H, form the algebra of bounded linear operators

$$B(H).$$

Physical observables are represented by suitable self-adjoint operators

$$\hat{A}=\hat{A}^{\dagger}.$$

The McGucken flow operator induces the quantum operator

$$\widehat{M}=i\hbar D_M,$$

while spacetime translations induce

$$\hat{p}_{\mu}=-i\hbar\partial_{\mu}.$$

Together these operators generate a physically meaningful subalgebra

$$A_M\subseteq B(H).$$

Result. Operator-algebra space is derived from McGucken Space by first deriving Hilbert space and then forming the algebra of transformations and observables acting on it.

Operator space	Derivation
$B(H)$	All bounded operators on McGucken-derived H
Self-adjoint observables	Physical measurement operators
$\widehat{M}=i\hbar D_M$	Quantum McGucken generator
A_M	McGucken-generated observable algebra

16.11 Summary of examples

The examples above support the stronger form of the paper's thesis:

The spaces of physics arise as descendants of McGucken Space under physically natural constructions.

Derived space	Derivation chain
Spacetime	$M_G\rightarrow\Phi_M^{-1}(0)\rightarrow M_{1,3}$
Light cone	$M_G\rightarrow M_{1,3}\rightarrow ds^2=0$

Derived space	Derivation chain
Configuration space	$M_G \to M_{1,3} \to \Sigma_t \to Q$
Phase space	$M_G \to Q \to T^{¿} Q$
Hilbert space	$M_G \to M_{1,3} \to V \to H$
Spinor space	$M_G \to \eta_{\mu\nu} \to Cl(1,3) \to S$
Gauge-bundle space	$M_G \to M_{1,3} \to P \to M_{1,3}$
Path space	$M_G \to \Sigma_M \to P(q_i, q_f)$
Fock space	$M_G \to H \to F(H)$
Operator algebra	$M_G \to H \to B(H) \to A_M$

These examples make the derivability principle concrete: the McGucken framework treats standard mathematical spaces not as independent foundations, but as successive constructions generated from the fourth-dimensional principle $dx_4/dt = ic$.

17. Foundational Priority and Minimality of McGucken Space

The preceding results show that McGucken Space generates the familiar spaces of physics. The stronger claim is proved: within the formal system adopted in this paper, McGucken Space is the most foundational physical space. The reason is stronger than derivational breadth: none of the derived spaces contains enough primitive structure to reconstruct it without adding the McGucken Principle as an extra axiom.

17.1 Derivability order

Define a derivability relation $\preceq$ on physical spaces by

$$X \preceq Y \Leftrightarrow X \in Der(Y).$$

Thus $X \preceq Y$ means that X is derivable from Y. In this notation, the Universal Derivability Principle states

$$X \preceq M_G \text{ for every } X \in PhysSpace.$$

The derivability relation is reflexive because $X \in Der(X)$. It is transitive because if $X \in Der(Y)$ and $Y \in Der(Z)$, then the derivation of X from Y can be composed with the derivation of Y from Z, giving $X \in Der(Z)$. Therefore $\preceq$ is a preorder on physical spaces.

17.2 Primitive signature of McGucken Space

The primitive signature of McGucken Space is

$$Sig(M_G) = \{E_4, x_4, t, i, c, \Phi_M = x_4 - ict, D_M = \partial_t + ic\,\partial_{x_4}, \Sigma_M\}.$$

Equivalently, McGucken Space contains four irreducible pieces of primitive physical data:

Primitive datum	Meaning
x_4	Fourth coordinate prior to projection
$dx_4/dt = ic$	Universal expansion law
$D_M = \partial_t + ic\,\partial_{x_4}$	Flow operator along the primitive fourth-dimensional expansion
Σ_M	Spherical propagation structure

These are not ordinary decorations added to spacetime or Hilbert space. They are the defining data from which spacetime, quantum amplitudes, wave propagation, and operator structures are derived.

17.3 Non-derivability from spacetime

Theorem 17.1 (McGucken Space is not derivable from Lorentzian spacetime alone). Lorentzian spacetime $M_{1,3}$ does not determine M_G unless the McGucken primitive signature is added as extra structure.

Proof. Lorentzian spacetime supplies a manifold with metric structure:

$$\left(M_{1,3}, g_{\mu\nu}\right).$$

It contains events, intervals, causal cones, and Lorentzian geometry. But it does not uniquely specify a prior Euclidean four-coordinate carrier E_4, a distinguished fourth coordinate x_4, the constraint function $\Phi_M = x_4 - ict$, the flow operator $D_M = \partial_t + ic\,\partial_{x_4}$, or the spherical propagation structure Σ_M as primitive generating data. Many different higher-dimensional or analytic structures can project to the same Lorentzian spacetime. Therefore the map

$$M_G \to M_{1,3}$$

is many-to-one at the level of primitive structure. A many-to-one projection has no unique inverse without additional assumptions. Hence $M_{1,3} \not\Rightarrow M_G$ unless the missing McGucken primitive signature is appended. ▫

17.4 Non-derivability from Hilbert space

Theorem 17.2 (McGucken Space is not derivable from Hilbert space alone). Hilbert space H does not determine M_G unless the McGucken primitive signature is added as extra structure.

Proof. A Hilbert space is a complete complex inner-product vector space. It determines linear superposition, inner products, norms, projections, and operator theory. But by itself it does not determine:

1. a unique underlying spacetime manifold $M_{1,3}$;
2. a unique fourth coordinate x_4;
3. the expansion law $dx_4/dt = ic$;
4. the constraint $\Phi_M = x_4 - ict$;

5. the McGucken flow D_M;
6. the spherical wavefront structure Σ_M.

Indeed, many inequivalent physical systems are represented on isomorphic Hilbert spaces. The Hilbert space encodes state geometry, not the unique generative origin of that geometry. Therefore the derivation

$$M_G \to M_{1,3} \to V \to H$$

forgets the primitive generative data that selected M_G. Since forgotten primitive data cannot be recovered from H alone, Hilbert space cannot derive McGucken Space without adding the McGucken Principle externally. ▫

17.5 Non-derivability from phase space, gauge bundles, and operator algebras

Theorem 17.3 (Derived spaces do not generate the source-space). Phase space, gauge-bundle space, spinor space, Fock space, path space, and operator-algebra space do not determine M_G unless the McGucken primitive signature is added as extra structure.

Proof. Each listed space is produced only after at least one information-losing construction:

Derived space	Information-losing step
Phase space $T^{¿}Q$	Requires prior choice of Q, which requires prior slicing of $M_{1,3}$
Spinor space S	Retains a Clifford representation, not the primitive x_4-flow
Gauge bundle $P \to M_{1,3}$	Retains internal fiber symmetry, not the unique source of $x_4 = ict$
Path space P	Retains histories in $M_{1,3}$, not the primitive carrier E_4
Fock space $F(H)$	Built after Hilbert completion and tensoring
Operator algebra A	Encodes transformations on states, not the generative origin of the state space

In every case, the construction starts from an already-derived spacetime, field, Hilbert space, or representation. These spaces may remember consequences of McGucken Space, but not the full primitive signature. Therefore none of them reconstructs M_G uniquely. ▫

17.6 Foundational maximality theorem

Theorem 17.4 (Foundational maximality of McGucken Space). In the derivability preorder $\le$, McGucken Space is a maximal foundation for the physical spaces considered in this paper:

$$\forall X \in PhysSpace, X \le M_G,$$

while for every standard derived physical space $X \ne M_G$,

$$M_G \npreceq X$$

unless the McGucken primitive signature is added to X as extra structure.

Proof. Equation (101) is the Universal Derivability Principle demonstrated by the derivation examples above. Equation (102) follows from Theorems 17.1, 17.2, and 17.3: spacetime, Hilbert space, phase space, spinor space, gauge-bundle space, path space, Fock space, and operator-algebra space each lacks at least one primitive item in $Sig(M_G)$. Since a derivation cannot recover primitive structure erased by projection, completion, quotienting, representation, or algebra formation without adding new axioms, M_G is not derivable from those spaces alone. Therefore McGucken Space is foundationally prior to them in the derivability order. ▫

17.7 Simplicity theorem

The preceding theorem establishes foundational priority. A separate question concerns simplicity.

Define the primitive-law complexity $C(X)$ of a physical source-space X to be the number of independent primitive physical laws required to generate its associated physical arenas. In the present framework,

$$C(M_G)=1,$$

because the entire construction begins from the single primitive physical law

$$\frac{dx_4}{dt}=ic.$$

Theorem 17.5 (Minimal primitive-law complexity). McGucken Space is the simplest possible physical source-space in the primitive-law sense:

$$C(M_G)=1\leq C(X)\ \text{for every nontrivial physical source-space}\ X.$$

Proof. A nontrivial physical source-space must contain at least one primitive physical law or generating relation; otherwise it generates no physical structure. Hence $C(X)\geq 1$ for every nontrivial source-space X. McGucken Space is generated by exactly one primitive law, $dx_4/dt=ic$. Therefore $C(M_G)=1$, which is the minimum possible nonzero primitive-law complexity. ▫

This theorem gives a precise meaning to the statement that McGucken Space is, by definition, the simplest possible physical space: it has the minimal possible number of primitive physical laws while still generating the standard spaces of physics.

17.8 Final foundational table

Foundation	Can derive standard spaces?	Can derive McGucken Space?	Foundational status
Lorentzian spacetime	Partially	No, lacks x_4, D_M, Σ_M	Derived event space
Phase space	Partially	No, requires prior configuration/spacetime	Derived classical-state space
Hilbert space	Partially	No, lacks unique spacetime and x_4-flow	Derived quantum-state space

Foundation	Can derive standard spaces?	Can derive McGucken Space?	Foundational status
Spinor space	Partially	No, only representation fiber	Derived representation space
Gauge-bundle space	Partially	No, requires base spacetime and internal symmetry choice	Derived interaction space
Fock space	Partially	No, requires prior Hilbert space	Derived many-body state space
Operator algebra	Partially	No, requires prior state space	Derived observable space
McGucken Space	Yes	Primitive	Foundational source-space

Thus the paper's central ordering is:

McGucken Space is not derived from the spaces of physics; the spaces of physics are derived from McGucke

17.9 Physical-reality explanation of the power of McGucken Space

The theorems above give the formal reason McGucken Space is foundational in the derivability order. There is also a natural physical reason for its unusual mathematical power: McGucken Space is built from the primitive physical reality identified by the theory, not from a downstream representation.

The linked McGucken Symmetry paper defines the McGucken Symmetry as the assertion that the fourth coordinate evolves according to $dx_4/dt = ic$, treats it as "a structural commitment of the geometry of spacetime," and presents it as the "father symmetry" from which Lorentz, Poincaré, Noether, gauge, quantum-unitary, CPT, diffeomorphism, supersymmetry, and duality symmetries descend ([1]). The linked McGucken Sphere paper defines the McGucken Sphere as the future null-conical/spherical wavefront structure generated from events and describes it as "spacetime's foundational atom," with spacetime composed of these McGucken Spheres ([2]).

This yields the following explanatory principle.

Principle 17.9 (physical-source explanation of mathematical power). A mathematical space has maximal foundational power when it is not a representation alone of physical states, events, fields, or observables, but directly encodes the primitive physical symmetry and primitive physical atom from which those states, events, fields, and observables are generated.

McGucken Space satisfies this principle because it contains both:

Foundational physical reality	Mathematical encoding in McGucken Space	Consequence
McGucken Symmetry	$dx_4/dt = ic$, $\Phi_M = x_4 - ict$	Lorentzian structure, symmetry descent, invariant speed
McGucken Sphere	Σ_M, null-spherical propagation	Light cones, wavefronts, path integrals, field propagation
Fourth-dimensional flow	$D_M = \partial_t + ic\,\partial_{x_4}$	Operator hierarchy, quantum generators, Wick structure
Primitive source-space	$M_G = \left(E_4, \Phi_M, D_M, \Sigma_M\right)$	Derivation of spacetime, Hilbert space, gauge bundles, Fock space, operator algebras

This explains why McGucken Space has more foundational reach than its mathematical peers. Hilbert space is powerful because it represents quantum states. Phase space is powerful because it represents classical states. Gauge-bundle space is powerful because it represents local internal symmetry. Twistor space and amplituhedron-like spaces are powerful because they reorganize scattering and null geometry. But in the McGucken framework, these spaces are downstream formal arenas. McGucken Space is upstream because it encodes the physical source itself: the fundamental symmetry $dx_4/dt = ic$ and the fundamental atom of spacetime, the McGucken Sphere.

Thus the formal asymmetry

$$PhysSpace \subseteq Der(M_G)$$

has a physical explanation: derivability follows the direction from physical source to mathematical representation, not the reverse.

18. Complete Space Hierarchy

The hierarchy is:

$$M_G \to M_{1,3} \to \{E \to M_{1,3}\} \to \Gamma(E) \to H \to F(H) \to A.$$

In words:

Stage	Construction	Result
1	Begin with $M_G = (E_4, \Phi_M, D_M, \Sigma_M)$	McGucken Space
2	Impose $\Phi_M = 0$	Lorentzian spacetime $M_{1,3}$
3	Attach fields/bundles	$E \to M_{1,3}$, $S \to M_{1,3}$, $P \to M_{1,3}$
4	Take sections	Classical fields, spinors, gauge fields
5	Add complex amplitudes and Born inner product	Hilbert space H
6	Add tensor products / occupation-number construction	Fock space $F(H)$
7	Add observables and generators	Operator algebra A

This is the precise sense in which ordinary spaces are “inside” McGucken Space: not always as subsets, but as descendants in a structured generative hierarchy.

19. Relation Table: Literal or Derived?

Object	Is it literally a subset of M_G?	Correct relation
E_4	Yes, as carrier component	Coordinate carrier
$C_M = \Phi_M^{-1}(0)$	Yes	Constraint surface
$M_{1,3}$	Not exactly; identified with projection of C_M	Derived spacetime
Light cone	No, subset of $M_{1,3}$	Derived null structure
Spatial slice	No, subset of $M_{1,3}$	Derived hypersurface
Worldline	No, curve in $M_{1,3}$	Derived path
Configuration space Q	No	Space of configurations over derived spacetime/slice
Phase space $T^{¿}Q$	No	Cotangent construction

Object	Is it literally a subset of M_G?	Correct relation
Hilbert space H	No	Completion of complex amplitude space over derived spacetime
Spinor space	No	Representation fiber over $M_{1,3}$
Gauge bundle	No	Fiber bundle over $M_{1,3}$
Fock space	No	Many-body quantum construction from H
Operator algebra	No	Algebra of transformations and observables on H

20. Central Theorem: Space-Operator Generation Chain

Theorem 20.1 (McGucken space-operator generation chain). Given McGucken Space

$$M_G=\left(E_4,\Phi_M,D_M,\Sigma_M\right),$$

the following space-operator chain is formally induced:

$$M_G\times D_M \xrightarrow{\Phi_M=0} M_{1,3}\times g \xrightarrow{\text{fields/bundles}} \Gamma\left(E\to M_{1,3}\right)\times\nabla \xrightarrow{\text{complex amplitudes + Born inner product}} H\times\{\hat{H},\hat{p},\hat{M}\} \xrightarrow{\text{tensor/Fock construction}} F\left(H\right.$$

Proof. The constraint $\Phi_M=0$ gives $x_4=ict$, which induces the Lorentzian interval and hence $M_{1,3}$ with metric signature g. The same primitive law gives the tangent source operator $D_M=\partial_t+ic\,\partial_{x_4}$. Fields, spinors, and gauge objects are sections of bundles over $M_{1,3}$. Connections arise when the source derivative is covariantized. Complex amplitudes arise from the i in the McGucken Principle. Spherical wavefront superposition supplies linearity. The Born rule supplies a positive quadratic norm and inner product. Completion gives H. Quantization of D_M gives $\hat{M}=i\hbar D_M$, with Hamiltonian and momentum sectors. Tensor products and occupation-number constructions give Fock space. Quantized and covariantized descendants generate the operator algebra A. ▫

Corollary 20.2 (space-operator unity). In the McGucken hierarchy, physical spaces and physical operators are not independent primitive categories. They are co-descendants of the single source relation $dx_4/dt=ic$.

Proof. Theorem 20.1 derives the spaces and their resident operators in the same chain from M_G and D_M, which themselves are co-generated from $dx_4/dt=ic$. Therefore spaces and operators are unified as co-descendant structures. ▫

21. Why This Matters

The standard view often begins with several separate arenas:

- spacetime for events;
- phase space for classical states;
- Hilbert space for quantum states;

- spinor spaces for fermions;
- gauge bundles for interactions;
- Fock space for quantum fields;
- operator algebras for observables.

The McGucken framework proposes a unifying generative arena:

$$M_G = \left(E_4 , \Phi_M , D_M , \Sigma_M \right).$$

From this one obtains:

Standard postulate/arena	McGucken source
Lorentzian spacetime	$x_4 = i c t$ projection
Complex quantum phase	i in $d x_4 / d t = i c$
Wave propagation	McGucken spherical wavefront
Hilbert space	Complex superposition + Born inner product + completion
Momentum operators	Translation generators on McGucken-derived amplitudes
Hamiltonian	Time-translation generator within $\widehat{M}$
Schrödinger equation	Quantum evolution theorem in linked McGucken chain
Dirac equation	Clifford square root of induced Lorentzian operator
Gauge theory	x_4-phase freedom and covariant D_M^A
Path integrals	Sum over McGucken Sphere chains
Fock space	Quantized field modes over McGucken-derived Hilbert space

This gives the framework its central philosophical and mathematical claim: McGucken Space is not a competitor to Hilbert space or spacetime. It is the source from which both become intelligible as different derived arenas.

22. Open Problems

Several tasks remain for full formal development:

1. Define McGucken Space globally on curved manifolds.
2. Specify analytic domains for D_M and $\widehat{M}$.
3. Prove self-adjointness or essential self-adjointness of $\widehat{M}$ under physical boundary conditions.
4. Derive the Born rule without auxiliary probability assumptions beyond McGucken spherical symmetry.
5. Prove the Hilbert-space completion theorem for interacting fields.
6. Define the precise functor from McGucken Space to Hilbert spaces.
7. Establish the gauge group selected by x_4-phase freedom.
8. Derive Fock-space statistics from the McGucken spinor/Clifford structure.
9. Connect the McGucken operator algebra to standard $C^{¿}$-algebraic quantum mechanics.
10. Determine experimental consequences distinguishing McGucken-derived Hilbert space from standard postulated Hilbert space.

23. Conclusion

McGucken Space is defined as

$$M_G = (E_4, \Phi_M, D_M, \Sigma_M)$$

with

$$\Phi_M = x_4 - ict, D_M = \partial_t + ic\,\partial_{x_4}.$$

It is the generative arena of the McGucken Principle.

Lorentzian spacetime is obtained as the constraint/projection

$$M_{1,3} \cong \Phi_M^{-1}(0).$$

Configuration space and phase space are then built over the derived spacetime. Spinor spaces and gauge bundles are representation and fiber structures over it. Hilbert space is not a literal subset of McGucken Space; rather, it is the completed complex inner-product state space of wavefunctions or sections over the McGucken-derived spacetime:

$$H \simeq L^2(M_{1,3}) \text{ or } H \simeq \Gamma_{L^2}(E \to M_{1,3}).$$

The final hierarchy is:

$$M_G \to M_{1,3} \to \text{fields and bundles} \to H \to F(H) \to A.$$

Thus McGucken Space is not another mathematical space beside Hilbert space, phase space, or spacetime. It is the source-space from which those spaces arise by constraint, projection, bundle formation, representation, quantization, and completion. The McGucken Principle generates not only the spaces, but also the operators that reside in them.

The formal conclusion is stronger. In the derivability preorder defined above,

$$X \preceq M_G \text{ for every physically meaningful space } X,$$

while

$$M_G \npreceq X$$

for the standard derived spaces X unless the McGucken primitive signature is added back into them. Thus McGucken Space is the most foundational space in the hierarchy: it generates spacetime, metric structure, Hilbert space, phase space, spinor space, gauge-bundle space, connection structure, Fock space, operator algebras, and the operators acting in those arenas, but none of those spaces generates McGucken Space.

It is also the simplest possible physical source-space in the primitive-law sense. Since every nontrivial physical source-space requires at least one primitive generating law, and since McGucken Space is generated by the single law

$$\frac{dx_4}{dt} = ic,$$

its primitive-law complexity is minimal:

$$C(M_G) = 1.$$

Therefore McGucken Space is both foundationally maximal and primitively minimal: maximal in derivational power, minimal in primitive assumptions. The final formal conclusion is the space-operator co-generation law:

$$\frac{dx_4}{dt} = ic \Rightarrow (M_G, D_M) \Rightarrow \text{the spaces and operators of fundamental physics.}$$

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