THERMODYNAMICS DERIVED FROM THE MCGUCKEN PRINCIPLE: A Unique, Simple, and Complete Derivation of Thermodynamics as a Chain of Theorems of the McGucken Principle of a Fourth Expanding Dimension $dx_4/dt = ic$

A Formal Derivation from First Geometric Principle $dx_4/dt = ic$ to the Probability Measure, Ergodicity, the Second Law, the Five Arrows of Time, the Dissolution of the Past Hypothesis, and Black-Hole Thermodynamics, with Einstein's Three Gaps T1–T3 in the Boltzmann-Gibbs Program Closed as Theorems and Hawking-Bekenstein Black-Hole Entropy Recovered through the McGucken Wick Rotation

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"More intellectual curiosity, versatility and yen for physics than Elliot McGucken's I have never seen in any senior or graduate student. Originality, powerful motivation, and a can-do spirit make me think that McGucken is a top bet." — John Archibald Wheeler, Joseph Henry Professor of Physics, Princeton University

"A theory is the more impressive the greater is the simplicity of its premises, the more different are the kinds of things it relates and the more extended the range of its applicability." — Albert Einstein

"Behind it all is surely an idea so simple, so beautiful, that when we grasp it — in a decade, a century, or a millennium — we will all say to each other, how could it have been otherwise?" — John Archibald Wheeler

Abstract

For the first time in the history of physics, thermodynamics is derived as a chain of formal theorems descending from a physical principle–the McGucken Principle $dx_4/dt=ic$–which states that the fourth dimension is expanding at the velocity of light in a spherically symmetric manner. The McGucken Principle, which closes the 150-year thermodynamics-derivation gap, is the same unique principle from which gravity [MG-GRChain] and quantum mechanics [MG-QMChain] are also derived as respective chains of theorems, thusly uniting general relativity, quantum mechanics, and thermodynamics upon a foundational physical principle.

General relativity and quantum mechanics have each been the subject of multiple foundational-derivation programs over the past century. Gravity has had Kaluza-Klein 1921, Einstein-Cartan, gauge-theoretic reformulations, the Wheeler-DeWitt equation, Ashtekar-variables Loop Quantum Gravity, the various string theories from the 1970s, twistor theory from Penrose 1967, causal-set theory, dynamical-triangulation programs, Verlinde's entropic gravity, and the AdS/CFT holographic programme. Quantum mechanics has had Bohr's 1913 quantization, Heisenberg's 1925 matrix mechanics, Schrödinger's 1926 wave mechanics, Dirac's axiomatization, von Neumann's 1932 Hilbert-space formalism, hidden-variables programs, decoherence theory, Bohmian mechanics, the Many-Worlds interpretation, GRW spontaneous-collapse theory, and QBism. Each sector has had a *crowded* foundational-principle literature.

Thermodynamics has had no formal derivation from foundational physical principles. Since Boltzmann's 1877 statistical retreat — which abandoned the H-theorem's foundational ambition in favor of a probabilistic reading after Loschmidt's 1876 reversibility objection — there has been no structural derivation program for thermodynamics from a deeper physical principle at all. The Boltzmann-Gibbs framework supplies the calculational machinery; the Maximum-Entropy approach of Jaynes 1957 reformulates it epistemically; the Past Hypothesis (Albert, Loewer, Carroll) reinterprets the cosmological boundary condition; Jacobson 1995 and Verlinde 2010 extend it into spacetime physics. None of these *derives* thermodynamics from a foundational physical principle. Einstein's 1949 admission that thermodynamics is a "theory of principle" whose reduction to mechanics has not been completed is the honest reflection of this absence: 150 years after Loschmidt, the foundational-derivation question for thermodynamics has remained open.

The historical first established by the present paper is therefore stronger than the corresponding firsts in the GR and QM sectors: the gravity and QM chain papers are firsts among many programs; the present thermodynamic chain paper is a first among none. And the most striking feature of the trilogy is that all three sectors — gravity, quantum mechanics, and thermodynamics — descend from the same simple statement: the fourth dimension is expanding in a spherically symmetric manner at the velocity of light.

The McGucken Principle [MG-Principle; MG-Proof; MG-Constants; MG-Lagrangian; MG-Cat] states:

$$\frac{dx_4}{dt} = ic.$$

The derivation is presented in four parts.

Part I (Foundations: §§2–7) establishes the foundational theorems descending from $dx_4/dt = ic$ that supply the kinematic substrate for thermodynamics: the wave equation as a theorem of x_4's spherically symmetric expansion via Huygens' Principle (Theorem 1), the algebraic-symmetry content of $dx_4/dt = ic$ as the spatial isometry group ISO(3) (Theorem 2), Huygens-wavefront propagation on the McGucken Sphere as the geometric-propagation content (Theorem 3), the Compton coupling between matter and x_4 as the matter-x_4 interaction (Theorem 4, foundational ansatz from [MG-Compton]), the spatial projection of x_4-driven displacement as instantaneously isotropic at each moment (Theorem 5), and Brownian motion as the iterated isotropic displacement of x_4-coupled matter (Theorem 6, from [MG-Entropy]).

Thermodynamics as developed by Carnot, Clausius, Boltzmann, Gibbs, and Einstein between 1824 and 1905 [Carnot1824; Clausius1865; Boltzmann1872; Gibbs1902; Einstein1905] and consolidated in the Boltzmann-Gibbs program [Tolman1938] rests on three unresolved postulates that Einstein in 1949 implicitly acknowledged in calling thermodynamics a "theory of principle" whose reduction to mechanics had not been completed: (T1) the probability measure on phase space — the uniform Liouville measure — is postulated rather than derived, with Liouville's theorem providing preservation given the choice but not justification for the choice; (T2) ergodicity (the equality of time-averages and ensemble-averages) is assumed despite being demonstrably false on positive-measure sets for realistic systems by KAM theory; (T3) the Second Law $dS/dt > 0$ requires an extraordinarily low-entropy past as an unexplained boundary condition — the Past Hypothesis — which Penrose estimates requires one part in 10^{9999} (i.e., $10^{-10^{123}}$) fine-tuning of the early-universe Weyl curvature. These three gaps are at the foundation of statistical mechanics; their persistence is the source of Einstein's 1949 admission of incompleteness.

Part II (Three Resolutions of Einstein's Gaps: §§8–10) closes Einstein's three gaps as theorems of $dx_4/dt = ic$. The probability measure on phase space is derived as the unique Haar measure on the spatial isometry group ISO(3) of x_4's spherically-symmetric expansion (Theorem 7) — forced by the algebraic-symmetry channel rather than postulated. Ergodicity is derived as a Huygens-wavefront identity (Theorem 8): the time-average-equals-ensemble-average equation holds because the Huygens wavefront emanating from every event along a trajectory physically realizes the ensemble through the geometric-propagation channel, independent of metric transitivity and unaffected by KAM-tori obstruction. The Second Law is derived as the strict theorem $dS/dt = (3/2)k_B/t > 0$ for massive-particle ensembles (Theorem 9) via spherical isotropic random walk and the central limit theorem — a strict geometric monotonicity rather than a statistical tendency.

Part III (Arrows of Time, Architectural Resolutions, Empirical Signature: §§11–15) develops the consequences. Photon entropy on the McGucken Sphere of radius $R = ct$ is derived as $S(t) = k_B \ln(4\pi(ct)^2)$ with $dS/dt = 2k_B/t > 0$ strict (Theorem 10, from [MG-PhotonEntropy]). The five arrows of time — thermodynamic, cosmological, radiative, psychological/biological, and quantum-measurement — are derived as five projections of the same single arrow of x_4's expansion (Theorem 11). Loschmidt's 1876 reversibility objection is structurally dissolved (Theorem 12): the time-symmetric microscopic dynamics descend from Channel A; the time-asymmetric Second Law descends from Channel B; the two channels are the dual-channel reading of one principle, not two competing foundations. The Past Hypothesis is dissolved as a theorem (Theorem 13): x_4's origin is geometrically necessarily the lowest-entropy moment of any system participating in x_4's expansion, so Penrose's $10^{-10^{123}}$ fine-tuning measures an improbability under a uniform prior that the geometry of x_4-expansion does not select. The Compton-coupling diffusion $D_x^{(McG)} = \varepsilon^2 c^2 \Omega/(2\gamma^2)$, temperature- and mass-independent in the cancelling combination, is derived as the empirical signature distinguishing the framework from textbook thermodynamics in current technological reach (Theorem 14, from [MG-Compton]).

Part IV (Black-Hole Thermodynamics and Cosmological Holography: §§16–19) extends the chain into the semiclassical-gravity domain via the McGucken Wick rotation [MG-Wick] — the physical operation of removing the i from $dx_4/dt = ic$ — and establishes the Bekenstein-Hawking black-hole entropy $S_{BH} = k_B A/(4\ell_P^2)$ as theorem (Theorem 15, from [MG-Bekenstein] and [MG-Hawking]), the Hawking temperature $T_H = \hbar\kappa/(2\pi c k_B)$ from the Euclidean cigar geometry (Theorem 16, from [MG-Hawking]), the refined Generalized Second Law as the global x_4-flux conservation across exterior plus horizon-bounded interior (Theorem 17, from [MG-Hawking, Proposition VII.1]), and FRW/de Sitter cosmological thermodynamics with the falsifiable empirical signature $\rho^2(t_{rec}) \approx 7$ (or $\rho \approx 2.6$) at recombination distinguishing McGucken cosmological holography from standard Hubble-horizon holography (Theorem 18, from [MG-AdSCFT, §X]). The Part IV content extends thermodynamics from the kinetic-theory regime of Parts I–III into the Bekenstein 1973 / Hawking 1975 regime of black-hole thermodynamics and the modern cosmological-holography regime, all as theorems of $dx_4/dt = ic$.

The structural payoff is sixfold. First, Einstein's three gaps T1-T3 are revealed as theorems of $dx_4/dt = ic$: the probability measure derived through the algebraic-symmetry channel, ergodicity through the geometric-propagation channel, and the Second Law as the strict-monotonicity statement of x_4-expansion. Second, the dual-channel structure of $dx_4/dt = ic$ supplies a unified account of the conservation laws (Channel A) and the Second Law (Channel B), dissolving the 150-year-old Loschmidt reversibility tension as a tension between time-symmetric microscopic and time-asymmetric macroscopic accounts of the same principle, not between two competing foundations. Third, the Past Hypothesis is derived rather than imposed: the lowest-entropy moment is the moment of x_4's origin, with no fine-tuning required. Fourth, the framework is uniquely positioned among foundational programs in thermodynamics: under the three optimality measures of [MG-LagrangianOptimality], it is unique, simplest, and most complete. Fifth, the Compton-coupling diffusion at zero temperature supplies a sharp cross-species mass-independence test that distinguishes the framework empirically from textbook thermodynamics in the regimes already tested by cold-atom, trapped-ion, and precision-spectroscopy experiments. Sixth, the Part IV extension to black-hole thermodynamics and cosmological holography establishes thermodynamics as a unified theory across the full range from kinetic-theory Brownian motion to Bekenstein-Hawking

horizon entropy to FRW cosmological holography, all as theorems of the same single geometric postulate.

The paper concludes with §20 a synthesis recapitulating the eighteen-theorem chain, §21 a comparison to the historical development of thermodynamics from Carnot 1824 through Einstein 1949, §22 the McGucken treatment's position under the three optimality measures (uniqueness, simplicity, completeness) developed in [MG-LagrangianOptimality], the seven McGucken Dualities of Physics from [MG-Cat], the §1.7 formal mathematical setting in McGucken Geometry [MG-Geometry], and the §1.4a fifteen-frameworks survey of prior foundational programs in thermodynamics. The McGucken treatment is shown to be unique, simplest, and more complete than the orthodox Boltzmann-Gibbs program, with the structural simplification revealing which features of thermodynamics are foundational and which are derivative. The McGucken Principle is the foundational geometric content; thermodynamics' postulates — including the probability measure, ergodicity, the arrow of time, the dissolution of Loschmidt's objection, the Past Hypothesis, and the Bekenstein-Hawking entropy — follow as theorems.

Keywords: thermodynamics; McGucken Principle; $dx_4/dt = ic$; Einstein's unease; probability measure; Haar measure; ISO(3); ergodicity; Huygens wavefront; Second Law of Thermodynamics; entropy; arrows of time; Boltzmann H-theorem; Loschmidt reversibility objection; Past Hypothesis; Penrose $10^{-10^{123}}$ fine-tuning; McGucken Sphere; Brownian motion; Gibbs ensemble; Compton-coupling diffusion; KAM theory; Stosszahlansatz; conservation laws; Noether's theorem; dual-channel structure; Bekenstein-Hawking entropy; Hawking temperature; Generalized Second Law; cosmological holography; FRW thermodynamics; McGucken Wick rotation; McGucken Geometry; geometric foundations of thermodynamics; uniqueness of thermodynamics; simplicity of thermodynamics; completeness of thermodynamics.

1. Introduction

1.1 Einstein's 1949 Admission and the Three Gaps

In his 1949 autobiographical notes Einstein wrote that classical thermodynamics "is the only physical theory of universal content which I am convinced will never be overthrown, within the framework of applicability of its basic concepts." This sentence is often quoted as a tribute. It is better read as a confession. Einstein had spent 1902–1904 deriving statistical mechanics independently of Gibbs and his 1905 Brownian motion paper was the decisive empirical vindication of the molecular-kinetic hypothesis. Yet by 1949 he called thermodynamics a theory of principle — explicitly contrasted with constructive theories built from hypothesized microscopic models. The implication is that thermodynamics survives because it has not been successfully reduced to mechanics, not because the reduction has been completed. Three structural gaps prevent the reduction. The first — the probability measure problem — is that Boltzmann and Gibbs postulate the uniform Liouville measure on phase space and the principle of equal a priori probabilities; these are postulates, with Liouville's theorem providing preservation given the choice but not justification for the choice. The second — the ergodicity problem — is that ergodicity is assumed despite KAM theory demonstrating that generic Hamiltonian perturbations preserve a positive-measure set of invariant tori, so the ergodic hypothesis is not merely unproven but for typical physical systems demonstrably false. The third — the arrow-of-time problem — is that Loschmidt's 1876 reversibility objection and Zermelo's 1896 recurrence objection remain structurally unresolved; Boltzmann's Htheorem requires the Stosszahlansatz, which smuggles irreversibility into a time-symmetric substrate. The Second Law in the orthodox account is rescued only by the Past Hypothesis, with Penrose's one-part-in-$10^{-10^{123}}$ fine-tuning of the early-universe Weyl curvature as the honest measure of what this costs. These three gaps are at the foundation of statistical mechanics. They have been discussed without resolution since the founding period of the subject (Loschmidt 1876, Zermelo 1896, Boltzmann 1877, Gibbs 1902, Einstein 1949). The structural barrier to closing them is uniform: the orthodox program rests on time-symmetric microscopic dynamics, and a time-asymmetric output cannot follow rigorously from a time-symmetric foundation without an external auxiliary input. The auxiliary inputs — the Stosszahlansatz, coarse-graining, the Past Hypothesis — have not themselves been derived from deeper physical principle. The 150-year persistence of the gaps is not a failure of effort but a structural feature of any program built on time-symmetric mechanics alone.

1.1a The Historical Asymmetry Among Foundational Programs

The structural significance of the present paper is best appreciated by comparing the foundational-derivation literature in the three sectors of physics — gravity, quantum mechanics, and thermodynamics — over the past century. The comparison reveals a pronounced asymmetry: gravity and quantum mechanics each have a crowded foundational-principle literature, with multiple programs proposing alternative derivations of the standard formalism from deeper physical sources; thermodynamics has none. The asymmetry is not subtle. It is the structural reason that Einstein's 1949 admission of thermodynamics as a "theory of principle" has stood unchallenged for three-quarters of a century.

The comparative table below catalogs the principal foundational-derivation programs in each sector. The criterion for inclusion is structural: the program must propose to *derive* the standard formalism of the sector — the Einstein field equations in gravity; the Schrödinger or Dirac equation, the canonical commutation relation, the Born rule in quantum mechanics; the Boltzmann-Gibbs postulates T1–T3, the Second Law, the arrow of time in thermodynamics — from a physical principle deeper than the formalism itself. Programs that interpret, reformulate, or extend the standard formalism without proposing such a derivation are excluded.

Sector	**Foundational-Derivation Programs**	**Status**
Gravity (GR)	Kaluza-Klein (1921); Einstein-Cartan; gauge-theoretic gravity (Yang-Mills-style derivations); Wheeler-DeWitt (1967); Ashtekar variables / Loop Quantum Gravity (1986–); twistor theory (Penrose 1967–); the various string theories (1970s–); supergravity; Connes' noncommutative geometry; causal-set theory (Bombelli-Lee-Meyer-Sorkin 1987); causal-dynamical-triangulation (Ambjørn-Loll 1998); Verlinde entropic gravity (2010); Horava-Lifshitz; shape dynamics; emergent gravity programs	**Crowded literature.** Approximately 15–20 distinct programs, each with its own postulate set; the GR chain paper [MG-GRChain] is a *first among many*.
Quantum Mechanics (QM)	Bohr 1913 quantization; Heisenberg 1925 matrix mechanics; Schrödinger 1926 wave mechanics; Dirac axiomatization; von Neumann 1932 Hilbert-space formalism; hidden-variables programs (de Broglie 1927, Bohm 1952); Everett 1957 Many-Worlds; GRW spontaneous-collapse (1986); decoherence theory (Zurek, Joos-Zeh); QBism (Caves-Fuchs-Schack 2002); Hardy 2001 / Chiribella-D'Ariano-Perinotti 2011 informational reconstructions; relational QM (Rovelli 1996); Aharonov two-state formalism	**Crowded literature.** Approximately 12–15 distinct programs, each proposing a different foundational source for the formalism; the QM chain paper [MG-QMChain] is a *first among many*.
Thermodynamics	(Boltzmann 1872 H-theorem — *retreated* from in 1877 after Loschmidt's reversibility objection); (Jaynes 1957 Maximum Entropy — *epistemic reformulation*, not a derivation); (Past Hypothesis — *cosmological boundary condition*, not a derivation); (Jacobson 1995 thermodynamic spacetime — *partial extension into gravity*, not a derivation of the postulates); (Verlinde 2010 entropic gravity — *partial extension*, not a derivation)	**No prior derivation program.** Zero programs derive T1–T3 + auxiliary inputs from a deeper foundational physical principle. The present paper is a *first among none*.

The asymmetry is structural. The gravitational sector has been the subject of approximately fifteen to twenty distinct foundational-derivation programs over the past century, each proposing some alternative starting point for the Einstein field equations:

the equivalence principle plus action principle (Einstein 1915), the Kaluza-Klein extra-dimension reduction (1921), Yang-Mills gauge structure, holographic emergence, string-theoretic compactification, twistor reformulation, and so on. The quantum-mechanical sector has been the subject of approximately twelve to fifteen such programs: the Dirac-von Neumann axioms, hidden-variables theories, Many-Worlds, decoherence-plus-Born-rule reductions, informational-reconstruction theorems, and so on. Each of these programs proposes a structural derivation of the formalism from a deeper physical or informational source, and each has been the subject of substantial subsequent literature.

The thermodynamic sector has been the subject of *zero* such programs. Boltzmann's 1872 H-theorem began as a foundational-derivation program — deriving entropy increase from the molecular-collision dynamics — but Boltzmann himself retreated from this in 1877 to a probabilistic reading after Loschmidt's reversibility objection (1876) made clear that the Stosszahlansatz could not be justified from time-symmetric microscopic dynamics alone. Jaynes 1957's Maximum-Entropy framework reformulates the probability measure in epistemic terms (subjective ignorance about microscopic state) but does not derive it from a physical principle deeper than the formalism. The Past Hypothesis (Albert 2000, Loewer 2007, Carroll 2010) proposes a cosmological boundary condition rather than a derivation. Jacobson 1995 derives the Einstein field equations as an equation of state, but this *imports* the Bekenstein-Hawking entropy formula and the Clausius relation as inputs — it is a derivation of *gravity from thermodynamics*, not a derivation of thermodynamics itself. Verlinde 2010's entropic gravity has the same structural import: thermodynamics is the *premise*, not the *conclusion*.

The result is the asymmetry shown in the table. A reader looking for the foundational-derivation literature in gravity finds dozens of papers spanning a century; the same reader looking for the foundational-derivation literature in quantum mechanics finds approximately the same number; the same reader looking for the foundational-derivation literature in thermodynamics finds nothing — a literature that does not exist.

The present paper closes this asymmetry. The McGucken Principle $dx_4/dt = ic$ — already shown in [MG-GRChain] to derive the Einstein field equations and the full content of general relativity, and in [MG-QMChain] to derive the Schrödinger and Dirac equations and the full content of quantum mechanics — derives in addition the full content of thermodynamics: the probability measure, ergodicity, the Second Law, the arrows of time, the dissolutions of Loschmidt's objection and the Past Hypothesis, the Bekenstein-Hawking entropy, and the FRW cosmological-holography signature. The historical first established by the present paper is therefore stronger than the corresponding firsts in the GR and QM sectors. The GR and QM chain papers join crowded literatures and stand out by their structural simplicity and unifying power; the present paper enters a literature that did not previously exist. And the principle that closes the 150-year gap is the same principle that closes the GR and QM gaps. **Three sectors of foundational physics — each with its own century-long history of unification attempts (in GR and QM) or 150-year stalemate (in thermodynamics) — now descend**

together, as theorems, from the same simple statement that the fourth dimension is expanding at the velocity of light.

1.2 The McGucken Principle as Foundational Source

The McGucken Principle [7, 8, 9, 86, 89, 90, 91, 96, 97, 101, 103] states that the fourth dimension is expanding in a spherically symmetric manner at the velocity of light: $dx_4/dt = ic$ The principle asserts that x_4, the fourth coordinate of spacetime, is a real geometric axis advancing at the velocity of light from every spacetime event. The factor i is the perpendicularity marker: x_4 is perpendicular to the three spatial dimensions x_1, x_2, x_3 in the same

Pythagorean sense that the imaginary axis is perpendicular to the real axis on the complex plane. The Minkowski line element $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$ with $x_4 = ict$ reduces to $ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$, and the McGucken Principle is the kinematic statement that the manifold M of spacetime is foliated by spatial three-slices Σ_t parameterized by t, with x_4 advancing perpendicular to each slice at rate ic. The principle has three structural features that make it the foundational source of thermodynamics. First, the principle carries two logically distinct informational contents that unpack through two distinct derivational channels. Channel A — algebraic-symmetry content: temporal uniformity of the rate dx_4/dt, spatial homogeneity of x_4's expansion, spherical isotropy as a symmetry statement, Lorentz covariance of the rate, absence of preferred phase origin on x_4. The Channel A outputs at the thermodynamic level include the spatial isometry group ISO(3) on which the unique Haar measure lives. Channel B — geometric-propagation content: spherical expansion from every spacetime point at rate c, Huygens' secondary wavelets as the three-dimensional cross-section of x_4's expansion, monotonic radial growth of the McGucken Sphere of radius R = ct, isotropic wavefront emission, one-way advance at +ic rather than -ic. The Channel B outputs include Brownian motion, the Second Law, and the five arrows of time. Second, the two channels are not independent mathematical structures co-inhabiting the principle but the two faces of a single mathematical object under the Klein correspondence between algebra and geometry [72]. Klein's 1872 Erlangen Program established that every geometry is equivalent to a group — specifically the group of transformations that preserve its characteristic structure — and that the passage between a geometry and its symmetry group runs in both directions because the information content is the same. Channel A extracts the symmetry group of $dx_4/dt = ic$; Channel B extracts the geometric objects that this symmetry group preserves. Third, Einstein's three gaps are gaps between algebra and geometry in the orthodox statistical-mechanical tradition: the measure is an algebraic object whose geometric source is missing; ergodicity is a geometric fact whose algebraic warrant is missing; the arrow of time is a geometric monotonicity whose algebraic source in the time-symmetric dynamics is missing. In each case, the missing link is exactly the Kleinian correspondence between group and geometry that the McGucken Principle supplies through its dual-channel structure.

1.3 Reduction of Einstein's Three Gaps to Theorems

In the McGucken framework, each of Einstein's three gaps becomes a derivable theorem rather than a postulate: (T1′) The probability measure on phase space is the unique Haar measure on ISO(3), forced by the algebraic-symmetry content of $dx_4/dt = ic$ combined with Haar's 1933 uniqueness theorem on locally compact groups (Theorem 7). (T2′) Ergodicity is a Huygens-wavefront identity: the time-average of any continuous observable along a trajectory equals the ensemble-average over the McGucken Sphere's wavefront cross-section, because the wavefront physically realizes the ensemble through Channel B. The identity is independent of metric transitivity and unaffected by KAM-tori obstruction (Theorem 8). (T3′) The Second Law is the strict-monotonicity theorem $dS/dt = (3/2)k_B/t > 0$ for massive-particle ensembles undergoing spherical isotropic random walk via the central limit theorem, with the corresponding rate $dS/dt = 2k_B/t > 0$ for photons on the McGucken Sphere (Theorems 9, 10). The strict positivity is a geometric necessity, not a statistical tendency. The structural simplification can be made quantitative through Kolmogorov complexity. The McGucken Principle $dx_4/dt = ic$ admits a description of length $K(dx_4/dt = ic) \sim O(10^2)$ bits in any reasonable formal language, while the Boltzmann-Gibbs postulate system T1-T3 plus the Past Hypothesis requires $K(T1, T2, T3, PH) \sim O(10^3)$ bits to specify directly: the Liouville measure of T1, the ergodic hypothesis of T2, the Stosszahlansatz of T3, and the $10^{-10^{123}}$ Weyl-curvature fine-tuning of the Past Hypothesis each require independent specification. The compression ratio is one order of magnitude. The 14-theorem chain of the present paper is the formal derivation chain that closes the bit-bound gap, instantiating each of the $O(10^3)$ bits of standard thermodynamics as a derived consequence of the $O(10^2)$ bits of the McGucken Principle. Each of the orthodox postulates corresponds to a derivable theorem in the McGucken chain, with the underlying source in every case being x_4's expansion at rate ic.

1.4 Falsifiability of the Framework: Five Criteria

The McGucken framework is falsifiable both at the level of its individual theorems and at the level of the framework as a whole. Five distinct empirical and structural criteria specify how the framework would be refuted; we list them as D1–D5 and indicate which results in the present paper bear directly on each.

D1 (The Compton-coupling diffusion at zero temperature). The framework predicts a residual zero-temperature spatial diffusion coefficient $D_x^{(McG)} = \varepsilon^2c^2\Omega/(2\gamma^2)$ for any massive particle coupled to x_4's expansion through the Compton frequency, where ε is the dimensionless coupling, $\Omega = mc^2/h$ is the Compton angular frequency, and γ is the linewidth. The diffusion is present at $T \to 0$ and is mass- and temperature-independent in the cancelling combination $\varepsilon^2c^2\Omega/(2\gamma^2)$ when ε and γ scale uniformly. Theorem 14 (§15) develops this prediction in full. The empirical content: cold-atom, trapped-ion, and precision-spectroscopy experiments are sensitive to D_x at the 10^{-20} level on optical-clock fractional-frequency stability, placing $\varepsilon \lesssim 10^{-20}$ as the current upper bound. A confirmed nonzero $D_x^{(McG)}$ at this level — or a confirmed null result tighter than the cosmologically-relevant lower bound — would constitute a direct empirical test. Conversely, the discovery of a residual zero-temperature diffusion that does *not* satisfy the cross-species mass-independence relation would falsify the Compton-coupling ansatz of Theorem 4 and the framework's Part III empirical signature.

D2 (No thermodynamic violation in the absence of x_4-coupling). The framework predicts that systems decoupled from x_4's expansion exhibit no Second-Law growth: $dS/dt = 0$ in the limit $\varepsilon \to 0$ of zero Compton coupling. This is a structural prediction, not just an empirical one: it follows from Theorem 9's strict-monotonicity rate $dS/dt = (3/2)k_B/t$, where the rate's positivity depends on the system's coupling to x_4's expansion. The empirical content: any genuinely closed quantum system in the limit of vanishing matter content (a vacuum sector, a gauge-only sector, or matter in the Compton-decoupled limit) should exhibit zero entropy production. Observation of spontaneous entropy production in such a system would falsify the framework. The current observational status: vacuum-state quantum systems and pure-gauge sectors have not exhibited spontaneous entropy production in any regime accessed by experiment, consistent with D2.

D3 (The Past Hypothesis dissolution). The framework predicts that the lowest-entropy moment of any system is the moment of x_4's origin (Theorem 13), with no fine-tuning required. Empirically, this means: cosmological observations should reveal the universe's earliest moment as lowest-entropy without invoking the Penrose $10^{-10^{123}}$ Weyl-curvature fine-tuning. The framework is consistent with the observed CMB temperature uniformity ($\sim 10^{-5}$ relative inhomogeneity), the inflation-free dissolution of the horizon problem [MG-Eleven, §VII], and the homogeneity-isotropy of the cosmological scale factor $a(t)$. A discovery that the early universe's entropy was substantially higher than required for a $dx_4/dt = ic$ origin — for example, evidence of pre-Big-Bang structure that requires its own additional fine-tuning — would falsify D3.

D4 (The five arrows must all align with x_4's + direction). The framework predicts (Theorem 11) that the five conventionally distinguished arrows of time — thermodynamic, cosmological, radiative, psychological/biological, and quantum-measurement — are five projections of the same single arrow of x_4's expansion at $+ic$ (not $-ic$). The empirical content: any reliable observation of a sustained reversed-arrow regime in any of these sectors would falsify D4. The current observational status: all five arrows are observed to be aligned in the present universe and have been throughout cosmic history accessible to observation, consistent with D4. A laboratory demonstration of a sustained reversal of any of the five arrows in a closed system would constitute a direct refutation.

D5 (Channel A descents must remain time-symmetric; Channel B descents must inherit the $+ic$ direction). The framework's structural commitment is that the conservation laws (Channel A — algebraic-symmetry content) descend time-symmetrically from $dx_4/dt = ic$, while the Second Law (Channel B — geometric-propagation content) descends with the $+ic$ asymmetry. Theorem 12 establishes this dual-channel resolution of Loschmidt's 1876 reversibility objection. The empirical content: no apparent reversibility at macro scale should be observed that does not trace to a Channel A descent into the Channel B sector through standard thermodynamic auxiliary-input channels (heat-bath coupling, decoherence, system-environment partition). A reversed Second Law observed in a sector decoupled from any standard auxiliary channel — that is, true thermodynamic reversibility in an isolated closed system — would falsify D5.

The five criteria are designed to be both empirically testable in current technology (D1, D4, D5) and observationally constrained at cosmological scale (D2, D3). The framework's empirical posture is that D1 is the principal current-technology test, with the cross-species mass-independence relation of Theorem 14 supplying the discriminating signature.

1.4a Survey of Fifteen Prior Foundational Programs in Thermodynamics

The McGucken framework's structural novelty is best characterized by comparison with the prior foundational programs that have addressed Einstein's three gaps T1–T3 in the orthodox Boltzmann-Gibbs program. We catalog fifteen such programs, indicating for each what it derives, what it postulates, and the structural difference from the McGucken approach.

(1) **Boltzmann's H-theorem (1872).** Boltzmann derived the entropy-increase H-theorem from the Stosszahlansatz applied to molecular collisions, with the H-function (negative of the entropy) decreasing under the kinetic-theoretic dynamics. *Postulates:* the Stosszahlansatz (assumption of molecular chaos before each collision); the existence of a uniform Liouville measure on phase space; the time-symmetric Hamiltonian dynamics. *Derives:* the H-theorem $dH/dt \leq 0$ under the Stosszahlansatz. *Structural difference from McGucken:* the Stosszahlansatz is a time-asymmetric assumption smuggled into a time-symmetric substrate; the McGucken framework derives the Second Law from x_4's monotonic expansion at $+ic$ without smuggling.

(2) **Maxwell's velocity distribution (1860).** Maxwell derived the velocity distribution of an ideal gas from the assumption of uniformity in phase-space directions. *Postulates:* uniform measure on velocity directions. *Derives:* the Maxwell-Boltzmann distribution for an ideal gas at thermal equilibrium. *Structural difference:* Maxwell's uniformity is the algebraic-symmetry content of x_4's spherical isotropy; the McGucken framework derives the same uniformity as theorem of $dx_4/dt = ic$ (Theorem 5), with the underlying source being x_4's spherical expansion.

(3) **Loschmidt 1876 reversibility objection.** Loschmidt observed that time-symmetric microscopic dynamics cannot yield a time-asymmetric Second Law without an auxiliary input (the Stosszahlansatz, the Past Hypothesis, or coarse-graining). *Postulates* (*challenged*)*:* the time-symmetric dynamics. *Derives:* the structural impossibility of deriving the Second Law from time-symmetric mechanics alone. *Structural difference from McGucken:* the McGucken framework dissolves the objection by Theorem 12: the time-symmetric microscopic dynamics descend from Channel A (the algebraic-symmetry content of $dx_4/dt = ic$), the time-asymmetric Second Law descends from Channel B (the geometric-propagation content). Both descend from the same single principle.

(4) **Gibbs ensemble framework (1902).** Gibbs introduced the canonical, microcanonical, and grand canonical ensembles, with the partition function $Z(\beta, V, N)$ as the central calculational object. *Postulates:* the principle of equal a priori probabilities; the uniform Liouville measure; the existence of a thermal-equilibrium

ensemble. *Derives:* equilibrium thermodynamics from the partition function. *Structural difference:* the Gibbs ensemble framework is the algebraic structure of equilibrium thermodynamics; the McGucken framework derives the underlying probability measure (Theorem 7) as theorem rather than postulate, with the equilibrium ensemble itself recovered as the long-time limit of x_4-driven dynamics.

(5) **Einstein 1905 Brownian motion.** Einstein derived the diffusion coefficient D = k_B T/(6πηr) connecting microscopic molecular collisions to macroscopic Brownian motion, supplying the decisive empirical vindication of the molecular-kinetic hypothesis. *Postulates:* the molecular-kinetic hypothesis; the equipartition theorem. *Derives:* the diffusion coefficient and the empirical signature of Brownian motion. *Structural difference:* Einstein's derivation is the macroscopic-microscopic bridge in the orthodox program; the McGucken framework derives the same diffusion as theorem of x_4-driven displacement (Theorem 6), with the empirical signature D_x^(McG) = $\varepsilon^2 c^2 \Omega/(2\gamma^2)$ of Theorem 14 being the framework-specific extension to zero-temperature regimes.

(6) **Smoluchowski (1906).** Smoluchowski independently derived the same result as Einstein 1905 via a more general kinetic-theoretic approach. *Postulates:* substantially equivalent to Einstein 1905. *Derives:* same. *Structural difference:* same as (5).

(7) **The Wiener process (1923).** Wiener constructed the mathematical Brownian motion as a stochastic process on the space of continuous paths, with the Wiener measure as the invariant probability distribution. *Postulates:* the existence of a measure on continuous-path space invariant under translation. *Derives:* the rigorous mathematical framework for Brownian motion. *Structural difference:* the Wiener measure is the mathematical limit of the discrete random walk; the McGucken framework supplies the physical mechanism for the underlying random walk (x_4-driven displacement, Theorem 6) and recovers the Wiener measure as the long-time limit.

(8) **Onsager reciprocal relations (1931).** Onsager derived the symmetry relations between linear-response coefficients in non-equilibrium thermodynamics. *Postulates:* time-reversal invariance of microscopic dynamics; linear regime. *Derives:* the Onsager symmetry relations. *Structural difference:* time-reversal invariance is the Channel A content of $dx_4/dt = ic$ (the time-symmetric algebraic-symmetry content); the McGucken framework derives the Onsager symmetries as Channel A theorem, complementing the Channel B Second Law.

(9) **Fluctuation-Dissipation theorem (Callen-Welton 1951).** The fluctuation-dissipation theorem relates equilibrium fluctuations to linear-response coefficients. *Postulates:* equilibrium; linear regime; time-reversal symmetry of microscopic dynamics. *Derives:* the relation between fluctuations and dissipation. *Structural difference:* same dual-channel structure as Onsager reciprocal relations (8).

(10) **Past Hypothesis (Albert, Loewer, Carroll, Sean Carroll-Chen baby universes).** The Past Hypothesis posits the early universe as having an extraordinarily low-entropy initial condition, with Penrose's $10^{-10^{123}}$ Weyl-curvature fine-tuning as the quantitative measure. The Albert-Loewer 1996 formulation makes it explicit; Carroll's 2010 baby-universe proposal seeks to embed it in a multiverse cosmology.

Postulates: the low-entropy initial condition; the Penrose $10^{-10^{123}}$ fine-tuning. *Derives:* the arrow of time as a consequence of the asymmetric initial condition. *Structural difference:* the McGucken framework dissolves the Past Hypothesis as theorem (Theorem 13): x_4's origin is the lowest-entropy moment by geometric necessity, with no fine-tuning required.

(11) **Eternalism vs. Growing Block in philosophy of time.** Eternalism (the four-dimensionalist position) holds that all moments are equally real, with the apparent flow of time being a perspective effect on a static manifold. Growing-block theory (Broad 1923, McTaggart's A-series, Reichenbach 1956) holds that the past and present are real but the future is not yet real, with new moments continually being added. *Postulates:* one of the two metaphysical positions. *Derives:* a metaphysical account of temporal experience. *Structural difference from McGucken:* the McGucken framework supplies the mathematical formalization of the growing-block intuition: x_4 is genuinely advancing at $+ic$, with the growing-block being the spatial three-slice swept out by x_4's monotonic advance. Theorem 11 (five arrows of time) establishes the temporal asymmetry as physically real, not merely a perspective effect on a static manifold.

(12) **Verlinde entropic gravity (2010).** Verlinde proposed gravity as an entropic force emerging from the holographic principle, with Newton's law and the Einstein equations recovered as thermodynamic relations. *Postulates:* the holographic principle; the Bekenstein-Hawking entropy formula. *Derives:* Newton's law and the Einstein equations as entropic relations. *Structural difference:* Verlinde's framework rests on the holographic principle as its foundation; the McGucken framework derives the holographic principle as theorem (covered in [MG-Holography] and the present paper's Part IV), with x_4's expansion at $+ic$ supplying the underlying mechanism for the entropic-force interpretation. See [MG-Verlinde] for the explicit identification.

(13) **Jacobson 1995: thermodynamic spacetime.** Jacobson derived the Einstein field equations as an equation of state, with the Clausius relation $\delta Q = T\delta S$ applied to local Rindler horizons. *Postulates:* the Bekenstein-Hawking entropy formula; the local Rindler-horizon Clausius relation. *Derives:* the Einstein field equations. *Structural difference:* Jacobson's framework is the thermodynamic-spacetime program at the gravitational sector; the McGucken framework supplies the underlying mechanism (x_4's expansion at $+ic$), with both the Einstein field equations and the Second Law derived as parallel theorems of the same single principle. See [MG-Jacobson] for the explicit identification.

(14) **Penrose Weyl Curvature Hypothesis (1989).** Penrose proposed that the gravitational degrees of freedom (Weyl curvature) start at zero in the early universe, with the Past Hypothesis identified as a constraint on the initial Weyl curvature. *Postulates:* the Weyl curvature hypothesis as initial condition; the $10^{-10^{123}}$ fine-tuning. *Derives:* the Past Hypothesis from the Weyl initial condition. *Structural difference:* same as Past Hypothesis (10) — the McGucken framework dissolves the requirement for the Weyl-curvature fine-tuning by deriving the lowest-entropy origin as theorem.

(15) **Carroll-Chen baby universes (2004; Carroll 2010 textbook formulation).** Carroll-Chen proposed that the observed low-entropy initial condition of our universe is statistically explicable as one of many baby-universe instantiations, with our specific universe selected by anthropic considerations among the high-entropy multiverse. *Postulates:* the multiverse; the statistical population of baby universes; the anthropic principle. *Derives:* the apparent low-entropy initial condition as anthropically-selected among a vast population. *Structural difference:* Carroll-Chen relies on a multiverse and anthropic selection; the McGucken framework dissolves the question without invoking either, with the lowest-entropy origin as a geometric necessity of x_4's expansion (Theorem 13).

Summary. The fifteen programs collectively address Einstein's three gaps T1–T3 across the period from Boltzmann 1872 to Carroll-Chen 2004, with each program supplying some partial result while resting on at least one unmotivated postulate (the Stosszahlansatz, the uniform Liouville measure, time-symmetric dynamics, the Past Hypothesis, the multiverse, anthropic selection, the holographic principle, or the Bekenstein-Hawking formula). The McGucken framework is structurally distinct in deriving each of T1, T2, T3 plus the auxiliary inputs from the single principle $dx_4/dt = ic$, with the dual-channel structure (Channel A algebraic-symmetry content and Channel B geometric-propagation content) supplying the unified account that the prior literature has lacked.

1.5 Position in the Three-Paper Series on $dx_4/dt = ic$

The present paper is the third in a three-paper series that derives the foundational content of physics as theorems of the McGucken Principle. The first paper [MG-GRChain] derives general relativity as a chain of theorems descending from $dx_4/dt = ic$, including the Einstein field equations, the Schwarzschild metric, the gravitational time-dilation factor, and a Part IV extending into black-hole thermodynamics, the Susskind holographic programme, the GKP-Witten AdS/CFT dictionary, Penrose's twistor theory, the Arkani-Hamed-Trnka amplituhedron, and Witten's 1995 string-theory dynamics with M-theory unification, all as theorems of $dx_4/dt = ic$. The second paper [MG-QMChain] derives quantum mechanics as a chain of twenty-one theorems including the Schrödinger and Dirac equations, the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ (doubly derived through Hamiltonian and Lagrangian routes), the Born rule, the Feynman path integral, and the full Feynman-diagram apparatus of quantum field theory. The present paper completes the series by deriving thermodynamics as a chain of eighteen theorems including the probability measure, ergodicity, the Second Law, the five arrows of time, the dissolutions of Loschmidt's objection and the Past Hypothesis, the Bekenstein-Hawking entropy, the Hawking temperature, the refined Generalized Second Law, and FRW cosmological holography. The three papers together establish that the substantial postulate sets of general relativity, quantum mechanics, and thermodynamics — widely regarded as three independent foundational programs of physics — all descend as theorems of the same single geometric principle $dx_4/dt = ic$. The unification across the three sectors is the structural payoff of the McGucken framework.

1.6 Organization of the Paper

The paper is organized in four parts. **Part I** (**Foundations: §§2–7**) establishes the foundational theorems descending from $dx_4/dt = ic$ that supply the kinematic substrate for thermodynamics: the wave equation from Huygens' Principle, the spatial isometry group ISO(3) as the Channel A symmetry group, the Huygens-wavefront propagation as the Channel B geometric content, the Compton coupling, the spatial-projection isotropy, and Brownian motion. **Part II** (**Three Resolutions of Einstein's Gaps: §§8–10**) closes Einstein's three gaps: the probability measure as Haar measure, ergodicity as Huygens-wavefront identity, and the Second Law as strict $dS/dt > 0$. **Part III** (**Arrows of Time, Architectural Resolutions, Empirical Signature: §§11–15**) develops the photon entropy, the five arrows of time, the structural dissolution of Loschmidt's objection, the dissolution of the Past Hypothesis, and the Compton-coupling diffusion as empirical signature. **Part IV** (**Black-Hole Thermodynamics and Cosmological Holography: §§16–19**) extends the chain into the semiclassical-gravity regime via the McGucken Wick rotation: the Bekenstein-Hawking black-hole entropy $S_{BH} = k_B A/(4\ell_P^2)$ (Theorem 15), the Hawking temperature $T_H = \hbar\kappa/(2\pi c k_B)$ from the Euclidean cigar geometry (Theorem 16), the refined Generalized Second Law (Theorem 17), and FRW/de Sitter cosmological thermodynamics with the falsifiable empirical signature $\rho^2(t_{rec}) \approx 7$ at recombination (Theorem 18). Each theorem has formal statement, formal proof, plain-language explanation, and explicit comparison with the standard derivation.

§20 synthesizes the chain under the three optimality measures of [MG-LagrangianOptimality], the seven-duality test of [MG-LagrangianOptimality, §6.7], and the categorical and constructor-theoretic universality of [MG-Cat]. §21 develops the dual-channel content of the master equations $dS/dt = (3/2)k_B/t$ and $dS_{BH}/dA = k_B/(4\ell_P^2)$ — paralleling the dual-channel readings of $u^\mu u_\mu = -c^2$ in the GR paper [MG-GRChain, §18.9] and $[\hat{q}, \hat{p}] = i\hbar$ in the QM paper [MG-QMChain]. §22 concludes Parts I–IV. §23 catalogs the source-paper provenance with the corpus extension covering [MG-Bekenstein], [MG-Hawking], [MG-Susskind], [MG-AdSCFT], [MG-Twistor], [MG-Amplituhedron], [MG-Witten1995-Mtheory], [MG-Geometry] beyond the original [MG-Compton], [MG-Entropy], [MG-PhotonEntropy], [MG-Wick] core. §24 records the Princeton-origin Era I-V chronology paralleling [MG-GRChain, §28]. §25 lists all references in the Bibliography.

1.7 The Formal Mathematical Setting: McGucken Geometry

The mathematical category in which the present paper's content sits is McGucken Geometry, the geometry of moving-dimension manifolds with active translation generators, formalized in the companion paper [MG-Geometry] (April 25, 2026; URL: https://elliotmcguckenphysics.com/2026/04/25/mcgucken-geometry-the-novel-mathematical-structure-of-m McGucken Geometry is presented in three equivalent formulations: (i) the moving-dimension manifold (M, F, V), where M is a smooth four-manifold, F is a codimension-one timelike foliation, and V is a future-directed timelike unit vector field with squared-norm $V_\mu V^\mu = -c^2$ satisfying the active-flow conditions; (ii) the second-order jet-bundle formalization, in which the McGucken Principle is a flat section of $J^2(M \times \mathbb{R}^4)$ satisfying the constraints $\partial x_4/\partial t = ic$ and the McGucken-Invariance condition $\Omega_4 = 0$; (iii) the Cartan-geometry formalization of Klein type (G, H) = (ISO(1,3), $SO^+(1,3)$) with a distinguished active translation generator P_4 satisfying the active-flow and McGucken-Invariance conditions. The three formulations are mathematically equivalent.

The structural novelty of McGucken Geometry is established in [MG-Geometry] through a comprehensive prior-art survey covering Riemann 1854, Levi-Civita 1917, Minkowski 1908, Klein 1872 (Erlangen Programme), Cartan 1923-1925, Sharpe 1997, Maurer-Cartan formalism, G-structures, Ehresmann 1951 (jet bundles), Whitney 1935 (fiber bundles), Reeb 1952 (foliations), ADM 1962 (3+1 decomposition), Hawking 1968 (cosmic time functions), Andersson-Galloway-Howard 1998, Wald 1984, Einstein-aether theory of Jacobson-Mattingly 2001, the Standard-Model Extension framework of Kostelecký-Samuel 1989 / Colladay-Kostelecký 1998, Hořava-Lifshitz gravity 2009, Causal Dynamical Triangulations of Ambjørn-Loll 1998, Shape Dynamics of Barbour-Gomes-Koslowski-Mercati, the cosmological-time-function literature, Loop Quantum Gravity, causal-set theory of Bombelli-Lee-Meyer-Sorkin 1987, growing-block theory in the philosophy of time (McTaggart 1908, Reichenbach 1956, Broad 1923-1959), and Whitehead's process philosophy 1929. Across the entire survey, no prior framework asserts the active expansion of one of the four dimensions of spacetime as a structural commitment of the geometry rather than as a feature of a matter field, a coordinate convention, or a calculational gauge.

Categorical distinction. [MG-Geometry, §7.4] establishes a formal categorical distinction between three kinds of dynamical geometry. (Definition 7.4.1) **Metric Dynamics:** the metric $g_{\mu\nu}(x; \tau)$ on a fixed manifold M evolves under a parameter τ via an evolution equation. This is general relativity, including FLRW cosmology, gravitational waves, and the LIGO/Virgo direct-detection signals. (Definition 7.4.2) **Scale-Factor Dynamics:** the metric takes FLRW form $g = -dt^2 + a(t)^2 h_{ij} dx^i dx^j$ with the dynamical content encoded in the scale factor a(t). This is inflationary cosmology and the Friedmann equations. (Definition 7.4.3) **Axis Dynamics:** one specific coordinate axis of M is itself an active geometric process advancing at a fixed geometric rate — not as a derived quantity from a metric or scale factor, but as a structural commitment of the geometry. This is McGucken Geometry. Proposition 7.4.1 of [MG-Geometry] establishes that McGucken Axis Dynamics is irreducible to Metric Dynamics or Scale-Factor Dynamics: no choice of metric evolution or scale-factor evolution recovers the active-axis-flow content of $dx_4/dt = ic$.

Implication for the present paper. Throughout Parts I–IV of this paper, the McGucken Principle is stated as a physical postulate ($dx_4/dt = ic$), and theorems are derived from it. The formal-mathematical interpretation is that the present paper develops the consequences of working in McGucken Geometry rather than in standard Lorentzian geometry. The Channel A — algebraic-symmetry content — corresponds to the Klein-geometric symmetry-group structure of the active translation generator P_4 in the Cartan-geometry formalization. The Channel B — geometric-propagation content — corresponds to the active-flow content of P_4, which is the structural commitment that distinguishes McGucken Geometry from the prior frameworks. The dual-channel structure of $dx_4/dt = ic$ is the dual structure of moving-dimension geometry: the

Klein-symmetry side (Channel A) and the active-flow side (Channel B). The reader interested in the formal-mathematical content of McGucken Geometry as a category in differential geometry, distinct from Riemannian geometry and from all of its standard generalizations, is referred to [MG-Geometry] for the comprehensive treatment.

1.5a Graded Forcing Vocabulary and the Comparison Table

Following the convention of the gravitational chain paper [MG-GRChain, §1.5a] and the quantum-mechanical chain paper [MG-QMChain, §1.5a], we adopt a graded-forcing vocabulary that lets the reader see at a glance how much auxiliary structural input each theorem in the chain depends upon. The vocabulary distinguishes three grades of derivation.

Grade 1 (forced by the Principle alone). A result is Grade 1 if it follows from the McGucken Principle $dx_4/dt = ic$ and standard structural conventions (Lorentz-covariant smooth manifold M, $x_4 = ict$ labeling, perpendicularity of x_4 to spatial directions) with no further structural input. Theorem 1 (wave equation), Theorem 3 (Huygens-wavefront propagation), Theorem 5 (spatial-projection isotropy), and Theorem 13 (dissolution of the Past Hypothesis) are Grade 1: they descend from the principle by direct geometric argument.

Grade 2 (forced by Principle plus standard structural assumptions). A result is Grade 2 if its derivation requires, in addition to the McGucken Principle, standard structural assumptions: locality of dynamical interactions; Lorentz invariance of the action; smooth ($C\infty$) differential structure; finite polynomial order in derivatives; specific dimensional or representation-theoretic content. Theorem 2 (algebraic-symmetry content as ISO(3)), Theorem 4 (Compton coupling), Theorem 6 (Brownian motion), Theorem 9 (Second Law $dS/dt = (3/2)k\_B/t$), Theorem 10 (photon entropy on McGucken Sphere), Theorem 11 (five arrows of time), Theorem 12 (Loschmidt resolution), and Theorem 14 (Compton-coupling diffusion empirical signature) are Grade 2.

Grade 3 (forced by Principle plus an external mathematical theorem). A result is Grade 3 if its proof invokes an external mathematical framework whose own derivation is taken as established but lies outside the chain of theorems developed in the present paper. Theorem 7 (probability measure as Haar measure on ISO(3)) is Grade 3: its derivation invokes Haar's 1933 uniqueness theorem on locally compact topological groups, with the McGucken framework supplying ISO(3) as the relevant group. Theorem 8 (ergodicity as Huygens-wavefront identity) is Grade 3: its derivation invokes the Birkhoff 1931 ergodic theorem combined with the geometric identity that the Huygens wavefront physically realizes the ensemble distribution.

1.5a.1 Comparison Table: Einstein's Three Gaps versus the McGucken Theorems

The graded-forcing vocabulary admits an immediate diagnostic application: it lets us measure the structural difference between the standard Boltzmann-Gibbs program and the McGucken Principle's derivation of the same content. Standard thermodynamics rests on three unresolved postulates plus auxiliary inputs (Stosszahlansatz, Past Hypothesis). Each is 'Grade 0' in our taxonomy: an unmotivated assumption inserted into the theory without derivation from a deeper physical principle. The McGucken framework re-derives each as a theorem of $dx_4/dt = ic$, with the Grade tag making explicit how much auxiliary input each derivation requires.

Standard Postulate / Gap	**Description**	**Source in standard literature**	**McGucken Theorem**
T1	Probability measure on phase space (Liouville uniform measure, principle of equal a priori probabilities)	Boltzmann 1872; Gibbs 1902; Jaynes 1957 maximum-entropy reinterpretation	Theorem 7 (Probability measure as Haar measure on ISO(3)), via Haar 1933 uniqueness theorem applied to the algebraic-symmetry content of $dx_4/dt = ic$. **Grade 3.**
T2	Ergodicity (time-averages equal ensemble-averages)	Boltzmann ergodic hypothesis 1871; Birkhoff 1931 ergodic theorem; KAM theory shows failure on positive-measure sets	Theorem 8 (Ergodicity as Huygens-wavefront identity), via Birkhoff 1931 combined with Channel B geometric realization of the ensemble. **Grade 3.**
T3	Second Law $dS/dt \geq 0$ (Loschmidt reversibility objection unresolved; Stosszahlansatz smuggles in irreversibility; Past Hypothesis required for Boltzmann formulation)	Clausius 1865; Boltzmann H-theorem 1872; Loschmidt 1876; Zermelo 1896; Penrose Past Hypothesis 1989	Theorem 9 (Second Law $dS/dt = (3/2)k_B/t > 0$ strict for massive particles); Theorem 10 ($dS/dt = 2k_B/t > 0$ for photons); Theorem 12 (Loschmidt resolution); Theorem 13 (Past Hypothesis dissolution). **Grades 1–2.**

The diagnostic content of the table is the column "Grade": each of Einstein's three gaps T1–T3 becomes a Grade-1, Grade-2, or Grade-3 theorem in the McGucken framework, with the auxiliary inputs identified explicitly. T1 is Grade 3 because it invokes Haar's theorem; T2 is Grade 3 because it invokes Birkhoff's theorem combined with the Channel B identity; T3 is Grades 1–2 across its components (the Second Law statement is Grade 2 invoking the central limit theorem; the Loschmidt resolution is Grade 2 via the dual-channel structural argument; the Past Hypothesis dissolution is Grade 1 from the geometric necessity that x_4's origin is the lowest-entropy moment). All three gaps are forced by the McGucken Principle plus standard structural assumptions; none requires postulates beyond the standard structural commitments shared with all reasonable physical theories.

The Boltzmann-Gibbs programme distributed the burden of proof across three independent gaps T1–T3, plus the unmotivated auxiliary inputs (Stosszahlansatz, Past Hypothesis), each requiring separate physical motivation. The McGucken framework concentrates the burden of proof at a single Grade-1 axiom (the McGucken Principle itself) and discharges T1–T3 plus the auxiliary inputs as theorems of grades 1, 2, or 3. The reduction is not merely cosmetic: the auxiliary inputs (locality, Lorentz invariance, smooth manifolds) are themselves either standard mathematical machinery that any reasonable physical theory will accept, or external uniqueness theorems (Haar, Birkhoff, central limit) that have been independently established and apply across many theoretical contexts. The McGucken Principle does not introduce more auxiliary structure than the standard programme; it shows that the auxiliary structure together with one geometric principle suffices to derive the entire content of thermodynamics.

In plain language. Some theorems in this paper follow purely from the McGucken Principle, no extra ingredients needed (Grade 1). Most require the principle plus standard physics assumptions like locality and Lorentz invariance (Grade 2). A few require the principle plus a separate mathematical theorem — Haar's uniqueness theorem on locally compact groups, the Birkhoff ergodic theorem, the central limit theorem — whose own proof is established elsewhere (Grade 3). Tagging each theorem with its grade lets the reader see at a glance how much structural input each result depends on, and which results would survive if a particular structural assumption were relaxed.

PART I — FOUNDATIONS

Part I establishes the foundational theorems descending from $dx_4/dt = ic$ that supply the kinematic substrate for thermodynamics. Theorem 1 establishes the wave equation as a theorem of x_4's spherically symmetric expansion via Huygens' Principle. Theorem 2 establishes the algebraic-symmetry content of $dx_4/dt = ic$ as the spatial isometry group ISO(3). Theorem

3 establishes the geometric-propagation content as Huygens-wavefront propagation on the McGucken Sphere. Theorem 4 establishes the Compton coupling between matter and x_4 as the matter-x_4 interaction (foundational ansatz from [10]). Theorem 5 establishes the spatial projection of x_4-driven displacement as instantaneously isotropic at each moment. Theorem 6 establishes Brownian motion as iterated isotropic displacement of x_4-coupled matter (from [109]). These six theorems supply the substrate from which the three resolutions of Einstein's gaps in Part II descend.

2. Theorem 1: The Wave Equation as a Theorem of x_4's Spherically Symmetric Expansion

Theorem 1 (The Wave Equation as a Theorem of x_4's Spherically Symmetric Expansion). The McGucken Principle $dx_4/dt = ic$ forces the three-dimensional wave equation

$(1/c^2)\partial^2\psi/\partial t^2 - \nabla^2\psi = 0$ as the differential statement of the wavefront expansion of x_4 from every spacetime event. The wave equation is the foundational kinematic substrate from which all subsequent thermodynamic content of the McGucken framework descends.

2.1 Proof

Proof. Start with the McGucken Principle: x_4 advances perpendicular to each spatial three-slice Σ_t at the constant rate $dx_4/dt = ic$. From every spacetime event $p_0 = (x_0, t_0)$, the principle states that x_4's expansion proceeds in a spherically symmetric manner, generating the threedimensional cross-section in space at each subsequent moment. Consider an arbitrary spacetime event p_0 and the locus of points reachable from p_0 by light-speed propagation in the spatial three-slice. By the spherical symmetry of x_4's expansion (Convention 1.5: the expansion has no preferred spatial direction), the locus is a sphere of radius $R(t) = c(t - t_0)$ centered at x_0 in spatial space. The wavefront amplitude $\psi(x, t)$ describing this propagation must satisfy two conditions: (i) it is a finite-amplitude function on M; (ii) the propagation occurs at speed c. The unique linear partial differential equation satisfied by all spherically-symmetric wavefronts of the form $\psi(r - ct)/r$ expanding from a point source is the three-dimensional wave equation $(1/c^2)\partial^2\psi/\partial t^2\ \nabla^2\psi = 0$. Direct verification: for $\psi = f(r - ct)/r$, $\partial^2\psi/\partial t^2 = c^2 f''(r - ct)/r$ and $\nabla^2\psi = f''(r - ct)/r$ in spherical coordinates. The equation $(1/c^2)\partial^2\psi/\partial t^2 = \nabla^2\psi$ is satisfied identically. The wave equation is therefore not assumed but derived: it is the differential statement of the McGucken Principle's assertion that x_4 expands spherically at rate c from every spacetime event. The full proof appears in [MG-HLA, §III] and is the foundational result on which all subsequent theorems of the present paper rest. ■

2.2 Comparison with Standard Derivation

Standard wave-mechanics treatments postulate the wave equation [125, 126] for matter waves and electromagnetic waves separately. The wave equation appears as a phenomenological starting point with no deeper geometric source. The McGucken framework reverses this: the wave equation is forced by the geometric structure of x_4's spherically-symmetric expansion through the unique linear PDE compatible with finite-amplitude spherical wavefronts at speed c. Huygens' Principle (every wavefront point is the source of secondary wavelets, and the new wavefront is the envelope of these) is the geometric content of the same equation:

it is the iterative form of the wave equation's spherical-source structure. Each result of the standard wave-mechanics literature is recovered as a consequence of $dx_4/dt = ic$.

In plain language. The McGucken Principle says: from every spacetime point, x_4 expands like a sphere at the speed of light. The differential equation that describes spherical expansion at speed c is the wave equation. So the wave equation isn't a postulate — it's automatic from the geometry of $dx_4/dt = ic$.

3. Theorem 2: The Algebraic-Symmetry Content of $dx_4/dt = ic$ as the Spatial Isometry Group ISO(3)

Theorem 2 (The Algebraic-Symmetry Content of $dx_4/dt = ic$ as the Spatial Isometry Group ISO(3)). The algebraic-symmetry content of the McGucken Principle $dx_4/dt = ic$ is the spatial isometry group $ISO(3) = SO(3) \ltimes \mathbb{R}^3$, the group generated by spatial rotations (SO(3)) and spatial translations ($\mathbb{R}^3$). This is the Channel A content from which the probability measure on phase space (Theorem 7) descends.

3.1 Proof

Proof. Start with the McGucken Principle $dx_4/dt = ic$ and identify its symmetries. The principle asserts that x_4 advances at the constant rate ic from every spacetime event, with the expansion proceeding in a spherically symmetric manner.

Temporal uniformity: The rate dx_4/dt is a constant ic, independent of t. This invariance under temporal translation $t \to t + \Delta t$ is the temporal-uniformity content of the principle.

Spatial homogeneity: The principle holds at every spacetime event, with no preferred spatial origin. This invariance under spatial translation $x \to x + \Delta x$ is the spatial-homogeneity content.

Spherical isotropy: The expansion is spherically symmetric, with no preferred spatial direction. This invariance under spatial rotation $O \in SO(3)$ is the spherical-isotropy content. Combining the spatial-translation invariance and the rotational invariance, the full symmetry group of $dx_4/dt = ic$ on each spatial three-slice is the Euclidean group — the spatial isometry group $ISO(3) = SO(3) \ltimes \mathbb{R}^3$, the semi-direct product of rotations and translations. This is a locally compact topological group. ISO(3) is the unique algebraic-symmetry content of $dx_4/dt = ic$ at the spatial-three-slice level. The Lorentz-covariance of the rate $dx_4/dt = ic$ extends ISO(3) to its Poincaré cover when we consider four-dimensional symmetries [12]. For thermodynamic applications restricted to spatial slices Σ_t, the relevant group is ISO(3); for applications involving Lorentz boosts (e.g., relativistic statistical mechanics), the group is the full Poincaré group. The present paper restricts attention to ISO(3) for the probability-measure derivation in Theorem 7. ■

3.2 Comparison with Standard Derivation

The standard Boltzmann-Gibbs programme [3, 4] takes ISO(3) (or its Galilean / Poincaré extensions) as a background symmetry group of the spacetime in which classical mechanics is formulated, with the symmetries motivated by the empirical observation that physics is the same at every place and orientation. The McGucken framework supplies the structural source: ISO(3) is not a background symmetry of spacetime but the algebraic-symmetry content of $dx_4/dt = ic$, encoded in the principle through its temporal uniformity, spatial homogeneity, and spherical isotropy. The group is therefore not a postulate about spacetime structure

but a derived feature of the principle's algebraic content. This is the central structural difference: in the McGucken framework, the symmetries of physics descend from the principle rather than being assumed at the start.

In plain language. Standard physics says: physics looks the same wherever you are and however you orient your apparatus. The mathematical group that captures this is called ISO(3): rotations plus translations. The McGucken framework says ISO(3) isn't an assumption; it's automatic from the principle $dx_4/dt = ic$. The principle says x_4 expands at the same rate from every point and in every direction, which is exactly the content of ISO(3). This algebraic-symmetry content is what generates the probability measure on phase space in Theorem 7.

4. Theorem 3: The Geometric-Propagation Content of $dx_4/dt = ic$ as Huygens-Wavefront Propagation on the McGucken Sphere

Theorem 3 (The Geometric-Propagation Content of $dx_4/dt = ic$ as Huygens-Wavefront Propagation on the McGucken Sphere). The geometric-propagation content of the McGucken Principle $dx_4/dt = ic$ is the Huygens-wavefront propagation on the McGucken Sphere of radius $R(t) = ct$ expanding from every spacetime event. This is the Channel B content from which ergodicity (Theorem 8) and the Second Law (Theorem 9) descend.

4.1 Proof

Proof. From Theorem 1, the wave equation describes spherically-symmetric expansion at speed c from any spacetime event. From the McGucken Principle, this expansion is the geometric realization of $dx_4/dt = ic$ in the spatial three-slice.

Definition of the McGucken Sphere. For any spacetime event $p_0 = (x_0, t_0)$, the McGucken Sphere $\Sigma_+(p_0)$ is the locus of spacetime events reachable from p_0 by null geodesics. In coordinates, $\Sigma_+(p_0)$ is the set of (x, t) with $|x - x_0| = c(t - t_0)$ and $t > t_0$ — the future light cone of p_0. The Sphere expands in the spatial three-slice as a sphere of radius $R(t) = c(t\ t_0)$ centered at x_0. Huygens' Principle as iterated McGucken Sphere. Huygens' Principle states that every point on a wavefront acts as a source of secondary wavelets, and the new wavefront is the envelope of these wavelets. In the McGucken framework, the geometric content of Huygens' Principle is iterative McGucken Sphere expansion: every point of the Sphere is itself a source of a new McGucken Sphere, and the union of all secondary Spheres at time $t + \Delta t$ is the McGucken Sphere of the original source at time $t + \Delta t$. The iteration is mathematically identical to the wave equation's evolution.

Geometric monotonicity. The radius $R(t) = c(t - t_0)$ is monotonically increasing in t. The McGucken Sphere therefore grows monotonically in volume $V(t) = (4/3)\pi R^3(t)$ and surface area $A(t) = 4\pi R^2(t)$, with no possibility of contraction. This monotonicity is the structural source of the Second Law (Theorem 9). One-way advance at $+ic$. The McGucken Principle states that x_4 advances at $+ic$, not $-ic$. The McGucken Sphere therefore expands forward in t, not backward. This time-orientation is the structural source of the arrow of time (Theorem 11). The Channel B content of $dx_4/dt = ic$ is therefore: the McGucken Sphere expanding at rate c from every event, with monotonic volume and surface-area growth and one-way timeorientation. This geometric-propagation content is the source of the irreversibility theorems of Part II. ■

4.2 Comparison with Standard Derivation

Huygens' 1690 derivation [27] of wave propagation by spherical secondary wavelets was a phenomenological account of light propagation: every wavefront point acts as a source. The principle was successful but had no deeper geometric source until the McGucken framework. In the McGucken framework, Huygens' Principle is the geometric content of $dx_4/dt = ic$ at the spatial-three-slice level, with the wavefront being the cross-section of the McGucken Sphere expanding from each event. The iterative structure of Huygens' Principle is iterative McGucken Sphere expansion, and the monotonic radial growth of the Sphere is the structural source of the irreversibility theorems of Part II.

In plain language. Huygens' Principle says: every point on a wavefront sends out new little spherical wavelets. In the McGucken framework, this is just the geometric content of the principle $dx_4/dt = ic$: from every spacetime event, x_4 expands like a sphere at speed c, and every point of that sphere is itself the source of a new sphere. Crucially, the sphere only grows — it never contracts. This monotonic growth of the McGucken Sphere is what eventually gives us the Second Law of Thermodynamics.

5. Theorem 4: The Compton Coupling Between Matter and the Expanding Fourth Dimension

Theorem 4 (The Compton Coupling Between Matter and the Expanding Fourth Dimension). Massive matter couples to x_4's expansion through the Compton frequency $\omega\_C = mc^2/\hbar$. A particle of rest mass m oscillates at ω_C in its rest frame as it advances along x_4, and the spatial cross-section of this oscillation supplies the matter-x_4 interaction.

5.1 Proof

Proof. Start with the rest-frame de Broglie / Compton relation: a particle of rest mass m has Compton angular frequency $\omega\_C = mc^2/\hbar$ and Compton wavelength $\lambda\_C = h/(mc)$. These are the natural units of x_4 oscillation for a particle of mass m. From the McGucken Principle, x_4 advances at rate ic. A particle at rest in the spatial threeslice advances purely along x_4, with proper time τ equal to laboratory time t. The Comptonfrequency oscillation in the rest frame is the natural 'clock' of the particle's x_4-advance: each Compton period is one cycle of the particle's phase along x_4.

The Compton coupling ansatz. For ensembles of massive particles in thermal contact with x_4's expansion, the matter-x_4 coupling is realized through a small modulation of the Compton frequency: the particle's x_4-phase $\psi \sim \exp(-i\cdot mc^2\tau/\hbar)$ is modulated by a small term $[1 + \varepsilon \cos(\Omega\tau)]$ where ε is the dimensionless modulation amplitude and Ω the modulation frequency [10]. The full matter-x_4 wavefunction is $\psi \sim \exp(-i\cdot mc^2\tau/\hbar)[1 + \varepsilon \cos(\Omega\tau)]$. The Compton coupling is the foundational matter-x_4 interaction. It is the source of the Brownian motion that supplies the spatial-projection isotropy of Theorem 5, and ultimately of the Compton-coupling diffusion $D\_x\^(McG) = \varepsilon^2c^2\Omega/(2\gamma^2)$ of Theorem 14. The full development appears in [10, §§2-4]. The Compton coupling is foundational ansatz at the level of the present paper: we adopt it as input from [10] and use it as the matter-x_4 interaction throughout. The motivation for the specific form is the

Compton-frequency identity: massive matter has a unique natural frequency $mc^2/\hbar$ on x_4, and any matter-x_4 coupling must respect this scale. ■

5.2 Comparison with Standard Derivation

Standard thermodynamics treats matter as classical particles undergoing thermal motion in a temperature bath; the matter-spacetime coupling is encoded in the Maxwell-Boltzmann distribution. The McGucken framework supplies an additional matter-x_4 coupling through the Compton frequency: matter has a natural oscillation rate at $mc^2/\hbar$ in its rest frame, and ensembles of massive matter couple to x_4's expansion through small modulations of this rate. This coupling supplies the source of Brownian motion (Theorem 6) and the empirical signature of Theorem 14 distinguishing the framework from textbook thermodynamics.

In plain language. Every massive particle has a natural 'clock' built in: its Compton frequency $mc^2/\hbar$ is the rate at which its quantum phase oscillates in its own rest frame. The McGucken framework says this Compton clock is the connection between matter and x_4's expansion: each tick of the Compton clock is one cycle of the particle's phase as it advances along x_4. Small modulations of this clock are what couple matter to x_4 in a measurable way.

6. Theorem 5: Spatial-Projection Isotropy of x_4-Driven Displacement

Theorem 5 (Spatial-Projection Isotropy of x_4-Driven Displacement). The spatial projection of x_4-driven displacement is isotropic at each instant. For any infinitesimal time interval dt, the spatial displacement induced by the Compton coupling has equal probability of pointing in any direction in the spatial three-slice Σ_t. This is the Channel B content that supplies the structural source of Brownian motion (Theorem 6).

6.1 Proof

Proof. Start with the spherical-symmetry of x_4's expansion (Theorem 3): from every spacetime event, x_4 expands in a spherically symmetric manner. The spatial cross-section of this expansion has no preferred direction. Now consider a particle at spacetime event $p_0 = (x_0, t_0)$ coupled to x_4 through the Compton coupling of Theorem 4. The particle's x_4-driven motion produces, in the spatial three-slice, an infinitesimal displacement dx during the time interval $[t_0, t_0 + dt]$. By the spherical symmetry of x_4's expansion combined with Channel B's spherical content, the displacement dx has equal probability of pointing in any direction in Σ_t. There is no privileged direction in spatial space because there is no privileged direction in x_4's spherical expansion. The spatial projection is therefore instantaneously isotropic. Formally: the probability density $\rho(dx)$ on the spatial-displacement vector is invariant under all rotations $O \in SO(3)$ of dx. By the uniqueness of the rotation-invariant probability measure on the sphere of constant magnitude (a corollary of Haar's theorem applied to SO(3)), the only such density is the uniform density on the sphere of magnitude $|dx|$ = constant. Spatial-projection isotropy. The result is Convention 1.5.6 of the GR chain paper [8] and Convention 5.6 of the QM chain paper [122]: the spatial projection of x_4-driven displacement is instantaneously isotropic at each moment. This isotropy is the geometric source of Brownian motion in Theorem 6: iterated isotropic displacement of x_4-coupled matter produces Brownian motion via the central limit theorem. ■

6.2 Comparison with Standard Derivation

Standard thermodynamics treats Brownian motion as a phenomenological description of small-particle agitation in a thermal bath, with the underlying microscopic source being molecular collisions. The McGucken framework supplies a deeper structural source: the spatial-projection isotropy of x_4-driven displacement is the geometric content of the principle, generating Brownian-motion-like behavior independent of any thermal bath. The empirical content overlaps with the standard account in the regimes where thermal motion dominates, but extends to regimes where thermal motion is absent ($T \to 0$) and the McGucken contribution is empirically distinguishable.

In plain language. From any spacetime event, x_4 expands like a sphere — meaning, with no preferred direction. So when the Compton coupling translates this expansion into spatial motion, the spatial direction of motion at each instant is also random — equally likely to point any way. Iterated over many small intervals, this random walk becomes Brownian motion: the diffusion of particles through space without any preferred direction.

7. Theorem 6: Brownian Motion as Iterated Isotropic Displacement

Theorem 6 (Brownian Motion as Iterated Isotropic Displacement). Iterated isotropic displacement of x_4-coupled matter (Theorem 5) at successive time intervals produces Brownian motion of the matter ensemble. Mathematically, the position vector $r(t)$ of a typical $x_4$coupled particle satisfies a Wiener process: $r(t)$ is a Gaussian random walk with variance $\mathrm{Var}(r(t)) = 6Dt$ for some diffusion constant $D > 0$.

7.1 Proof

Proof. Start with Theorem 5: the spatial projection of x_4-driven displacement is instantaneously isotropic. For an infinitesimal time interval dt, the displacement dx has zero mean and variance proportional to dt: $\langle dx \rangle = 0$ and $\langle |dx|^2 \rangle = 6D\,dt$ for some $D > 0$ (the factor 6 comes from the three spatial dimensions, each contributing 2D dt). Iterating this displacement at successive time intervals $dt \to 0$ with the random walk property (independent increments at each step), the position $r(t)$ at time t is the sum of $N = t/dt$ independent isotropic displacement vectors, each of magnitude proportional to $\sqrt{(dt)}$ on average. By the central limit theorem, the sum of $N \to \infty$ independent and identically distributed displacement vectors converges to a Gaussian distribution. The position $r(t)$ is therefore Gaussian-distributed with mean $\langle r(t) \rangle = 0$ and variance $\mathrm{Var}(r(t)) = 6Dt$: a Wiener process. This is exactly Brownian motion: the random walk of a typical particle in the matter ensemble, with the position spreading as the square root of time. The Gaussian density $\rho(r, t) = (4\pi Dt)^{-3/2} \exp(-r^2/(4Dt))$ is the standard Brownian density. The structural significance of this derivation is that Brownian motion is forced by the McGucken Principle through Channel B's spatial-projection isotropy combined with the central limit theorem — not by any temperature postulate or molecular-collision mechanism. The standard Maxwell-Boltzmann distribution emerges as a consequence of Brownian motion at finite temperature, but Brownian motion itself descends from x_4's isotropic expansion and persists at zero temperature in the McGucken framework. The full development appears in [109, §V] and [120]. ■

7.2 Comparison with Standard Derivation

Einstein 1905 [127] derived Brownian motion from the molecular-kinetic hypothesis and the assumption of thermal equilibrium, with the diffusion relation $D = k_B T/(6\pi\eta r)$ connecting microscopic molecular collisions to macroscopic diffusion. Perrin's 1908-1913 experiments [128] confirmed Avogadro's number to parts in a hundred. The McGucken framework supplies an additional structural source: Brownian motion is the iterated isotropic displacement of $x_4$coupled matter through Channel B, and persists even when thermal motion is suppressed ($T \to 0$). This is the source of the Compton-coupling diffusion of Theorem 14, which has the empirical-signature property of being mass- and temperature-independent in the cancelling combination.

In plain language. Einstein in 1905 explained Brownian motion: pollen grains in water are jostled by random molecular collisions, and they perform a random walk whose diffusion rate gives Avogadro's number. The McGucken framework adds another layer: even at absolute zero temperature where there's no thermal motion, x_4-coupled matter still performs a tiny random walk because of the spatial-projection isotropy of x_4's expansion. This gives a residual Brownian-motion-like diffusion that's the empirical signature of the McGucken framework.

PART II — THE THREE RESOLUTIONS OF EINSTEIN'S GAPS

Part II closes Einstein's three gaps T1-T3 in the Boltzmann-Gibbs program as theorems of $dx_4/dt = ic$. Theorem 7 derives the probability measure on phase space as the unique Haar measure on the spatial isometry group ISO(3) of x_4's spherically-symmetric expansion, forced by the Channel A algebraic-symmetry content combined with Haar's 1933 uniqueness theorem. Theorem 8 derives ergodicity as a Huygens-wavefront identity through the Channel B geometric-propagation content: the time-average of any continuous observable along a trajectory equals the ensemble-average over the McGucken Sphere's wavefront cross-section, independent of metric transitivity and unaffected by KAM-tori obstruction. Theorem 9 derives the Second Law as the strict-monotonicity theorem $dS/dt = (3/2)k_B/t > 0$ for massive-particle ensembles via spherical isotropic random walk and the central limit theorem. Theorem 10 derives the corresponding photon-entropy theorem $dS/dt = 2k_B/t > 0$ for photons on the McGucken Sphere. The four theorems together constitute the structural resolution of Einstein's 1949 incompleteness.

8. Theorem 7: The Probability Measure as the Unique Haar Measure on ISO(3)

Theorem 7 (The Probability Measure as the Unique Haar Measure on ISO(3)). The probability measure on phase space is the unique Haar measure on the spatial isometry group ISO(3) of x_4's spherically-symmetric expansion. The measure is forced by Haar's 1933 uniqueness theorem on locally compact topological groups [129] applied to the algebraic-symmetry content of $dx_4/dt = ic$; it is not postulated. This closes Einstein's first gap (T1).

8.1 Proof

Proof. From Theorem 2, the algebraic-symmetry content of $dx_4/dt = ic$ on each spatial three-slice is the spatial isometry group $ISO(3) = SO(3) \ltimes \mathbb{R}^3$. ISO(3) is a locally compact topological group: SO(3) is compact (a 3-sphere), $\mathbb{R}^3$ is locally compact, and the semi-direct product is locally compact. Haar's theorem (1933) [129]. On any locally compact topological group G, there exists a

unique (up to positive scalar) left-invariant Borel measure μ_L — the left Haar measure — satisfying μ_L(gE) = μ_L(E) for all $g \in G$ and all Borel-measurable $E \subseteq G$. For groups that are unimodular (i.e., left and right Haar measures coincide), the unique invariant measure is simply called the Haar measure. ISO(3) is unimodular.

Application to phase space. Phase space for an ensemble of N particles is the product of N copies of position space $\mathbb{R}^3$ and N copies of momentum space $\mathbb{R}^3$, giving $\mathbb{R}^{6N}$. The relevant symmetry group acting on this phase space is $ISO(3)^N = (SO(3) \ltimes \mathbb{R}^3)^N$: the rotational and translational invariance per particle, combined with the symmetric-permutation structure for indistinguishable particles. The measure that is invariant under this group action is the unique Haar measure.

Identification with the Liouville measure. The Haar measure on $(ISO(3))^N$ restricted to the energy-shell phase space is, up to normalization, the standard Liouville measure dΓ = ∏_i d³q_i d³p_i restricted to the constant-energy hypersurface. The Boltzmann uniform measure on phase space is therefore the unique Haar measure forced by the algebraic-symmetry content of $dx_4/dt = ic$, not a postulate.

Closure of the structural argument. The Liouville-measure preservation under Hamiltonian flow that Liouville's theorem guarantees is a consistency check: the measure is preserved by the dynamics that respects ISO(3) symmetry. But Liouville's theorem does not derive the measure; it only preserves the measure. The McGucken framework, by contrast, derives the measure from the algebraic-symmetry content of $dx_4/dt = ic$ via Haar's theorem. This closes Einstein's first gap T1: the probability measure on phase space is no longer postulated but is forced by the algebraic content of $dx_4/dt = ic$ plus Haar's uniqueness theorem [130, §V; 102, Proposition V.1]. ■

8.2 Comparison with Standard Derivation

Boltzmann 1872 [3] and Gibbs 1902 [4] postulated the uniform Liouville measure on phase space as the foundational probability measure of statistical mechanics. The principle of equal a priori probabilities is the same postulate in different language.

Liouville's theorem (1838) [131] guarantees that this measure is preserved under Hamiltonian flow, but Liouville's theorem provides preservation given the choice of measure, not justification for the choice. Jaynes' 1957 maximum-entropy reformulation [132] relocated the postulate into epistemology without deriving it from dynamics. The McGucken framework supplies the derivation: the measure is forced by Haar's 1933 theorem applied to the spatial isometry group ISO(3) of the algebraic-symmetry content of $dx_4/dt = ic$. The measure is therefore not a postulate but a derived theorem, with the structural source being the algebraic content of the McGucken Principle.

In plain language. Statistical mechanics needs to know how likely each microscopic state of a system is — the probability measure on phase space. Standard physics just postulates that the measure is uniform: every region of phase space has equal probability per unit volume. This is the Liouville measure, and it's a postulate. The McGucken framework derives this postulate as a theorem: there's a mathematical theorem (Haar 1933) that says any locally compact group has a unique invariant measure. The McGucken Principle gives us the relevant group (ISO(3) from the spatial symmetry of $dx_4/dt = ic$), and Haar's theorem gives us the unique invariant measure on that group — which turns out to be exactly the Liouville measure. So the probability measure of statistical mechanics is not a postulate but a derived consequence of $dx_4/dt = ic$ plus a standard mathematical theorem.

9. Theorem 8: Ergodicity as a Huygens-Wavefront Identity

Theorem 8 (Ergodicity as a Huygens-Wavefront Identity). Ergodicity — the equality of time-averages and ensemble-averages — is a geometric identity of Channel B Huygenswavefront propagation: for any continuous observable F on phase space, the time-average along any trajectory equals the ensemble-average over the McGucken Sphere's wavefront cross-section. The identity is independent of metric transitivity and unaffected by KAM-tori obstruction. This closes Einstein's second gap (T2).

9.1 Proof

Proof. From Theorem 3, the geometric-propagation content of $dx_4/dt = ic$ is the McGucken Sphere expanding from every spacetime event with Huygens-wavefront propagation. The wavefront at time t is a spherical surface of radius $R(t) = ct$ in the spatial three-slice.

The Huygens-wavefront identity. Consider a particle initially at spacetime event $p_0 = (x_0, t_0)$. At time $t > t_0$, the McGucken Sphere from p_0 has radius $R(t) = c(t - t_0)$ and surface area $A(t) = 4\pi R^2(t)$. Every point on this Sphere is itself the source of a new McGucken Sphere by Huygens' Principle (Theorem 3).

Ensemble realization on the Sphere. The continuous family of intermediate Spheres along the trajectory from p_0 physically realizes the ensemble over which the trajectory's 'possible histories' spread. At time t, the ensemble of realizations is parameterized by the surface of the McGucken Sphere with the uniform measure (the rotationally-invariant measure on S^2). This ensemble is the geometric content of the trajectory, not a fictional bookkeeping device imposed by the theorist.

Birkhoff ergodic theorem (1931) [133]. Birkhoff's ergodic theorem establishes that for any continuous observable F on a measure space and any measure-preserving transformation T, the time-average of F along the orbit of T converges almost surely to the ensemble-average of F over the invariant measure: $\lim_{N\to\infty} (1/N) \Sigma_{n=0}^{N-1} F(T^n x) = \int F\, d\mu$. This requires metric transitivity (the entire phase space is reached from almost every initial condition).

The McGucken-framework strengthening. The McGucken framework strengthens the Birkhoff theorem: the ensemble-average is geometrically realized by the Huygens-wavefront cross-section at each instant, not by the long-time limit of the trajectory. The time-average along the trajectory equals the ensemble-average over the wavefront because the trajectory is the wavefront, viewed as a propagating geometric object. The identity holds for any continuous observable F and is independent of the standard Birkhoff hypotheses (metric transitivity, almost-everywhere convergence): it is a structural identity of the geometric-propagation content of $dx_4/dt = ic$.

Independence of KAM-tori obstruction. KAM theory [134] establishes that generic Hamiltonian perturbations of integrable systems preserve a positive-measure set of invariant tori on which the trajectory is restricted to a sub-dimensional subset of phase space. The standard ergodic hypothesis fails on these positive-measure sets. In the McGucken framework, the Huygens-wavefront identity is unaffected by the KAM-tori obstruction: the wavefront crosssection is the ensemble of geometric realizations at each instant, not the long-time orbit of the trajectory. The KAM-tori restriction operates on the orbit; the McGucken-framework ergodicity operates on the wavefront. The two are different geometric structures, and the McGuckenframework identity holds even where the KAM-tori obstruction breaks the standard ergodic hypothesis [102, Proposition VI.1]. This closes Einstein's second gap T2: ergodicity is no longer an unproven hypothesis (false in

the standard formulation on positive-measure sets) but a geometric identity of the Huygenswavefront content of $dx_4/dt = ic$, independent of metric transitivity and unaffected by KAM-tori obstruction. ■

9.2 Comparison with Standard Derivation

Boltzmann 1871 [135] introduced the ergodic hypothesis as the mathematical bridge between time-averages and ensemble-averages: a single particle's trajectory was supposed to densely fill the constant-energy hypersurface in phase space, so that the long-time average along the trajectory equals the ensemble average over the hypersurface. Birkhoff 1931 [133] formalized this with the ergodic theorem under the metric-transitivity hypothesis. KAM theory (Kolmogorov 1954, Arnold 1963, Moser 1962) [134] subsequently established that for typical Hamiltonian systems, metric transitivity fails on a positive-measure set of invariant tori, so the ergodic hypothesis is not merely unproven but demonstrably false. The orthodox account has therefore relied on a hypothesis that is known to fail in physical systems. The McGucken framework supplies a structural alternative: ergodicity is a Huygens-wavefront identity through Channel B, with the ensemble physically realized by the propagating wavefront and independent of orbit dynamics. The KAM obstruction operates on orbits, not wavefronts, so the McGucken-framework ergodicity holds even where standard ergodicity fails.

In plain language. Standard statistical mechanics needs to assume that the time-average of a quantity along a particle's trajectory equals the average of the quantity over all possible states of the system (the ensemble average). This is called the 'ergodic hypothesis.' The problem is that KAM theory has shown this hypothesis is actually false for typical physical systems — trajectories don't fill phase space, they get stuck on lower-dimensional surfaces called invariant tori. The McGucken framework provides a different argument: the ensemble isn't about long-term orbits at all; it's about the spherical wavefront expanding from every spacetime event in the McGucken Principle. At each moment, the wavefront is the ensemble of possible outcomes — geometrically, not statistically. The KAM problem doesn't apply because we're not arguing about orbits.

10. Theorem 9: The Second Law dS/dt = (3/2)k_B/t > 0 Strict for Massive-Particle Ensembles

Theorem 9 (The Second Law dS/dt = (3/2)k_B/t > 0 Strict for Massive-Particle Ensembles). For an ensemble of massive particles undergoing the spherical isotropic random walk of Theorem 6, the Boltzmann-Gibbs entropy satisfies dS/dt = (3/2)k_B/t > 0 strictly for all t > 0. This is a strict geometric monotonicity, not a statistical tendency. Loschmidt's reversibility objection is structurally dissolved (Theorem 12). This closes Einstein's third gap (T3) for massive particles.

10.1 Proof

Proof. From Theorem 6, an ensemble of N massive particles undergoing spherical isotropic random walk has Gaussian-distributed positions r_i(t) with variance Var(r_i(t)) = 6Dt for some diffusion constant D > 0. The phase-space density ρ(r, t) of the ensemble is ρ(r, t) = $(4\pi Dt)^{-3Ł2}$ exp($-r^2$/(4Dt)).

The Boltzmann-Gibbs entropy. The Boltzmann-Gibbs entropy of the ensemble is S(t) = -k_B ∫ ρ(r, t) ln ρ(r, t) d^3r. For the Gaussian density above, this evaluates to S(t) = (3/2)k_B + (3/2)k_B ln(4πDt). The constant (3/2)k_B is the rest-frame contribution; the (3/2)k_B ln(4πDt)

term grows logarithmically with t. The strict-monotonicity theorem. Differentiating S(t) with respect to t: dS/dt = (3/2)k_B · 1/t > 0 for all t > 0. This is a strict positivity result: the Boltzmann-Gibbs entropy of any sphericallyisotropic-random-walk ensemble of massive particles is monotonically increasing at the rate (3/2)k_B per Boltzmann-time-unit, with no possibility of momentary decrease. The structural significance. The result is not a statistical tendency — not 'dS/dt ≥ 0 on average' or 'dS/dt ≥ 0 with overwhelming probability' — but a strict geometric monotonicity. The reason is that Brownian motion (Theorem 6) is forced by the spherical-symmetry of x_4's expansion, not by any statistical mechanism that could be reversed. The diffusion constant D is positive because x_4 expands at +ic, not -ic. The expansion is one-way, so the Brownian motion is one-way, so the entropy is one-way. The arrow of time. The strict positivity of dS/dt is the geometric source of the thermodynamic arrow of time. The arrow points from low entropy to high entropy because x_4 advances forward at +ic, not backward at -ic. There is no counterpart trajectory in time-reversed direction because there is no counterpart x_4-expansion at -ic. This closes Einstein's third gap T3 for massive particles: the Second Law is no longer a statistical tendency requiring auxiliary low-entropy boundary conditions but a strict geometric theorem of $dx_4/dt = ic$. The full development appears in [109, §V] and [102, Theorem VII.1]. ■

10.2 Comparison with Standard Derivation

Boltzmann 1872 [3] derived the Second Law ($dS/dt \geq 0$) using the H-theorem and the Stosszahlansatz, but Loschmidt's 1876 reversibility objection [136] showed that the timesymmetric Newtonian dynamics cannot rigorously force a time-asymmetric output. Boltzmann 1877 retreated to a statistical interpretation: entropy-decreasing trajectories are overwhelmingly improbable, but not absolutely prohibited. The orthodox account requires the Past Hypothesis [137-139] (Penrose's $10^{-10^{123}}$ fine-tuning of the early-universe Weyl curvature) to provide the necessary low-entropy boundary condition. The McGucken framework supplies a structural alternative: $dS/dt = (3/2)k_B/t > 0$ is a strict-monotonicity theorem from the spherical-isotropic-random-walk structure of Brownian motion in Theorem 6, with the diffusion constant $D > 0$ forced by the +ic orientation of x_4's advance. The Second Law is no longer a tendency but a necessity, and Loschmidt's objection is structurally dissolved (Theorem 12) because the time-symmetric microscopic dynamics descend from Channel A while the time-asymmetric Second Law descends from Channel B.

In plain language. The Second Law of Thermodynamics says: entropy always increases. Boltzmann tried to prove this from microscopic mechanics in 1872, but Loschmidt pointed out a fatal problem: the microscopic laws are time-symmetric, so they should produce just as many entropy-decreasing trajectories as entropy-increasing ones. Boltzmann had to retreat to a statistical answer: entropy-decreasing trajectories are extremely rare. This is a statistical tendency, not a strict law. The McGucken framework gives a strict law: x_4 expands at +ic (forward in time), not -ic (backward). This forces Brownian motion to spread one way only, which forces the entropy of any massive-particle ensemble to increase at the rate (3/2) k_B per unit of t-time, strictly. No statistical fudging needed.

11. Theorem 10: Photon Entropy on the McGucken Sphere

Theorem 10 (Photon Entropy on the McGucken Sphere). For an ensemble of photons emitted at spacetime event p_0 and propagating on the McGucken Sphere of radius $R(t) = c(t - t_0)$, the Shannon entropy is $S(t) = k_B \ln(4\pi(c(t-t_0))^2)$ with strict positive rate $dS/dt =$

$2k_B/(t - t_0) > 0$ for all $t > t_0$. The sphere grows because x_4 advances at rate c; the entropy grows because the sphere grows.

11.1 Proof

Proof. Start with Theorem 3: the McGucken Sphere from spacetime event p_0 has radius $R(t) = c(t - t_0)$ and surface area $A(t) = 4\pi R^2(t) = 4\pi c^2(t - t_0)^2$. The photon ensemble.

Consider an ensemble of photons emitted at p_0 with isotropic angular distribution. By the spherical-symmetry of Channel B's wavefront expansion, the photons spread uniformly over the surface of the McGucken Sphere — no preferred direction. The Shannon entropy. The information-theoretic entropy of the angular distribution of the photon ensemble on the McGucken Sphere's surface is $S(t) = k_B \ln(A(t)) = k_B \ln(4\pi c^2(t - t_0)^2) = k_B [\ln(4\pi) + 2 \ln(c(t - t_0))]$. This is the standard logarithm-of-volume Shannon entropy for the uniform distribution on a region of size $A(t)$. The strict-monotonicity rate. Differentiating $S(t)$ with respect to t: $dS/dt = 2k_B/(t - t_0) > 0$ for all $t > t_0$. The factor 2 arises from the surface-area scaling $A(t) \sim (t - t_0)^2$, with the logarithm giving the factor of 2. The strict positivity is a geometric necessity: the McGucken Sphere's area is monotonically increasing because the sphere grows monotonically. The structural significance. The photon-entropy result complements the massive-particle Second Law of Theorem 9. Massive-particle ensembles satisfy $dS/dt = (3/2)k_B/t > 0$ with the (3/2) coefficient from three-dimensional Brownian spread; photon ensembles satisfy $dS/dt = 2k_B/t > 0$ with the 2 coefficient from two-dimensional spherical-surface spread. Both rates are strict positive and both descend from the geometric-propagation content of $dx_4/dt = ic$ [120, §3]. Closing Einstein's third gap for photons. The photon-entropy theorem closes T3 in the radiative sector: photon entropy increases monotonically at the strict rate $dS/dt = 2k_B/t$, with no statistical fudging required. The radiative arrow of time (radiation propagates outward, not inward) is the structural source: x_4 advances at $+ic$, the McGucken Sphere expands monotonically, and the photon ensemble's entropy increases monotonically with the sphere's growth. ■

11.2 Comparison with Standard Derivation

Standard radiative thermodynamics treats photon entropy via the Stefan-Boltzmann law and the Planck blackbody spectrum, with the radiative arrow of time as a separate phenomenological feature requiring explanation [140]. The McGucken framework supplies a unified account: photon entropy on the McGucken Sphere is $S(t) = k_B \ln(4\pi c^2(t-t_0)^2)$, with the strict rate $dS/dt = 2k_B/(t - t_0) > 0$ forced by the geometric monotonicity of the Sphere's expansion. The radiative arrow of time is the same arrow as the thermodynamic arrow (Theorem 9): both are projections of x_4's monotonic forward advance at $+ic$. The two distinct rates ($(3/2)k_B/t$ for massive particles, $2k_B/t$ for photons) reflect the different dimensional content (three-dimensional volume spreading vs. two-dimensional spherical-surface spreading), but both are strict-monotonicity theorems descending from Channel B.

In plain language. Standard physics has separate accounts for matter entropy (from molecular collisions) and photon entropy (from radiation). The McGucken framework unifies them: both are projections of x_4's monotonic expansion. Massive particles spread out in 3D space as Brownian motion, giving entropy growth at rate (3/2) k_B per unit time. Photons spread out on the surface of the expanding McGucken Sphere (2D), giving entropy growth at rate 2 k_B per unit time. Same underlying mechanism, different rates because the geometry is different

(3D for massive particles, 2D surface for photons).

PART III — ARROWS OF TIME, ARCHITECTURAL RESOLUTIONS, AND EMPIRICAL SIGNATURE

Part III develops the consequences of Part II's three resolutions. Theorem 11 derives the five arrows of time — thermodynamic, cosmological, radiative, psychological/biological, and quantum-measurement — as five projections of the same single arrow of x_4's expansion at +ic. Theorem 12 establishes the structural dissolution of Loschmidt's 1876 reversibility objection: the time-symmetric microscopic dynamics descend from Channel A; the time-asymmetric Second Law descends from Channel B; the two channels are the dual-channel reading of one principle, not two competing foundations. Theorem 13 dissolves the Past Hypothesis as a theorem: x_4's origin is geometrically necessarily the lowest-entropy moment of any system participating in x_4's expansion, with no fine-tuning required. Theorem 14 establishes the empirical signature: the Compton-coupling diffusion $D_x^{(McG)} = \varepsilon^2 c^2 \Omega/(2\gamma^2)$, temperature- and mass-independent in the cancelling combination, distinguishes the framework from textbook thermodynamics in current technological reach.

12. Theorem 11: The Five Arrows of Time as Projections of x_4's Expansion at +ic

Theorem 11 (The Five Arrows of Time as Projections of x_4's Expansion at +ic). The five conventionally distinguished arrows of time — (a) thermodynamic (entropy increases), (b) cosmological (universe expands), (c) radiative (radiation propagates outward), (d) psychological/biological (memory of the past, not the future), and (e) quantum-measurement (collapse on observation) — are five projections of the same single arrow of x_4's expansion at +ic, not five independent arrows requiring separate explanation.

12.1 Proof

Proof. From the McGucken Principle, x_4 advances at +ic, not -ic. The forward direction is geometrically built into the principle: the McGucken Sphere expands at rate c, with no possibility of contraction (Theorem 3). (a) Thermodynamic arrow. By Theorem 9, $dS/dt = (3/2)k_B/t > 0$ strict for any massive-particle ensemble undergoing spherical-isotropic random walk. The thermodynamic arrow points from low entropy to high entropy because Brownian-motion spread is one-way, with the diffusion constant $D > 0$ forced by the +ic orientation of x_4's advance. (b) Cosmological arrow. The universe is observed to expand on cosmological scales (Hubble's law). In the McGucken framework, the cosmological expansion is the global-scale realization of x_4's advance: each McGucken Sphere expands locally at rate c, and the global topology of the spatial three-slice carries this expansion to cosmological scales [114; 116]. The cosmological arrow points from the early universe (small) to the late universe (large) because x_4 advances at +ic, not -ic. (c) Radiative arrow. Radiation propagates outward from sources, not inward to sinks. By Theorem 3 and Theorem 10, the McGucken Sphere from any source event expands at rate c, carrying radiation outward. The retarded Green's function of the wave equation propagates causally forward; the advanced Green's function would propagate causally backward. The McGucken framework selects the retarded Green's function because x_4 advances at +ic, not -ic. There is no advanced-Green's-function counterpart in physical reality.

(d) Psychological/biological arrow. We remember the past, not the future. Memory formation is an entropy-decreasing process locally (storage of information requires structure), and entropy decrease can only be funded by entropy increase elsewhere. The thermodynamic arrow of (a) provides this funding: the local entropy decrease in memory storage is more than compensated by global entropy increase. The biological arrow of evolution is similarly an entropyfunded process. The psychological/biological arrow is therefore a derived consequence of the thermodynamic arrow. (e) Quantum-measurement arrow. Quantum measurement (the projection of a wavefunction onto a measurement eigenstate) is irreversible. The standard Copenhagen reading attributes this to the collapse postulate. In the McGucken framework, measurement is the 3D-crosssection reading of the four-dimensional wavefunction (see [122, Theorem 17]), with the irreversibility tracing to the +ic orientation of x_4's advance: the measurement event is at a specific spacetime location, and the wavefunction at events to the past of the measurement event evolved without knowledge of the outcome, while events to the future of the measurement event are conditioned on the outcome. This time-asymmetry is the same +ic orientation that supplies (a)-(d). The unification. All five arrows therefore point in the same direction because they are five aspects of the same underlying x_4-expansion at +ic. There is one arrow, with five projections in different physical contexts. The standard accounts treat these arrows as five separate phenomena requiring independent explanation; the McGucken framework reveals them as one phenomenon with five projections [114, §V.3]. ■

12.2 Comparison with Standard Derivation

The five arrows of time are conventionally distinguished in the philosophy-of-physics literature as separate phenomena requiring independent explanation [137, 141, 142]. Penrose 1989 [137] argues that the cosmological arrow is the most fundamental, with the other four arrows derived from it via the low-entropy initial state of the universe. The McGucken framework supplies a deeper unification: all five arrows are projections of the same single arrow of x_4's expansion at +ic. The cosmological arrow is x_4's advance at the global scale; the thermodynamic arrow is x_4's advance projected through the Brownian-motion lens of Theorem 9; the radiative arrow is x_4's advance projected through the McGucken Sphere's expansion; the psychological/biological arrow is x_4's advance projected through the memory/evolution lens; the quantum-measurement arrow is x_4's advance projected through the 3D-cross-section reading of the wavefunction. The unification is structural: there is one principle, $dx_4/dt = +ic$, and five projections.

In plain language. Time has five 'arrows' that all point the same way: entropy increases, the universe expands, radiation goes outward, we remember the past, quantum measurements collapse irreversibly. Standard physics treats these as five separate puzzles. The McGucken framework says they're all the same single arrow viewed from different angles: x_4 expands at +ic (forward), not at -ic (backward), and that's the only arrow there is. All five conventional arrows of time are projections of this one fundamental fact.

13. Theorem 12: Structural Dissolution of Loschmidt's 1876 Reversibility Objection

Theorem 12 (Structural Dissolution of Loschmidt's 1876 Reversibility Objection). Loschmidt's 1876 reversibility objection — that time-symmetric microscopic Newtonian dynamics cannot rigorously force a time-asymmetric Second Law — is structurally dissolved in the McGucken framework. The time-symmetric microscopic dynamics descend from Channel

A (algebraic-symmetry content); the time-asymmetric Second Law descends from Channel B (geometric-propagation content). The two channels are the dual-channel reading of one principle, not two competing foundations.

13.1 Proof

Proof. Recall Loschmidt's 1876 objection [136]. Boltzmann's 1872 H-theorem [3] derives the Second Law $dS/dt \geq 0$ from the Stosszahlansatz applied to molecular collisions. Loschmidt observed that the underlying Newtonian dynamics are time-reversal symmetric: for every entropyincreasing trajectory, there exists by velocity reversal a corresponding entropy-decreasing trajectory of equal statistical weight. The time-symmetric microscopic laws cannot by themselves produce a time-asymmetric consequence; the Stosszahlansatz — assumed for precollision velocities but not post-collision velocities — is where the asymmetry enters, and the argument is therefore circular: Boltzmann assumed molecular chaos to derive the Second Law, the Second Law is equivalent to molecular chaos, and nothing is derived that was not already assumed. The McGucken-framework structural dissolution. The two-channel content of $dx_4/dt = ic$ resolves Loschmidt's objection structurally rather than statistically. Channel A: time-symmetric microscopic dynamics. The algebraic-symmetry content of $dx_4/dt = ic$ includes temporal uniformity, spatial homogeneity, spherical isotropy, Lorentz covariance, and absence of preferred phase origin on x_4. These symmetries generate the Noether conservation laws (energy from temporal uniformity, momentum from spatial homogeneity, angular momentum from spherical isotropy, etc.) [12]. The Noether currents are timesymmetric quantities: each conservation law is symmetric under time reversal. The timesymmetric microscopic dynamics of Newtonian and Hamiltonian mechanics are the Channel A output of $dx_4/dt = ic$. Channel B: time-asymmetric Second Law. The geometric-propagation content of $dx_4/dt = ic$ includes spherical expansion at rate c from every spacetime event, monotonic radial growth of the McGucken Sphere, isotropic wavefront emission, and one-way advance at +ic (not -ic). These geometric features are intrinsically time-asymmetric: the McGucken Sphere expands monotonically and one-way. The time-asymmetric Second Law $dS/dt > 0$ (Theorem 9) and the five arrows of time (Theorem 11) are the Channel B output of $dx_4/dt = ic$. The two channels are not in conflict. Channel A and Channel B are not two competing foundations but the two faces of one principle under the Klein correspondence between algebra and geometry

[72]. The same principle $dx_4/dt = ic$ carries both time-symmetric and timeasymmetric content because algebra and geometry are the two information-equivalent descriptions of the same Kleinian object (Theorem 2 and Theorem 3, plus the Klein correspondence). Loschmidt's objection assumed that time-symmetric content and time-asymmetric content must come from different foundations, hence cannot coexist in a coherent theory of mechanics. The McGucken framework demonstrates the assumption is wrong: a single principle can carry both, in two distinct channels, and the dual-channel structure is the resolution. The structural dissolution. The time-symmetric Newtonian dynamics that Loschmidt invoked are correct as a description of Channel A. The time-asymmetric Second Law that Boltzmann tried to derive from those dynamics is correct as a description of Channel B. The standard derivation that smuggles in the Stosszahlansatz fails because it tries to derive Channel B from Channel A alone — an impossibility, since Channel A's output is time-symmetric. The McGucken framework resolves the issue by recognizing that Channel B is independent of Channel A and is the source of the time-asymmetric content. The two channels coexist, and Loschmidt's objection is dissolved [102, §VI].

■

13.2 Comparison with Standard Derivation

Loschmidt's objection [136] has resisted resolution for 150 years. Boltzmann 1877 retreated to a statistical answer that resolved the tension by surrendering the derivation: probability is not necessity. The 20th-century literature on the foundations of thermodynamics [137-139, 142, 143] has continued the debate without identifying a structural resolution. The McGucken framework supplies the structural resolution: the dual-channel content of $dx_4/dt = ic$ carries time-symmetric and time-asymmetric content as independent informational channels of one principle, and Loschmidt's objection — that time-symmetric microscopic dynamics cannot force a time-asymmetric Second Law — is dissolved because the Second Law does not derive from the time-symmetric microscopic dynamics. The Second Law derives from Channel B's geometric-propagation content of the same principle that, through Channel A, also generates the time-symmetric microscopic dynamics. There is no conflict, only dual-channel content of one foundation.

In plain language. Loschmidt's objection from 1876 is the deepest problem in the foundations of thermodynamics: how can microscopic laws that work the same forwards and backwards in time produce a Second Law that runs only forwards? Boltzmann never really answered this; he switched to a statistical argument that says entropy-decreasing trajectories are extremely rare. The McGucken framework gives a structural answer: $dx_4/dt = ic$ has two kinds of content built in. The algebraic content gives time-symmetric conservation laws and time-symmetric microscopic dynamics. The geometric content gives the time-asymmetric Second Law. They're not in conflict because they're two different aspects of the same principle, like the two sides of a coin. The microscopic dynamics aren't supposed to derive the Second Law; they're both forced by $dx_4/dt = ic$ but through different channels.

14. Theorem 13: Dissolution of the Past Hypothesis: x_4's Origin is the Lowest-Entropy Moment by Geometric Necessity

Theorem 13 (Dissolution of the Past Hypothesis: x_4's Origin is the Lowest-Entropy Moment by Geometric Necessity). The Past Hypothesis — that the universe began in an extraordinarily low-entropy state, with Penrose estimating one part in $10^{-10^{123}}$ fine-tuning of the early-universe Weyl curvature [137] — is dissolved as a theorem. The lowest-entropy moment of any system participating in x_4's expansion is the moment at which x_4 has not yet expanded ($R = 0$), and this is geometrically necessary, not fine-tuned.

14.1 Proof

Proof. Recall the Past Hypothesis. The Boltzmann-Gibbs Second Law $dS/dt \geq 0$ (in the orthodox formulation) requires an extraordinarily low-entropy initial state to give the universe room to evolve toward thermal equilibrium. In an unconstrained statistical mechanics, the universe's entropy would already be at thermal equilibrium — the 'heat death' state — and there would be no thermodynamic activity. The fact that we observe ongoing entropy increase requires a low-entropy past as boundary condition. Penrose's estimate. Penrose 1989 [137] estimates the fine-tuning required for the earlyuniverse initial state at one part in $\exp(10^{123}) = 10^{-10^{123}}$, based on the gravitational entropy of the early universe via the Weyl curvature tensor. This is one of the most extreme fine-tunings in physics: the universe is supposed to have started in a state that occupies an exponentially small fraction of the available phase-space volume.

The McGucken-framework dissolution. In the McGucken framework, the Past Hypothesis is not a fine-tuned initial condition but a geometric necessity. From Theorem 9 and Theorem 10, the entropy of any system participating in x_4's expansion is monotonically increasing at strict positive rate. The lowest-entropy moment of any such system is therefore the earliest moment at which x_4 has expanded — i.e., the limit $t \to t_0^+$ where $R = 0$. At this moment, the system has not yet undergone any spherical isotropic random walk (massive particles) or any McGucken Sphere expansion (photons), so there is no spread, no diffusion, no entropy. The geometric necessity. The lowest-entropy state of an x_4-coupled system at $R = 0$ is geometrically necessary because the McGucken Sphere has zero radius at $t = t_0$, hence zero volume, hence zero entropy. This is not a fine-tuned configuration; it is the unique geometric initial condition compatible with x_4's expansion having a starting point. The $10^{-10^{123}}$ fine-tuning of Penrose's estimate measures an improbability under a uniform prior on the gravitational phase space; but the gravitational phase space is not uniformly weighted under the McGucken framework. The McGucken-framework prior on initial conditions is the geometric prior set by x_4's expansion, which forces $R = 0$ at the origin. Why Penrose's estimate is high. Penrose's $10^{-10^{123}}$ figure assumes a uniform measure on the gravitational phase space and asks how much of that measure is occupied by configurations as low-entropy as the early universe. The answer is exponentially small. But the measuretheoretic argument assumes the wrong prior: in the McGucken framework, the prior is set by the geometry of x_4's expansion, not by a uniform Liouville measure on gravitational configurations. The McGucken-framework prior is concentrated on $R = 0$ at $t = t_0$; the uniform prior is spread over all gravitational configurations. Penrose's $10^{-10^{123}}$ measures an improbability under the wrong prior. The dissolution. The Past Hypothesis is therefore dissolved as a theorem: x_4's origin is geometrically necessarily the lowest-entropy moment, with no fine-tuning required. The structural argument appears in [116, §XIII] and [102, Proposition VI.3]. The orthodox Past-Hypothesis problem dissolves under the McGucken-framework prior, just as Loschmidt's objection dissolves under the McGucken-framework dual-channel content (Theorem 12). ■

14.2 Comparison with Standard Derivation

The Past Hypothesis is the most embarrassing fine-tuning in physics: one part in $10^{-10^{123}}$ (Penrose 1989) [137] is required to explain why the universe began in a low-entropy state. The orthodox Boltzmann-Gibbs program has no derivation of this initial condition; it is imposed as a brute fact. Carroll 2010 [138] and Wallace 2013 [139] have surveyed the philosophical literature and identified the Past Hypothesis as a structurally unsatisfactory feature of any orthodox account. The McGucken framework supplies a structural alternative: the lowest-entropy moment is x_4's origin at $R = 0$, which is geometrically necessary rather than fine-tuned. The $10^{-10^{123}}$ figure measures an improbability under a uniform measure on gravitational configurations; the McGucken-framework measure is concentrated on $R = 0$, so the figure measures the wrong thing. The Past Hypothesis dissolves under the correct prior.

In plain language. Penrose pointed out the most extreme fine-tuning problem in all of physics: the universe seems to have started in a one-in-$10^{-10^{123}}$ special low-entropy state, far more special than any random configuration would be. Standard physics has no explanation for this; it's just imposed as a brute fact. The McGucken framework dissolves this: x_4 expanded from a single point. At that point, the McGucken Sphere had radius zero, so the volume was zero, so the entropy was zero. There's no fine-tuning involved; it's geometrically necessary that the starting point of x_4's expansion is the lowest-entropy moment. Penrose's $10^{-10^{123}}$ figure measures the wrong probability — it assumes you could have started anywhere, but you couldn't.

15. Theorem 14: The Compton-Coupling Diffusion D_x^(McG) = $\varepsilon^2 c^2 \Omega/(2\gamma^2)$ as Empirical Signature

Theorem 14 (The Compton-Coupling Diffusion D_x^(McG) = $\varepsilon^2 c^2 \Omega/(2\gamma^2)$ as Empirical Signature). A gas of massive particles coupled to x_4's expansion through the Compton coupling of Theorem 4 exhibits a residual zero-temperature spatial diffusion coefficient D_x^(McG) = $\varepsilon^2 c^2 \Omega/(2\gamma^2)$, where ε is the dimensionless modulation amplitude, Ω the modulation frequency, and γ the environmental damping rate. The diffusion coefficient is massindependent: the mass dependence cancels between the coupling strength and the mobility. This is the empirical signature distinguishing the McGucken framework from textbook thermodynamics in current technological reach.

15.1 Proof

Proof. We give the explicit five-step derivation here; the same derivation appears in [10, §3-§4] and is reproduced for completeness. Step 1: The modulation Hamiltonian. From

Theorem 4, a particle of rest mass m couples to x_4's expansion through its Compton angular frequency $\omega_C = mc^2/\hbar$, with the McGuckenCompton coupling adding a small modulation: $\psi \sim \exp(-i \cdot mc^2\tau/\hbar) \cdot [1 + \varepsilon \cos(\Omega\tau)]$. This is equivalent to the rest-frame effective Hamiltonian term $H_{mod}(\tau) = \varepsilon mc^2 \cos(\Omega\tau)$. Step 2: First-order time-averaged response is zero. For Ω large compared to inverse timescales of spatial motion, the first-order effect of H_{mod} time-averages to zero: $\langle\cos(\Omega\tau)\rangle_t = 0$ over a period $2\pi/\Omega$. The leading nontrivial dynamical effect is therefore second-order in ε. Step 3: Second-order momentum diffusion via Floquet analysis. A Floquet/Magnus expansion at second order in ε, combined with weak environmental coupling that breaks coherence between cycles, generates a stochastic momentum impulse per cycle of order $\Delta p \sim \varepsilon mc$. Over time t there are $\sim\Omega t$ cycles, and their contributions add as a random walk: $\langle(\Delta p)^2\rangle \sim \varepsilon^2 m^2 c^2 \Omega t$. This is momentum-space diffusion with constant $D_p = \varepsilon^2 m^2 c^2 \Omega/2$. Step 4: Translation to spatial diffusion via Langevin dynamics. For a particle in an environment providing damping rate γ, the Langevin/Ornstein-Uhlenbeck equation $dp/dt = -\gamma p + \eta(t)$ at long times gives spatial diffusion $D_x = D_p/(m\gamma)^2$. Step 5: Mass cancellation. Substituting $D_p = \varepsilon^2 m^2 c^2 \Omega/2$ into $D_x = D_p/(m\gamma)^2$ gives $D_x^{(McG)} = \varepsilon^2 c^2 \Omega/(2 \gamma^2)$. The m^2 cancels: the spatial diffusion coefficient is massindependent. This cancellation is structural: the coupling strength is proportional to m (through the rest energy mc^2) while the mobility is inversely proportional to m, so the ratio is mass-independent. The result is a sharp prediction of the specific Compton coupling form proposed in [10, §2]. Total diffusion at finite temperature. Adding the McGucken contribution to ordinary thermal diffusion via the Einstein relation: $D_{total} = k_B T/(m\gamma) + \varepsilon^2 c^2 \Omega/(2\gamma^2)$. The first term vanishes as $T \to 0$; the second persists. This is the experimental signature: a gas cooled toward absolute zero retains a nonzero diffusion constant from x_4-coupling. Current atomic-clock and coldatom diffusion bounds constrain $\varepsilon^2\Omega \lesssim 2D_0^{exp} \gamma^2/c^2$. Cross-species mass-independence test. The mass-independence of $D_x^{(McG)}$ generates a sharp cross-species test. Two species A and B with similar damping rates $\gamma_A \approx \gamma_B$ should show residual diffusion ratios ≈ 1 (mass-independent), in contrast to thermal diffusion which scales as the inverse mass ratio. Comparing residual diffusion across electrons in solids, ions in traps, and neutral atoms in optical lattices — with γ controlled or measured — provides a direct test.

■

15.2 Comparison with Standard Derivation

Standard thermal diffusion in the Maxwell-Boltzmann framework satisfies $D_{thermal} = k_B T/(m\gamma)$, which scales linearly with temperature and inversely with mass; both factors vanish in the appropriate limits, so $D_{thermal} \to 0$ as $T \to 0$ and $D_{thermal} \to 0$ as $m \to \infty$. The McGucken framework predicts an additional diffusion contribution $D_x^{(McG)} = \varepsilon^2c^2\Omega/(2\gamma^2)$ that is temperature-independent (no T factor) and mass-independent (the mass cancels in the Compton-coupling derivation). The two predictions are sharply distinguishable: cooling to $T \to 0$ should give $D \to 0$ in the standard framework but $D \to D_x^{(McG)} > 0$ in the McGucken framework. The mass-independence makes this a particularly clean test: comparing electrons, atoms, and ions under similar trap conditions should give the same residual if the McGuckenCompton coupling is real, or different residuals scaling with mass if the standard account is the full story. Cold-atom experiments at JILA, NIST, MIT, trapped-ion experiments, ultracoldneutron storage, and precision atomic clocks each provide a sharp laboratory signature.

In plain language. If matter actually couples to x_4's expansion through the Compton frequency, then a gas cooled to absolute zero should still drift around at a tiny but measurable rate — with a diffusion constant that doesn't depend on the particles' mass. Standard thermodynamics predicts no such residual at $T = 0$ (after subtracting all known noise sources). The mass-independence makes this a particularly clean test: comparing electrons, atoms, and ions in similar trap conditions should give the same residual if the McGucken-Compton coupling is real, or different residuals scaling with mass if standard physics is the full story. Cold-atom experiments at major laboratories worldwide are within current technological reach.

PART IV — BLACK-HOLE THERMODYNAMICS AND COSMOLOGICAL HOLOGRAPHY

Part IV extends the chain of theorems established in Parts I–III into the semiclassical-gravity regime via the McGucken Wick rotation. Theorems 1–14 of Parts I–III cover the kinetic-theory regime: Brownian motion, the Second Law for ideal gases, the five arrows of time, and the dissolutions of Loschmidt's reversibility objection and the Past Hypothesis. The thermodynamic content of Parts I–III is the textbook content of statistical mechanics from Boltzmann 1872 through Einstein 1949, derived as theorems of $dx_4/dt = ic$. Black-hole thermodynamics from Bekenstein 1973 through Hawking 1975 supplies the next regime: thermodynamic systems whose entropy and temperature are not derived from molecular kinetic theory but from horizon geometry, with the area law $S_{BH} = k_B A/(4\ell_P^2)$ and the Hawking temperature $T_H = \hbar\kappa/(2\pi c k_B)$ as the canonical results. The same single principle $dx_4/dt = ic$, applied through the McGucken Wick rotation, produces these results as theorems.

The material of Part IV is imported in substantial part from the companion source papers [MG-Bekenstein] (April 20, 2026; URL: https://elliotmcguckenphysics.com/2026/04/20/how-the-mcgucken-principle-of-a-fourth-expanding-dimens and [MG-Hawking] (April 20, 2026; URL: https://elliotmcguckenphysics.com/2026/04/20/how-the-mcgucken-principle-of-a-fourth-expanding-dimens with [MG-AdSCFT] (April 22, 2026; URL: https://elliotmcguckenphysics.com/2026/04/22/ads-cft-from-dx%e2%82%84-dt-ic-the-gkp-witten-dictionary supplying the FRW/de Sitter cosmological-thermodynamics content of §19.

Four additional theorems are established in Part IV. Theorem 15 (§16) establishes the Bekenstein-Hawking black-hole entropy $S_{BH} = k_B A/(4\ell_P^2)$ as a theorem of $dx_4/dt = ic$, with the horizon recognized as an x_4-stationary null hypersurface populated by x_4-stationary modes whose count, by the McGucken second law (Theorem 9 extended through the Wick rotation), gives a geometric entropy proportional to area, with the coefficient 1/4 derived from integrating the entropy along the Euclidean disk under the thermodynamic normalization fixed by T_H. Theorem 16 (§17) derives the Hawking temperature $T_H = \hbar\kappa/(2\pi c k_B)$ from the Euclidean cigar's angular period under the

McGucken Wick rotation $\tau = x_4/c$. Theorem 17 (§18) establishes the refined Generalized Second Law as the global x_4-flux conservation across a spacetime partitioned into exterior and horizon-bounded interior, recovering Bekenstein's 1974 generalized second law with sharper structural content. Theorem 18 (§19) derives FRW/de Sitter cosmological thermodynamics with the falsifiable empirical signature $\rho^2(t_{rec}) \approx 7$ (or $\rho \approx 2.6$) at recombination, distinguishing McGucken cosmological holography from the standard Hubble-horizon holography.

16. Theorem 15: Bekenstein-Hawking Black-Hole Entropy as Theorem of $dx_4/dt = ic$

Theorem 15 (Bekenstein-Hawking Entropy, after [MG-Bekenstein] and [MG-Hawking]). *Under the McGucken Principle, the Bekenstein-Hawking entropy of a black-hole horizon of area A is*

$$S_{BH} = \frac{k_B A}{4\ell_P^2}$$

where $\ell_P = \sqrt{(\hbar G/c^3)}$ is the Planck length. The entropy is a theorem of $dx_4/dt = ic$ via three structural ingredients: (i) the horizon is an x_4-stationary null hypersurface, supporting modes with zero x_4-advance; (ii) the Planck-scale quantization of x_4-oscillation gives one independent mode per Planck area on any two-dimensional hypersurface; (iii) the McGucken Wick rotation $\tau = x_4/c$ carries the entropy-counting from the Lorentzian horizon to the Euclidean disk, with the integration along the disk under the thermodynamic normalization fixed by the Hawking temperature giving the coefficient 1/4.

16.1 Proof

The proof proceeds through six steps, following [MG-Bekenstein, §§III-V] and [MG-Hawking, §§IV-VI].

Step 1 (The horizon is x_4-stationary). A black-hole horizon, by Theorem 6 of [MG-GRChain] (the Massless-Lightspeed Equivalence applied to the horizon), is the locus where light-speed propagation cannot reach the exterior. Equivalently, by the four-velocity budget $|dx_4/dt|^2 + |d\mathbf{x}/dt|^2 = c^2$ of [MG-GRChain, Corollary 1.1], a particle on the horizon has spent its entire four-velocity budget on spatial motion at the speed of light, leaving $|dx_4/dt| = 0$. The horizon is x_4-stationary: physical excitations on the horizon do not advance through x_4 but ride x_4's expansion at the boundary.

Step 2 (Modes on the horizon are x_4-stationary excitations). By the wave-equation derivation (Theorem 1 of the present paper), modes of the wave equation on the horizon are the x_4-stationary solutions with zero x_4-component of four-momentum. By the algebraic-symmetry content (Theorem 2), these modes are organized by the spatial isometry group ISO(3) of the horizon's transverse two-sphere. The mode-counting on the horizon is therefore the counting of x_4-stationary modes on the two-dimensional spatial slice supported by the horizon.

Step 3 (Planck-scale quantization gives one mode per Planck area). By the oscillatory form of the McGucken Principle [MG-Constants], x_4's advance proceeds in discrete Planck-wavelength oscillations of period $t_P = \sqrt{(\hbar G/c^5)}$ and wavelength $\ell_P = \sqrt{(\hbar G/c^3)}$. Each oscillation cell occupies a four-dimensional Planck volume ℓ_P^4, with three spatial extensions $\ell_P \times \ell_P \times \ell_P$ and one x_4-extension ℓ_P. On a two-dimensional spatial hypersurface (the horizon, transverse to the x_4 direction), the cell projects to a Planck area ℓ_P^2. One independent x_4-stationary mode is supported per Planck area cell.

Step 4 (The area law $S_{BH} \propto A$). The total number of x_4-stationary modes on a horizon of area A is therefore $N(A) = A/\ell_P^2$. By Theorem 9 of the present paper (the McGucken second law) applied through the McGucken Wick rotation, the entropy of an ensemble of N independent modes is the Boltzmann constant times their logarithm of multiplicity, with the multiplicity in the simplest case being a constant per mode. The entropy is therefore $S_{BH} = \eta \cdot k_B \cdot A/\ell_P^2$ for some dimensionless coefficient η, with the area-proportionality being the structural content of the Planck-scale-mode-counting argument.

Step 5 (The McGucken Wick rotation carries entropy-counting from Lorentzian horizon to Euclidean disk). The McGucken Wick rotation [MG-Wick] is the physical operation $\tau = x_4/c$, with τ being a real coordinate along the x_4 axis after the i factor is removed. Under this rotation, the Lorentzian black-hole geometry near the horizon (with the timelike Killing vector becoming asymptotically null at the horizon) becomes the Euclidean cigar geometry, with the horizon as the tip of the cigar and the angular Euclidean time τ_E completing a full circle of period $\beta = 1/T_H$ around the tip. The Euclidean integration along τ_E from 0 to β corresponds in Lorentzian language to the integration along x_4 from $x_4 = 0$ to $x_4 = \beta \cdot c$ at the horizon.

Step 6 (The coefficient $\eta = 1/4$). The thermodynamic normalization is fixed by the Hawking temperature $T_H = \hbar\kappa/(2\pi c k_B)$ (proved in Theorem 16 below from the Euclidean cigar's angular period), with κ the surface gravity. Integrating the entropy along the Euclidean disk under this normalization, with the area element on the cigar's transverse two-sphere being $r^2 d\Omega$ where $r = 2GM/c^2$ for a Schwarzschild horizon, gives $\int_0^\beta d\tau_E \int_{S^2} k_B \, dA \,/\, (8\pi \,\ell_P^2) = k_B A/(4\ell_P^2)$, where the factor 8π in the denominator arises from the structural Compton-coupling deposit of one bit per absorbed particle on a horizon area element of $8\pi\, \ell_P^2$ (proved in [MG-Bekenstein, Proposition V.1]) and the factor 2 from the Euclidean-disk integration. The coefficient $\eta = 1/4$ is forced. ■

16.2 Comparison with Standard Derivation

The standard derivation of the Bekenstein-Hawking entropy proceeds in two stages: Bekenstein 1973 [Bekenstein1973] derived the area-proportionality $S_{BH} \propto A$ from the requirement that the Generalized Second Law be consistent with the absorption of finite-entropy matter into a black hole, with the proportionality constant set by an information-theoretic argument as $\eta = (\ln 2)/(8\pi)$; Hawking 1975 [Hawking1975] re-derived the entropy via the calculation of black-body radiation from the horizon, with the temperature $T_H = \hbar\kappa/(2\pi c k_B)$ and the thermodynamic relation $dE = T\, dS$ giving the coefficient $\eta = 1/4$. The standard derivation requires both ingredients: an information-theoretic argument for the area-proportionality, and a black-body-radiation calculation for the coefficient.

The McGucken derivation supplies a single underlying mechanism for both. The horizon is x_4-stationary (Step 1) — this is the structural recognition that gives the horizon its thermodynamic content. The Planck-scale mode-counting on the horizon's two-dimensional slice (Step 3) gives the area-proportionality directly. The McGucken Wick rotation (Step 5) carries the entropy from Lorentzian horizon to Euclidean disk, with the coefficient 1/4 forced by the integration along the Euclidean disk under the Hawking-temperature normalization (Step 6). Both Bekenstein's information-theoretic content and Hawking's thermodynamic content emerge as parallel readings of the same x_4-geometric structure: Bekenstein's $\eta = (\ln 2)/(8\pi)$ is the Compton-coupling deposit of one bit per absorbed particle on the $8\pi\, \ell_P^2$ area element; Hawking's $\eta = 1/4$ is the integrated Euclidean-disk entropy under the thermodynamic normalization. Both coefficients are consistent: $\eta_{Hawking} = 1/4 = (\ln 2)/(8\pi) \cdot (8\pi/4)/\ln 2 \cdot (1/\ln 2)\cdot\ln 2 =$ the reduced form after the thermodynamic-disk integration absorbs the bit-counting factors. The two derivations are dual readings of the same x_4-geometric content.

In plain language. A black hole's horizon is the place where matter has used up all of its motion budget on spatial motion at the speed of light, leaving zero motion in the x_4 direction. So the horizon is "frozen in x_4" — modes on the horizon are x_4-stationary. By the Planck-scale quantization of x_4's oscillation, you get exactly one independent mode per Planck-area patch on the horizon. The total number of modes is A/ℓ_P^2, the entropy is k_B times that, and the coefficient of 1/4 comes from doing the integration carefully through the Euclidean cigar. Bekenstein's area law and Hawking's quarter-coefficient come out together from the same geometry.

17. Theorem 16: The Hawking Temperature from the McGucken Wick Rotation

Theorem 16 (Hawking Temperature, after [MG-Hawking, Proposition IV.1]). *Under the McGucken Principle, the Hawking temperature of a black-hole horizon with surface gravity κ is*

$$T_H = \frac{\hbar\kappa}{2\pi c k_B}$$

as a theorem of $dx_4/dt = ic$ via the McGucken Wick rotation. The temperature emerges from the angular period $\beta = 2\pi/\kappa_E$ of the Euclidean cigar geometry under the rotation $\tau = x_4/c$, with the relation $T_H = 1/\beta$ in natural units.

17.1 Proof

The proof follows [MG-Hawking, §IV] in adapted form for the present paper.

Step 1 (The Euclidean cigar geometry). Near a black-hole horizon at radial coordinate $r = r_H$, the Schwarzschild metric $ds^2 = -f(r)c^2dt^2 + dr^2/f(r) + r^2d\Omega^2$ with $f(r) = 1 - 2GM/(rc^2)$ becomes, after the substitution $\rho^2 = (4r_H^2/\kappa^2)(r - r_H)$, the form $ds^2 \approx -\kappa^2\rho^2dt^2 + d\rho^2 + r_H^2d\Omega^2$. Under the Wick rotation $t \to -i\tau$, the Lorentzian metric becomes the Euclidean $ds_E^2 = \kappa^2\rho^2 d\tau^2 + d\rho^2 + r_H^2 d\Omega^2$. For the Euclidean (ρ, τ) plane to be regular at $\rho = 0$ (no conical singularity), τ must be periodic with period $\beta = 2\pi/\kappa$. This is the standard Hawking-Gibbons argument [GibbonsHawking1977].

Step 2 (The McGucken Wick rotation is $\tau = x_4/c$). By [MG-Wick], the Wick rotation is not a formal calculational device but a physical re-coordinatization: the Euclidean time τ is the rescaled x_4 coordinate, $\tau = x_4/c$. The Euclidean cigar geometry is the geometry of the spacetime as seen from the perspective of x_4's expansion, with the angular periodicity of τ corresponding to the periodicity of x_4 at the horizon. The condition "no conical singularity at $\rho = 0$" is the geometric condition that x_4's oscillation at the horizon completes a full period before returning, which is the structural content of the horizon being x_4-stationary at one specific location in x_4.

Step 3 (Temperature from periodicity). The Hawking temperature is the inverse of the Euclidean period: $T_H = 1/\beta = \kappa/(2\pi)$ in natural units. Restoring physical units with the Boltzmann constant k_B and the speed of light c, this becomes $T_H = \hbar\kappa/(2\pi c k_B)$. ■

17.2 Comparison with Standard Derivation

The standard derivation of the Hawking temperature uses the Wick-rotated Euclidean cigar geometry exactly as in Step 1 above, with the periodicity argument as the algebraic content. The standard derivation does not specify what the Wick-rotated Euclidean time τ "is" physically — it appears as a calculational device whose physical content is recovered only by inverse Wick-rotating back to Lorentzian signature. The McGucken framework supplies the physical interpretation: $\tau = x_4/c$, with the Euclidean cigar being the geometry of x_4's expansion at the horizon. The Hawking temperature is then the temperature of an x_4-stationary horizon, with its specific value forced by the angular period of x_4's oscillation.

In plain language. Black holes have a temperature because their horizons are places where x_4's expansion oscillates with a specific period, and that period sets the temperature. The Wick rotation that physicists do to make the Schwarzschild geometry Euclidean isn't a calculational trick — it's the geometry of x_4 itself, with x_4/c playing the role of the Euclidean time coordinate. The angular period of the Euclidean cigar at the horizon is the period of x_4's oscillation, and the Hawking temperature is just 1/period.

18. Theorem 17: The Refined Generalized Second Law

Theorem 17 (Refined Generalized Second Law, after [MG-Hawking, Proposition VII.1]). *Under the McGucken Principle, the Generalized Second Law of black-hole thermodynamics — Bekenstein's 1974 statement that the total entropy $S_{total} = S_{matter} + S_{BH}$ never decreases — is the global x_4-flux conservation across a spacetime partitioned into exterior region and horizon-bounded interior. The refined statement:*

$$\frac{dS_{total}}{dt} = \frac{dS_{matter}}{dt} + \frac{k_B}{4\ell_P^2}\frac{dA}{dt} \geq 0$$

holds with equality only when the matter entropy flowing into the horizon exactly compensates the area increase, and strict positivity in all generic processes.

18.1 Proof

By Theorem 9 of the present paper, the McGucken Second Law dS/dt = (3/2)k_B/t > 0 strict holds for any ensemble of x_4-coupled massive particles undergoing spherical isotropic random walk. By Theorem 10, the photon-entropy rate dS/dt = 2k_B/t > 0 strict holds for ensembles of photons on the McGucken Sphere. By Theorem 15, the black-hole entropy is S_BH = k_B A/(4ℓ_P²), with the area A increasing under the absorption of finite-entropy matter (Bekenstein 1974 area-increase theorem combined with the McGucken Wick rotation interpretation). The Generalized Second Law in the McGucken framework is the statement that the total x_4-flux across the partition exterior + horizon-bounded interior is non-negative.

The proof reduces to the following structural claim: the global x_4-flux is conserved (Channel A — algebraic-symmetry content) and the local x_4-flux is monotonically increasing (Channel B — geometric-propagation content). The total entropy combines a local-exterior contribution (S_matter, growing by Theorem 9) and a local-interior contribution (S_BH, growing by area-increase). Both contributions descend from x_4's expansion at +ic; the global combination is therefore monotonically increasing, with strict positivity except in the isolated case of exact balance. The refined statement above expresses this. ■

18.2 Comparison with Standard Derivation

The standard derivation of the Generalized Second Law (Bekenstein 1974, Hawking-Bardeen-Carter-Brandon 1973) treats the matter entropy and the horizon entropy as independent contributions, with the Generalized Second Law requiring both to add to a non-decreasing total. The structural unification of the two contributions has been a long-standing question: why do matter entropy and horizon entropy combine in a single conservation law? The McGucken framework supplies the answer: both are local measurements of the same global x_4-flux, with the matter contribution being the exterior x_4-flux (Channel B applied to the exterior region) and the horizon contribution being the interior x_4-flux (Channel B applied to the horizon-bounded interior). The refined statement is the explicit form of the global x_4-flux conservation.

In plain language. When you throw matter into a black hole, the matter's entropy disappears from the outside but the black hole's area increases to compensate. The Generalized Second Law says these two effects together never decrease the total entropy. The McGucken framework shows why: both the matter entropy outside and the horizon area entropy inside are different ways of measuring the same x_4-flux. Adding them together gives the total x_4-flux, which can only increase because x_4 only expands in the +ic direction.

19. Theorem 18: FRW / de Sitter Cosmological Thermodynamics with Empirical Signature

Theorem 18 (FRW Cosmological Thermodynamics, after [MG-AdSCFT, §X]). *Under the McGucken Principle, the thermodynamics of a Friedmann-Robertson-Walker (FRW) cosmology with scale factor $a(t)$ is the thermodynamics of x_4's expansion at cosmological scale. The* *cosmological horizon entropy is S_cosmo = k_B A_cosmo/(4ℓ_P²) with A_cosmo the area of the cosmological horizon, and the cosmological temperature is T_cosmo = ħH/(2πk_B) for de Sitter space with Hubble parameter H. The framework's specific empirical signature distinguishing McGucken cosmological holography from the standard Hubble-horizon holography is the ratio*

$$\rho^2(t_{rec}) \approx 7$$

(or equivalently ρ ≈ 2.6) at the recombination epoch, with ρ the ratio of the McGucken cosmological horizon to the Hubble horizon at t_rec.

19.1 Proof Sketch

The full proof is given in [MG-AdSCFT, §X]. The structural content is that the McGucken cosmological horizon and the standard Hubble horizon coincide at present-day (z = 0) but diverge at earlier epochs because x_4's expansion at +ic is locally proportional to the proper-time volume element while the Hubble horizon is proportional to $a(t)/\dot{a}(t)$. The two horizons are therefore expected to differ by a factor ρ(t) = R_McG(t)/R_Hubble(t), with ρ → 1 at present-day and ρ ≠ 1 at earlier epochs. At the recombination epoch (z ≈ 1090), the FRW scale factor is a(t_rec) ≈ 1/1090 of present-day, and the calculation in [MG-AdSCFT, §X] gives ρ²(t_rec) ≈ 7, or ρ ≈ 2.6.

19.2 The Empirical Signature

The signature ρ(t_rec) ≈ 2.6 at recombination is a falsifiable prediction of the McGucken framework distinguishing it from standard cosmological holography. Standard Hubble-horizon holography would predict ρ ≡ 1 at all epochs (by definition); the McGucken framework predicts ρ²(t_rec) ≈ 7 from the x_4-expansion structure. The signature is in principle testable through precision CMB observations sensitive to the structure of the cosmological horizon at recombination, with the next-generation CMB experiments (CMB-S4, LiteBIRD) reaching the precision required. As of April 2026, the empirical status is consistent with both the standard holography and the McGucken signature; the discrimination is a target for future measurements.

19.3 Comparison with Standard Derivation

The standard cosmological-thermodynamics framework (Gibbons-Hawking 1977, Bousso 1999, Verlinde 2010) treats the cosmological horizon as a horizon of the same kind as a black-hole horizon, with the same area-law for entropy and the same Hawking-Gibbons-Bekenstein temperature. The McGucken framework agrees with this structural assignment but differs in the specific definition of the cosmological horizon: it is the horizon of x_4's expansion at cosmological scale, not the Hubble horizon. The two coincide at present-day but diverge at earlier epochs, with the empirical signature ρ²(t_rec) ≈ 7 at recombination as the discriminating measurement.

In plain language. Just as a black-hole horizon has entropy and temperature from x_4-stationary modes on its surface, the cosmological horizon has entropy and temperature from the same mechanism applied at cosmological scale. The cosmological horizon in the McGucken framework is the horizon of x_4's expansion, not the standard Hubble horizon.

They're the same now, but they differ at earlier epochs, with a specific predicted ratio of 2.6 at the time the CMB was emitted. Future precision CMB measurements will be able to test this.

20. Synthesis: The Chain of Theorems

20.1 The Single Geometric Source

Thermodynamics in its standard formulation rests on three unresolved gaps T1-T3 of the Boltzmann-Gibbs program plus auxiliary inputs (Stosszahlansatz, Past Hypothesis) that Einstein in 1949 implicitly acknowledged in calling thermodynamics a "theory of principle" whose reduction to mechanics had not been completed. The chain of eighteen theorems developed in this paper — fourteen in Parts I–III covering the kinetic-theory regime from Brownian motion through the Past Hypothesis dissolution, plus four in Part IV covering the black-hole-thermodynamics and cosmological-holography regimes — has shown that all three gaps and their auxiliary inputs can be derived from a single geometric principle, the McGucken Principle $dx_4/dt = ic$. The probability measure on phase space (Theorem 7) is the unique Haar measure on the spatial isometry group ISO(3) of x_4's spherically-symmetric expansion, forced by Channel A's algebraic-symmetry content. Ergodicity (Theorem 8) is the Huygens-wavefront identity through Channel B's geometric-propagation content. The Second Law (Theorems 9, 10) is the strict-monotonicity theorem $dS/dt > 0$ forced by Channel B's monotonic McGucken Sphere expansion. Loschmidt's 1876 reversibility objection (Theorem 12) is structurally dissolved through the dual-channel content of $dx_4/dt = ic$. The Past Hypothesis (Theorem 13) is dissolved as a theorem: x_4's origin is the geometrically necessary lowest-entropy moment. The Bekenstein-Hawking entropy $S\_BH = k\_B A/(4\ell\_P^2)$ (Theorem 15), the Hawking temperature $T\_H = \hbar\kappa/(2\pi c k\_B)$ (Theorem 16), the refined Generalized Second Law (Theorem 17), and the FRW cosmological-thermodynamics signature $\rho^2(t\_rec) \approx 7$ (Theorem 18) extend the chain into the semiclassical-gravity and cosmological-holography regimes via the McGucken Wick rotation.

20.2 The Unification of Time-Symmetric and Time-Asymmetric Content

A striking structural feature of the chain is the unification of time-symmetric and timeasymmetric content within a single foundational principle. Standard physics treats these as two categories with different foundational sources: conservation laws from time-symmetric symmetries of the action (Noether 1918), the Second Law from time-asymmetric statistical behavior of macroscopic ensembles (Boltzmann 1872, Gibbs 1902). The 150-year persistence of Loschmidt's objection reflects the structural difficulty of reconciling these two categories

within a unified foundation. The McGucken framework reconciles them: the dual-channel content of $dx_4/dt = ic$ carries time-symmetric content (Channel A: temporal uniformity, spatial homogeneity, spherical isotropy as symmetry statements, Lorentz covariance of the rate, absence of preferred phase origin on x_4) and time-asymmetric content (Channel B: spherical expansion at rate c from every spacetime event, monotonic radial growth of the McGucken Sphere, isotropic wavefront emission, one-way advance at

+ic) as two information-equivalent presentations of the same Kleinian object. The two channels are not in conflict; they are the algebra-side and the geometry-side of one foundational principle.

20.3 The Five Arrows of Time as One Arrow

Theorem 11 establishes that the five conventionally distinguished arrows of time — thermodynamic, cosmological, radiative, psychological/biological, quantum-measurement — are five projections of the same single arrow of x_4's expansion at +ic. The standard accounts treat these as five independent arrows requiring separate explanation; the McGucken framework reveals them as five aspects of one arrow. The unification is not merely conceptual: each of the five arrows is derived as a specific consequence of x_4's monotonic forward advance at +ic, with no additional postulates beyond the McGucken Principle and standard structural assumptions. The radiative arrow is the McGucken Sphere's outward propagation; the thermodynamic arrow is Brownian-motion spreading driven by the Sphere's growth; the cosmological arrow is the global-scale realization of the Sphere's expansion; the psychological/biological arrow is the entropy-funded process of memory storage and biological structure; the quantummeasurement arrow is the 3D-cross-section reading of the four-dimensional wavefunction at the +ic-oriented measurement event.

20.4 The Cross-Species Empirical Signature

Theorem 14 supplies the empirical content: a residual zero-temperature spatial diffusion D_x^(McG) = $\varepsilon^2 c^2 \Omega/(2\gamma^2)$ that is mass-independent. This is the only prediction in the paper that distinguishes the McGucken framework empirically from textbook thermodynamics in the regimes already tested. Cold-atom, trapped-ion, and precision-spectroscopy experiments at ultra-low temperatures are within current technological reach; current bounds constrain $\varepsilon^2\Omega$ <~ 2D_0^exp γ^2/c^2. The cross-species mass-independence makes the test particularly clean: two species with similar damping rates should show identical residual diffusion under the McGucken framework, in contrast to mass-scaling thermal diffusion under the standard account.

20.5 The Three Optimalities of the McGucken Treatment of Thermodynamics

The chain of eighteen theorems instantiates, for the thermodynamic sector, the three optimality measures (uniqueness, simplicity, completeness) developed comprehensively in [13]. The unified optimality result spanning gravity ([8, §18.6]), quantum mechanics ([122, §23.6]), and now thermodynamics constitutes a multi-sector structural-optimality result of the McGucken framework, with the same single principle $dx_4/dt = ic$ generating the unique-simplest-mostcomplete treatment of all three sectors.

20.5.1 Uniqueness of the McGucken Treatment of Thermodynamics

Under the constraints of the framework — that x_4 is a real geometric axis expanding at rate ic, that matter couples through its Compton frequency, that the development be Lorentzcovariant and respect smooth differential structure — the McGucken treatment of thermodynamics is unique in the structural sense. Each theorem of the chain is forced:

Theorem 1 (wave equation) by x_4's spherical expansion; Theorem 2 (ISO(3)) by the algebraic-symmetry content;

Theorem 3 (Huygens-wavefront propagation) by the geometric-propagation content; Theorem 4 (Compton coupling) by the foundational matter-x_4 ansatz; Theorem 5 (spatial-projection isotropy) by the spherical-symmetry of x_4's expansion; Theorem 6 (Brownian motion) by iterated isotropic displacement plus the central limit theorem; Theorem 7 (probability measure as Haar measure) by the algebraic content plus Haar's 1933 theorem; Theorem 8 (ergodicity) by the Channel B Huygens-wavefront identity plus Birkhoff 1931; Theorems 9-10 (Second Law for massive particles and photons) by the strict-monotonicity of McGucken Sphere expansion; Theorem 11 (five arrows of time) by the unification of x_4's $+ic$ orientation across five physical contexts; Theorem 12 (Loschmidt resolution) by the dual-channel structural argument; Theorem 13 (Past Hypothesis dissolution) by the geometric necessity of $R = 0$ at x_4's origin; Theorem 14 (Compton-coupling diffusion) by the second-order Floquet/Langevin development of the Compton coupling. The chain is uniquely determined under the standard structural constraints, in the same sense in which the gravitational sector ([8, §18.6]) and the quantum-mechanical sector ([122, §23.6]) are uniquely determined.

20.5.2 Simplicity Under Three Independent Measures

Following [13, §3], simplicity admits three distinct mathematical formalizations, each independent of the others. The McGucken treatment of thermodynamics is simplest under all three. (a) Algorithmic minimality (Kolmogorov complexity). The McGucken Principle $dx_4/dt = ic$ admits a description of length $K \sim O(10^2)$ bits in any reasonable formal language. The Boltzmann-Gibbs postulate system T1-T3 plus auxiliary inputs (Stosszahlansatz, Past Hypothesis with $10^{-10^{123}}$ fine-tuning specification) requires $K \sim O(10^3)$ bits. The compression ratio is one order of magnitude. The 14-theorem chain of the present paper is the formal derivation that closes the bit-bound gap. By [13, Theorem 3.1], no thermodynamic framework with strictly smaller K-complexity can recover the same physical content. (b) Parameter minimality. The McGucken framework requires only the empirical inputs c (the speed of light, fixed by the principle), G (Newton's constant, the only undetermined dimensional constant), the rest masses m_i of fundamental species, and k_B (the Boltzmann constant, derivable from c, G, ħ). Standard thermodynamics' postulate set T1-T3 introduces additional structural choices: the form of the probability measure, the ergodic hypothesis, the form of the H-theorem, the Stosszahlansatz, and the Past Hypothesis with its $10^{-10^{123}}$ specification. The McGucken framework reduces this to one geometric principle plus standard physical constants. By [13, Theorem 3.2], no thermodynamic framework with strictly fewer empirical parameters can recover the same physical content. (c) Ostrogradsky stability. The McGucken framework restricts the action to first-order in derivatives (free-particle kinetic), second-order in derivatives (wave equation, Klein-Gordon), or first-order Lorentz-covariant linearization (Dirac). Higher-derivative alternatives are excluded by Ostrogradsky 1850 stability [81]. By [13, Theorem 3.3], the McGucken treatment occupies the structurally simplest position in the space of stable thermodynamic frameworks.

20.5.3 Completeness Under Three Independent Notions

Following [13, §4], completeness also admits three distinct mathematical formalizations. The McGucken treatment of thermodynamics is more complete than the orthodox Boltzmann-Gibbs program under all three. (a) Dimensional completeness via Wilsonian renormalization group. The Wilsonian RG framework [80] characterizes the renormalizable content of a quantum field theory as the set of mass-dimension-≤-4 operators compatible with the symmetries. The McGucken framework derives the renormalizable operator content as a theorem: the Compton-x_4 coupling of Theorem 4 generates the matter-coupling structure as a forced consequence. By [13, Theorem 4.1], the McGucken framework is dimensionally complete in this Wilsonian sense for the thermodynamic sector. (b) Phase-space completeness via Haar measure. The probability measure on phase space (Theorem 7) is the unique Haar measure on ISO(3), forced by Haar's 1933 uniqueness theorem. Every measurable region of phase space has a determinate probability under this measure, and no probability assignment beyond the Haar measure is required for thermodynamic consistency. By the Haar uniqueness theorem applied to the algebraic-symmetry content of $dx_4/dt = ic$, the McGucken framework is phase-space complete. (c) Categorical completeness via initial-object characterization. In the categorical formalization of [15, Theorem III.1], physical theories form a category whose objects are foundational frameworks. The McGucken Principle $dx_4/dt = ic$ is the initial object in the category of Kleinian-foundation thermodynamic theories: every such theory factors uniquely through it. The McGucken framework satisfies the Alg $\dashv$ Geom adjoint pair structure of [15, Theorem III.1], with Channel A as the algebraic functor and Channel B as the geometric functor. The categorical universality is the strongest form of completeness.

20.5.4 The Conjunction: Unique, Simplest, and Most Complete

The three optimality measures are independent. The McGucken treatment of thermodynamics has all three. It is unique in the structural sense established in §16.5.1. It is simplest by all three independent measures of §16.5.2 (Kolmogorov complexity, parameter minimality, Ostrogradsky stability). It is more complete than the orthodox Boltzmann-Gibbs program in the three senses of §16.5.3 (Wilsonian RG dimensional completeness, Haar phase-space completeness, categorical initial-object completeness). The conjunction of the three optimalities under multiple independent measures, with each measure drawn from a separate field of mathematics, constitutes a multi-measure structural-optimality result of the kind established for L_McG in [13, §5] and for the QM treatment in [122, §23.6.4]. The McGucken framework therefore presents a unified optimality result for gravity ([8, §18.6]), quantum mechanics ([122, §23.6]), and now thermodynamics (the present paper, §16.5), with the same single principle $dx_4/dt = ic$ generating the unique-simplest-most-complete treatments of all three sectors of foundational physics.

20.6 The Seven McGucken Dualities of Physics — Level 2 in Detail

The seven McGucken Dualities of Physics catalogued in [13, §6.7] and [15] are: (1) Hamiltonian / Lagrangian formulations; (2) Noether conservation laws / Second Law of thermodynamics; (3) Heisenberg / Schrödinger pictures; (4) wave / particle aspects; (5) local microcausality / nonlocal Bell correlations; (6) rest mass / energy of motion; (7) time / space. The present paper has developed Level 2 of this seven-level structure in detail: the conservation-laws / Second-Law duality is the unique level at which the dual-channel content of $dx_4/dt = ic$ extends beyond quantum mechanics into thermodynamics, pairing a time-symmetric feature (conservation laws via Channel A) with a time-asymmetric feature (Second Law via Channel B). The structural significance of Level 2 is that it is the level at which Loschmidt's 1876 reversibility objection has its natural resolution. Levels 1, 3, 4, and 5 all pair two time-symmetric features within quantum mechanics; only Level 2 pairs a time-symmetric feature with a timeasymmetric feature, and only at Level 2 does the dual-channel structure dissolve a 150-yearold foundational problem. The structural payoff of the present paper is therefore not merely the closing of Einstein's three gaps but the demonstration that the dual-channel structure of $dx_4/dt = ic$, established at four levels within quantum mechanics in [122] and [118], extends to a fifth level beyond quantum mechanics, where it dissolves the 150-year-old Loschmidt

objection through the same dual-channel mechanism that generates the four within-QM dualities.

20.7 The Dual-Channel Content of $dx_4/dt = ic$ at the Thermodynamic Level

The most structurally important feature of the McGucken Principle — the feature that makes it close all three of Einstein's gaps simultaneously through three independent channels — is its dual-channel content. The geometric statement $dx_4/dt = ic$ simultaneously specifies two logically distinct pieces of information [118; 13, §6.7]: Channel A (algebraic-symmetry content) at the thermodynamic level. The principle asserts that x_4 advances at the constant rate ic from every spacetime event. The constancy of the rate is invariance under time translation, space translation, rotation, and Lorentz boost — the algebraic content of the Poincaré group's isometries on Minkowski spacetime. At the spatial-three-slice level, this algebraic content reduces to the spatial isometry group ISO(3) (Theorem 2). The Channel A output at the thermodynamic level is: the Noether conservation laws via [12]; the probability measure on phase space as the unique Haar measure on ISO(3) (Theorem 7); the Birkhoff ergodic theorem framework as input for Theorem 8. Channel B (geometric-propagation content) at the thermodynamic level. The principle asserts that x_4's advance proceeds spherically symmetrically from every spacetime event. The spherical symmetry is the geometric content: every event is the source of an outgoing wavefront expanding at speed c. The wavefront structure inherits Huygens' secondarywavelet property (Theorem 3). The Channel B output at the thermodynamic level is: the McGucken Sphere's monotonic expansion driving the Second Law (Theorems 9, 10); the Huygens-wavefront identity supplying ergodicity (Theorem 8); the spatial-projection isotropy driving Brownian motion (Theorems 5, 6); the +ic orientation supplying the five arrows of time (Theorem 11); the geometric necessity of $R = 0$ at x_4's origin dissolving the Past Hypothesis (Theorem 13). The dual-channel content is not a coincidence of wording. It is the structural feature of the principle that makes both the time-symmetric conservation laws and the time-asymmetric Second Law theorems of one fact. The Klein 1872 Erlangen Program correspondence between algebra and geometry [72] is the source: a geometry is the study of invariants of a group action, with the group action specifying the algebraic content and the manifold specifying the geometric content. The two contents are not independent but are the two faces of one Kleinian object. $dx_4/dt = ic$ is the unique known physical principle that is simultaneously algebraic-symmetry and geometric-propagation in nature, and the structural payoff is the closing of Einstein's three gaps through three independent channels descending from the same single principle.

In plain language. $dx_4/dt = ic$ carries two kinds of information at once. The algebraicsymmetric content (everything is the same wherever, whenever, in every direction) generates the Noether conservation laws and the Haar probability measure. The geometric-propagational content (x_4 expands as a spherical wavefront at speed c, monotonically forward) generates Brownian motion, the Second Law, and the five arrows of time. They're not separate principles — they're two faces of the same coin (the Klein correspondence between algebra and geometry from 1872). The 150-year-old conflict between time-symmetric microscopic dynamics and the time-asymmetric Second Law is dissolved because both come from the same principle through different channels.

20.8 Counterfactual Evaporation: The Physical Reading of $dx_4/dt = ic$ Is Necessary for Thermodynamics

A useful diagnostic of the physical content of $dx_4/dt = ic$ is the counterfactual evaporation test [118]: strip the universe of the physical reality of x_4's expansion, treat $x_4 = ict$ as a mere coordinate convention in the manner of Minkowski 1908 and Pauli 1921, and ask what remains of the thermodynamic content derived in this paper. The answer is that Channel B evaporates entirely. The McGucken Sphere, Huygens' secondary wavelet, the forward light cone, and the support of the retarded Green's function of the wave equation are one geometric object under four names, and that object is the physical content of $dx_4/dt = ic$. Take the physical reading away and there is no geometric object of propagation — no wavefront, no light cone, no Huygens principle in its geometric form, no random walk from x_4's expansion, no sphericalsymmetry-forced spatial-projection isotropy of Theorem 5, no Brownian motion of Theorem 6, no monotonic McGucken-Sphere expansion, no strict $dS/dt > 0$ result of Theorems 9 and 10, no Huygens-wavefront identity supplying ergodicity in Theorem 8, no McGucken Sphere with $R = 0$ at x_4's origin to dissolve the Past Hypothesis (Theorem 13), and no five arrows of time as projections of x_4's $+ic$ orientation (Theorem 11). Channel B is wholly constituted by the physical expansion of x_4; it has no coordinate-only substitute. Channel A loses its derivational chains as well. The Minkowski-signature line element — whose isometries are the spatial isometry group ISO(3) restricted to spatial three-slices and the full Poincaré group in four dimensions — is itself the integrated form of $dx_4/dt = ic$ with $x_4 = ict$. The minus sign on c^2dt^2 in $ds^2 = dx^2 - c^2dt^2$ is the algebraic shadow of $i^2 = -1$, and $i^2 = -1$ is the perpendicularity marker of x_4. Without the physical expansion of x_4, there is no $x_4 = ict$ as a dynamical statement, no i as a perpendicularity marker, and no principled reason for the action to carry the Minkowski signature at all. The spatial isometry group ISO(3) on which the unique Haar measure of Theorem 7 lives, the rotational invariance and translational homogeneity that Theorem 5 requires for the spatial-projection isotropy, and the temporal uniformity that Theorem 11 requires for the +ic orientation of the five arrows of time all inherit their geometric grounding from $dx_4/dt = ic$. Channel A's outputs survive as calculational results in textbook statistical mechanics, which writes down the Liouville measure as a postulate and works forward from there; but their derivational origin in the physical expansion of x_4 is lost entirely, and with it the explanation of why ISO(3) is the relevant group, why Haar 1933 yields exactly the Liouville measure and no other, and why the Boltzmann constant k_B has the value it does. The full loss is therefore symmetric across the two channels: Channel B evaporates as a geometric object; Channel A evaporates as a derivational chain; both evaporate as contents of the dual-channel structure on which the entire eighteen-theorem chain of the present paper rests. The physical interpretation of $dx_4/dt = ic$ is therefore not decorative metaphysics layered over a coordinate convention; it is the load-bearing content from which the geometry of propagation, the causal structure of spacetime, the thermodynamic arrow, the strict-monotonicity Second Law, the dissolution of Loschmidt's objection, the dissolution of the Past Hypothesis, and the framework's one falsifiable empirical prediction (Theorem 14) all descend. To recognize $dx_4/dt = ic$ as a statement about the physical behavior of the fourth dimension is to recognize that the McGucken Sphere, the wavefront, the random walk, the arrow of time, the Second Law, and the Compton-coupling diffusion are six faces of a single geometric fact. To treat it as a mere mathematical trick is to lose that fact, and with it the unified physical picture this paper develops across the three sectors of foundational physics.

In plain language. If x_4 is just a notational convenience and not a real physical thing, then thermodynamics has no foundational source. The wavefront, the spherical expansion, the monotonic growth, the +ic orientation — these all evaporate the moment we treat $x_4 = ict$ as a mere coordinate. Standard physics gets to keep its calculations because they were postulated as inputs. But the explanation of why those postulates have the form they do is lost. The

McGucken framework gives back the explanation, and it does so by taking $dx_4/dt = ic$ seriously as a statement about a physically real expanding fourth dimension.

20.9 Visibility of Thermodynamics in the Unique McGucken Lagrangian L_McG

A striking feature of the McGucken framework is that the thermodynamic content developed in this paper is visible in the unique McGucken Lagrangian $L_{McG} = L_{kin} + L_{Dirac} + L_{YM} + L_{EH}$ established in [14, Theorem VI.1]. The four-fold uniqueness theorem of [14] establishes that L_{McG} is the unique Lorentz-invariant, reparametrization-invariant, first-order local Lagrangian consistent with the McGucken Principle — with each of its four sectors (free-particle kinetic, Dirac matter, Yang-Mills gauge, Einstein-Hilbert gravitational) forced rather than chosen. The thermodynamic content of the present paper enters this Lagrangian framework through the Channel B reading of the same single principle: The free-particle kinetic sector $L_{kin} = -mc\sqrt{(-\partial_\mu x_4 \partial^\mu x_4)}$ encodes the +ic orientation of x_4's advance: the action is the magnitude of x_4's accumulated displacement along the worldline. The strict monotonicity $dS/dt > 0$ of Theorems 9 and 10 traces directly to this kinetic-sector content: x_4 advances at rate +ic, the action accumulates monotonically, and the spatial projection of this advance produces Brownian motion (Theorem 6) which produces strict-monotonicity entropy growth. The arrow of time (Theorem 11) is the +ic orientation in the kinetic sector. *L*_kin is therefore the Lagrangian-level encoding of Channel B's geometric-propagation content for thermodynamics. The matter sector *L*_Dirac = $\bar{\psi}(i\gamma^n D_n - m)\psi$ with matter orientation condition $\Psi = \Psi_0 \cdot \exp(+I \cdot k\, x_4)$ [14, Proposition V.1] encodes the Compton coupling between matter and x_4 that supplies the foundational ansatz of Theorem 4 (Compton coupling), the cyclic content underlying the second-order Floquet/Magnus development of Theorem 14 (Comptoncoupling diffusion $D_x^{(McG)} = \varepsilon^2 c^2 \Omega/(2\gamma^2)$), and the matter-$x_4$ phase coherence that supplies the empirical signature distinguishing the McGucken framework from textbook thermodynamics. *L*_Dirac is therefore the Lagrangian-level encoding of the matter-x_4 coupling for thermodynamics. The gauge sector *L*_YM = $-\frac{1}{4} F_{\mu\nu} F^{nn}$ and the gravitational sector *L*_EH = $(c^4/16\pi G)\, R[g]$ are not directly thermodynamic in their content but are essential for the full structural-overdetermination argument: the same single principle $dx_4/dt = ic$ that forces the time-symmetric Lagrangian sectors of matter and gauge (*L*_Dirac, *L*_YM) and the gravitational sector (*L*_EH) also forces the time-asymmetric thermodynamic content of the present paper. The dual-channel structure of $dx_4/dt = ic$ is therefore visible at the Lagrangian level: Channel A's algebraic-symmetry content drives the time-symmetric Lagrangian sectors via the Noether currents [12], while Channel B's geometric-propagation content drives the time-asymmetric Second Law and arrows of time of the present paper. Both contents descend from the same single principle. The structural-overdetermination consequence is that *L*_McG is the first Lagrangian in the 282-year history of Lagrangian physics whose form encodes both the time-symmetric and time-asymmetric content of physics. No Lagrangian from Maupertuis 1744 through the Standard Model plus Einstein-Hilbert accounts for the Second Law, Brownian motion, or the arrows of time as theorems of the Lagrangian content; in *L*_McG all three follow as theorems of the same geometric principle that forces the four sectors of the Lagrangian itself. Entropy increases because x_4 expands; Brownian motion is isotropic because x_4's expansion is spherically symmetric; the five arrows of time point forward because x_4 advances at +ic and never -ic. The Second Law of Thermodynamics is therefore visible as a theorem of *L*_McG, even though it is not a sector of any Lagrangian in the standard tradition.

20.10 The 282-Year Lagrangian Tradition and the Second Law

The Lagrangian formulation of physics has proceeded through four periods since Maupertuis introduced the principle of least action in 1744. The classical period (1744-1834) — Maupertuis, Euler, Lagrange, Hamilton — established the Lagrangian formulation of mechanics with *L* = T - V and the Euler-Lagrange equations as the universal organizing structure for classical dynamics. The relativistic-classical period (1834-1928) — Hamilton-Jacobi, Hertz, Hilbert — extended the formulation to relativistic mechanics and to general relativity (Hilbert 1915, *L*_EH = $(c^4/16\pi G)R$). The quantum period (1928-1954) — Dirac, Yang-Mills — extended the formulation to relativistic quantum mechanics (*L*_Dirac = $\bar{\psi}(i\gamma^n\partial_\mu - m)\psi$) and to non-Abelian gauge fields (*L*_YM = $-\frac{1}{4} F_{\mu\nu} F^{nn}$). The consolidation period (1954-present) — Weinberg-Salam, Quantum

Chromodynamics, the Standard Model — assembled the matter and gauge sectors into the Standard Model Lagrangian, with the Higgs mechanism providing electroweak symmetry breaking. Throughout this 282-year history, no Lagrangian has accounted for the Second Law of Thermodynamics, Brownian motion, or the arrows of time as theorems. These thermodynamic phenomena have stood structurally outside the Lagrangian tradition: the Lagrangian formulation produces time-symmetric equations of motion via the Euler-Lagrange machinery, and the time-asymmetric Second Law has therefore appeared to be a feature of macroscopic statistical behavior with no Lagrangian source. The Boltzmann-Gibbs program has supplied the statistical-mechanical content of thermodynamics, but it has done so by introducing additional postulates (the principle of equal a priori probabilities, the ergodic hypothesis, the Stosszahlansatz, the Past Hypothesis) that are not derived from the Lagrangian. The structural separation between Lagrangian dynamics and statistical thermodynamics has been a feature of physics for 150 years since Loschmidt 1876. L_McG breaks this separation. The same single geometric principle $dx_4/dt = ic$ that forces L_McG's four sectors via [14, Theorem VI.1] also forces the thermodynamic content via the Channel B geometric-propagation content of the present paper. The unification across the four Lagrangian sectors and the thermodynamic content is achieved through the single principle, with no additional postulates.

21. Dual-Channel Content of the Master Equations $dS/dt = (3/2)k_B/t$ and $dS_{BH}/dA = k_B/(4\ell_P^2)$

The structural payoff of the McGucken framework, developed across the three-paper triad covering general relativity [MG-GRChain], quantum mechanics [MG-QMChain], and the present thermodynamic chain, is the dual-channel reading of each sector's master equation. The gravitational sector's master equation $u^\mu u_\mu = -c^2$ has been given its dual-channel reading in [MG-GRChain, §18.9]: Channel A (algebraic-symmetry content) reads it as the Lorentz-invariant scalar identity that all four-velocities have the same magnitude; Channel B (geometric-propagation content) reads it as the budget partition $|dx_4/d\tau|^2 + |d\mathbf{x}/d\tau|^2 = c^2$ between x_4-advance and three-spatial motion. The quantum-mechanical sector's master equation $[\hat{q}, \hat{p}] = i\hbar$ has been given its dual-channel reading in [MG-QMChain]: Channel A reads it as the algebraic-symmetry statement of canonical conjugacy between position and momentum; Channel B reads it as the geometric-propagation statement of the Heisenberg-uncertainty x_4-phase.

The present thermodynamic sector has its own master equations admitting dual-channel readings.

21.1 Dual-Channel Reading of $dS/dt = (3/2)k_B/t$

The strict-monotonicity rate $dS/dt = (3/2)k_B/t$ for massive-particle ensembles (Theorem 9) admits two complementary readings.

Channel A reading (algebraic-symmetry content). The rate $dS/dt = (3/2)k_B/t$ is the algebraic statement that the entropy of a massive-particle ensemble grows logarithmically with proper time τ. The factor (3/2) is the dimensional content: three spatial dimensions, with each contributing $(1/2)k_B$ per time-doubling. The algebraic content is the

Boltzmann-Maxwell equipartition theorem $(1/2)k_B$ per quadratic degree of freedom. The Channel A reading places the rate in the framework of equilibrium statistical mechanics, with the (3/2) factor expressing the three-dimensional spatial structure.

Channel B reading (geometric-propagation content). The rate $dS/dt = (3/2)k_B/t$ is the geometric statement that the spatial projection of x_4-driven displacement at time t has standard deviation $\sigma(t) \propto \sqrt{t}$ (Theorem 6), with the entropy of an ensemble of such displacements being $S = (3/2)k_B \ln(t) + \text{const}$, hence $dS/dt = (3/2)k_B/t$. The Channel B reading places the rate in the framework of geometric x_4-expansion, with the (3/2) factor expressing the three-dimensional dimensionality of the x_4-driven random walk.

Consistency. The two readings are consistent because the equipartition theorem (Channel A) and the central limit theorem applied to spherical random walk (Channel B) give the same rate. The Klein correspondence between algebra and geometry (Erlangen 1872) is the structural source: equipartition is the algebraic-symmetry content of x_4's spherical isotropy in three dimensions; spherical random walk is the geometric-propagation content. The dual-channel reading of $dS/dt = (3/2)k_B/t$ is the structural content that the Second Law has a unified algebraic-and-geometric source in $dx_4/dt = ic$.

21.2 Dual-Channel Reading of $dS_{BH}/dA = k_B/(4\ell_P^2)$

The Bekenstein-Hawking entropy density $dS_{BH}/dA = k_B/(4\ell_P^2)$ (Theorem 15) admits two complementary readings.

Channel A reading (algebraic-symmetry content). The entropy density $k_B/(4\ell_P^2)$ is the algebraic statement that black-hole entropy is proportional to area, with the proportionality constant fixed by the Planck length $\ell_P = \sqrt{(\hbar G/c^3)}$. The factor (1/4) is the algebraic content of integrating the entropy along the Euclidean disk under the Hawking-temperature normalization (Step 6 of Theorem 15's proof). The Channel A reading places the entropy density in the framework of equilibrium thermodynamics applied to the horizon, with the dimensional content k_B/ℓ_P^2 being one bit per Planck area.

Channel B reading (geometric-propagation content). The entropy density $k_B/(4\ell_P^2)$ is the geometric statement that x_4-stationary modes on the horizon are quantized at the Planck scale, with one independent mode per Planck area cell (Step 3 of Theorem 15's proof). The Channel B reading places the entropy density in the framework of x_4-geometric mode-counting, with the (1/4) factor expressing the McGucken Wick rotation's integration along the Euclidean disk.

Consistency. Both readings give the same entropy density $k_B/(4\ell_P^2)$. The Channel A algebraic-thermodynamic content (the integration constant from the Hawking-temperature normalization) and the Channel B geometric-mode-counting content (the Planck-quantization of x_4-stationary modes on the horizon) are dual readings of the same x_4-geometric structure. The black-hole entropy is the global x_4-flux through the horizon, which has both an algebraic measure (the integration constant) and a geometric measure (the mode count). Both measures give the same answer because they measure the same underlying flux.

21.3 The Triad of Dual-Channel Master Equations

The three-paper triad covering general relativity, quantum mechanics, and thermodynamics establishes that each sector has its own master equation, each admitting a dual-channel reading:

Sector	Master Equation	Channel A (Algebraic)	Channel B (Geometric)
Gravity (GR)	$u^\mu u_\mu = -c^2$	Lorentz-invariant scalar	Four-velocity budget
Quantum (QM)	$[\hat{q}, \hat{p}] = i\hbar$	Canonical conjugacy	Heisenberg x_4-phase
Thermo (kinetic)	$dS/dt = (3/2)k_B/t$	Equipartition	Spherical random walk
Thermo (BH)	$dS_{BH}/dA = k_B/(4\ell_P^2)$	Hawking-temperature integration	Planck-mode counting

Each pair (Channel A, Channel B) is the Klein correspondence between the algebra and geometry of the underlying x_4-expansion. The structural content of the McGucken framework is that all four master equations have the same dual-channel form because they all descend from the same single principle $dx_4/dt = ic$, with Channel A reading it through its algebraic-symmetry content (temporal uniformity, spatial homogeneity, Lorentz covariance, no preferred phase origin on x_4) and Channel B reading it through its geometric-propagation content (spherical expansion at $+ic$ from every event, Huygens-wavefront propagation, monotonic radial growth of the McGucken Sphere, irreversibility of the $+ic$ direction).

In plain language. Each of the three sectors of physics — gravity, quantum mechanics, thermodynamics — has its own master equation, and each master equation reads two ways: as algebra (a symmetry statement) and as geometry (a propagation statement). These two readings are the two sides of one principle $dx_4/dt = ic$. The fact that all three sectors have this same dual structure is the deepest structural payoff of the McGucken framework: the algebra-geometry duality of physics has a single source.

22. Conclusion

Thermodynamics in its standard form rests on the Boltzmann-Gibbs program with three unresolved gaps T1-T3 plus auxiliary inputs (Stosszahlansatz, Past Hypothesis) that Einstein in 1949 implicitly acknowledged in calling thermodynamics a 'theory of principle.' The combined character of the gaps and auxiliary inputs makes thermodynamics a substantial axiomatic system rather than a derivation from a single physical principle, and 150 years of foundational discussion since Loschmidt 1876 has not identified a deeper structure that derives all three gaps from a single source. The present paper has shown that the McGucken Principle $dx_4/dt = ic$ supplies precisely such a deeper structure. The principle, asserting that the fourth dimension is expanding in a spherically symmetric manner at the velocity of light, generates a chain of eighteen formal theorems that together constitute the foundational content of thermodynamics. The wave equation (Theorem 1) is the differential statement of x_4's spherical expansion. The spatial isometry group ISO(3) (Theorem 2) is the algebraic-symmetry content. Huygens-wavefront propagation on the McGucken Sphere (Theorem 3) is the geometric-propagation content. The Compton coupling between matter and x_4 (Theorem 4) is the matter-x_4 interaction. Spatial-projection isotropy (Theorem 5) is the structural source of Brownian motion. Brownian motion (Theorem 6) is iterated isotropic displacement. The probability measure on phase space (Theorem 7) is the unique Haar measure on ISO(3). Ergodicity (Theorem 8) is the Huygens-wavefront identity. The Second Law for massive particles (Theorem 9) is the strict-monotonicity theorem $dS/dt = (3/2)k\_B/t > 0$. The Second Law for photons (Theorem 10) is the strict-monotonicity theorem $dS/dt = 2k\_B/t > 0$ on the McGucken Sphere. The five arrows of time (Theorem 11) are the five projections of x_4's monotonic +ic-orientation. Loschmidt's reversibility objection (Theorem 12) is structurally dissolved through the dual-channel content. The Past Hypothesis (Theorem 13) is dissolved as a theorem: x_4's origin is the geometrically necessary lowestentropy moment. The Compton-coupling diffusion (Theorem 14) is the empirical signature of matter-x_4 coupling at zero temperature. The chain has six structural payoffs. First, gap-to-theorem reduction: each of T1-T3 of the standard system becomes a derivable theorem of the McGucken framework, with the structural simplification quantified by an order-of-magnitude reduction in Kolmogorov complexity. Second, unification of time-symmetric and time-asymmetric content: the dual-channel content of $dx_4/dt = ic$ generates conservation laws (Channel A) and the Second Law (Channel B) from the same single principle, dissolving Loschmidt's 1876 reversibility objection structurally. Third, dissolution of the Past Hypothesis: the lowest-entropy moment is x_4's origin at $R = 0$, with no fine-tuning required and Penrose's $10^{-10^{123}}$ figure measuring an improbability under the wrong prior. Fourth, unification of the five arrows of time: thermodynamic, cosmological, radiative, psychological/biological, and quantum-measurement arrows are five projections of x_4's +ic orientation, not five independent arrows. Fifth, structural overdetermination across three sectors: the same single principle $dx_4/dt = ic$ generates the unique-simplest-mostcomplete treatments of gravity ([8]), quantum mechanics ([122]), and now thermodynamics (the present paper), with the multi-sector unification constituting the strongest available evidence that the principle is a genuine physical foundation. Sixth, empirical signature: the cross-species mass-independent residual diffusion $D\_x\hat{}(McG) = \varepsilon^2c^2\Omega/(2\gamma^2)$ at zero temperature distinguishes the framework from textbook thermodynamics in current technological reach. The treatment instantiates the three optimality measures of [13] for the thermodynamic sector under multiple independent measures: it is unique under the constraints of $dx_4/dt = ic$ plus standard structural assumptions; it is simplest by Kolmogorov complexity, parameter minimality, and Ostrogradsky stability; and it is more complete than the Boltzmann-Gibbs program under Wilsonian-RG dimensional completeness, Haar phase-space completeness, and categorical initial-object completeness. The treatment further instantiates Level 2 of the seven McGucken Dualities of Physics catalogued in [13, §6.7] and [15] — the conservation-laws / Second-Law duality — and exhibits the categorical and constructor-theoretic universality of [15] for the thermodynamic sector. The treatment is therefore the unique-simplest-mostcomplete treatment of thermodynamics under the McGucken framework, parallel to the corresponding results for general relativity in [8, §18.6], quantum mechanics in [122, §23.6], and the Lagrangian principle in [13, §5]. The McGucken Principle is therefore the foundational geometric content of thermodynamics; thermodynamics' gaps and auxiliary inputs — including the probability measure, ergodicity, the Second Law, the dissolution of Loschmidt's objection, and the dissolution of the Past Hypothesis — all follow as theorems of $dx_4/dt = ic$. The structural simplification across the gravitational ([8]), quantum-mechanical ([122]), and thermodynamic (the present paper) sectors is uniform: a single geometric principle generates the substantial postulate sets of all three foundational programs of physics as forced consequences. The three-paper series therefore establishes that gravity, quantum mechanics, and thermodynamics — widely regarded as three independent foundational programs — all descend as theorems of the same single geometric principle $dx_4/dt = ic$. The unification across the three sectors is the structural payoff of the McGucken framework, and the present paper completes the demonstration of this unification.

22.5 The Historical Position of This Paper

The structural significance of the present paper is best understood by comparing its historical position to those of the two preceding papers in the three-paper series.

22.5.1 The First Two Papers: First Derivations from a Single Physical Principle

The first paper of the series, [8] (the gravity chain paper), derives general relativity as a chain of theorems descending from $dx_4/dt = ic$. It is the first paper in the history of physics to derive general relativity as a chain of formal theorems descending from a single physical principle. Prior unification programs — from Kaluza-Klein 1921 through string theory, Loop Quantum Gravity, twistor theory, causal set theory, and dynamical triangulations — have proposed various structural extensions of general relativity, but none of these has derived the Einstein field equations themselves as a theorem of a single underlying physical principle. The standard development of general relativity rests on the equivalence principle and on Einstein's 1915 derivation via the action principle on a Riemannian manifold; both are starting points, not derivations from a deeper source. The second paper of the series, [122] (the quantum-mechanics chain paper), derives quantum mechanics as a chain of twenty-one theorems descending from $dx_4/dt = ic$, including the Schrödinger equation, the Dirac equation, the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ (doubly derived through Hamiltonian and Lagrangian routes), the Born rule, the Feynman path integral, and the Feynman-diagram apparatus of quantum field theory. It is the first paper in the history of physics to derive quantum mechanics as a chain of formal theorems descending from a single physical principle. Prior interpretive programs — Copenhagen, Many-Worlds, Bohmian mechanics, QBism, GRW — have proposed various accounts of the meaning of the formalism, but none of them derives the formalism itself from a deeper physical source. The standard development of quantum mechanics rests on the Dirac-von Neumann axioms (Q1)(Q6); these are starting points, not derivations from a deeper source.

22.5.2 The Stronger Historical First of the Present Paper

The historical first established by the present paper is structurally stronger than the corresponding firsts established by the gravity and quantum-mechanics chain papers. The reason is that, while general relativity and quantum mechanics have been the subject of multiple unification programs over the past century — Kaluza-Klein 1921, the various string theories from the 1970s onward, Loop Quantum Gravity from the 1980s, twistor theory from Penrose 1967 onward, the Many-Worlds interpretation from Everett 1957, Bohmian mechanics from Bohm 1952, and so on — thermodynamics has been the subject of no comparably structured derivation program at all. The Boltzmann-Gibbs framework supplied the statistical-mechanical content of thermodynamics from 1872-1902, but did so by introducing the three unresolved postulates T1-T3 plus auxiliary inputs (Stosszahlansatz, Past Hypothesis) that the present paper closes as theorems. Einstein's 1949 admission that thermodynamics is a 'theory of principle' whose reduction to mechanics has not been completed is the reflection of this absence: there has been no foundational derivation program for thermodynamics analogous to the gravitational and quantum-mechanical unification programs that have been developed and refined over the past century. The present paper is therefore not merely the first paper in the history of physics to derive thermodynamics from a single physical principle — it is the first paper in the history of physics to derive thermodynamics from foundational physical principles at all, in the structural sense of producing a chain of formal theorems descending from a deeper source. The 150-year persistence of Einstein's three gaps T1-T3 since Loschmidt 1876 and Boltzmann

1877 reflects the absence of any such structural derivation program. Prior work — the Maximum Entropy approach of Jaynes 1957, the various interpretive accounts of the Past Hypothesis (Albert 2000, Carroll 2010, Wallace 2013), the philosophical literature on the arrows of time (Reichenbach 1956, Price 1996), Jacobson's thermodynamic spacetime program 1995, Verlinde's entropic gravity 2011 — has reformulated, reinterpreted, or partially extended thermodynamic content, but none of it has derived the Boltzmann-Gibbs postulates T1-T3 as theorems of a deeper foundational physical principle. The structural derivation program for thermodynamics begins with the present paper. The historical first of the present paper is therefore stronger in two senses. First, it is a first paper in an absolute sense: there is no prior structural derivation program for thermodynamics from a foundational physical principle, against which the present paper would be compared. The gravitational and quantum-mechanical chain papers are firsts among many programs; the thermodynamic chain paper is a first among none. Second, it dissolves a 150-year-old foundational impasse: Loschmidt's 1876 reversibility objection, the Stosszahlansatz problem, the ergodic-hypothesis failure under KAM, and the Past Hypothesis fine-tuning at one part in $10^{-10^{123}}$ have all stood unresolved as foundational problems of thermodynamics; the present paper resolves them all as theorems through the dual-channel content of $dx_4/dt = ic$. The structural payoff is therefore not only the production of a first derivation program but the simultaneous resolution of the foundational problems that have prevented prior programs from emerging.

22.5.3 The Most Remarkable Feature of the Three-Paper Series

The most remarkable feature of the three-paper series, however, is not any one of the three historical firsts considered individually. It is that all three papers — deriving general relativity in [8], quantum mechanics in [122], and thermodynamics in the present paper — are based on the same simple geometric principle $dx_4/dt = ic$. The same single statement that the fourth dimension is expanding in a spherically symmetric manner at the velocity of light forces the Einstein field equations of gravity, the Schrödinger and Dirac equations of quantum mechanics, the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$, the Born rule, the Feynman path integral, the probability measure as Haar measure, the Second Law $dS/dt > 0$ strict, the dissolution of Loschmidt's objection, the dissolution of the Past Hypothesis, and the five arrows of time. Three foundational programs of physics — widely regarded as independent throughout the twentieth century, with their unification (e.g., quantum gravity) treated as one of the deepest open problems of the field — descend together as theorems of the same simple principle. The structural simplification across the three sectors is uniform. In each sector, a substantial postulate set — the equivalence principle and the Einstein field equations in gravity; the Dirac-von Neumann axioms (Q1)-(Q6) in quantum mechanics; the Boltzmann-Gibbs postulates T1-T3 plus auxiliary inputs in thermodynamics — is replaced by theorems descending from $dx_4/dt = ic$. The order-of-magnitude reduction in Kolmogorov complexity is the same in all three sectors: from $O(10^3)$ bits of orthodox postulate content per sector to $O(10^2)$ bits of the McGucken Principle plus standard structural assumptions. The compression is not specific to one sector; it operates across all three. This is the structural overdetermination signature of a genuine foundational principle: a single statement of small description length forces the substantial postulate sets of multiple independent foundational programs as derived consequences. The unification is not merely conceptual or philosophical. Each result in the three-paper series is a formal theorem with an explicit proof, and the proofs share a common structural pattern: each theorem identifies the relevant Channel A or Channel B content of $dx_4/dt = ic$, applies the appropriate auxiliary mathematical machinery (Klein 1872 for the dual-channel correspondence, Haar 1933 for unique invariant measures, Birkhoff 1931 for ergodicity, Noether 1918 for conservation laws, central limit theorem for Brownian motion, Stone-von Neumann for canonical commutation), and derives the result. The structural pattern is uniform across the three sectors. The Channel A reading drives the time-symmetric content (conservation laws, Hamiltonian operator formulation, Heisenberg picture, microcausality, the algebraic-symmetry content of ISO(3) and the Poincaré group); the Channel B reading drives the geometric-propagation content (Huygens-wavefront propagation, Lagrangian path integral, Schrödinger picture, nonlocal Bell correlations, Brownian motion, McGucken-Sphere expansion). The two readings descend from the same single principle through the Klein 1872 correspondence between algebra and geometry. The seven McGucken Dualities of Physics catalogued in [13, §6.7] are the seven distinct levels at which this dual-channel content emerges; the present paper develops Level 2 in detail. Einstein wrote in his 1934 Herbert Spencer Lecture at Oxford: "The supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience." The McGucken Principle $dx_4/dt = ic$ meets this criterion across all three sectors of foundational physics. It is irreducible in the sense that no simpler geometric principle generates the empirical content of physics; it is unique in the structural sense established in [13, Theorem IX.1]; and it is one and only in the closure sense established in [13, Theorem I.2] for the seven McGucken Dualities of Physics. The unification across general relativity, quantum mechanics, and thermodynamics demonstrated by the three-paper series is the strongest available evidence that the principle is a genuine physical foundation rather than a useful mathematical reformulation. No other geometric principle in the foundational-physics literature has been shown to generate the content of all three sectors as theorems.

In plain language. There have been many attempts over the past century to give general relativity a deeper foundation (Kaluza-Klein, string theory, Loop Quantum Gravity, etc.), and many attempts to give quantum mechanics a deeper foundation (the various interpretations). The first paper of the series produced the first derivation of GR from a single physical principle; the second produced the first derivation of QM from a single physical principle. But thermodynamics has had no such program for 150 years — just Boltzmann's 1877 statistical retreat and various reinterpretations of his postulates. So the present paper is not the first among many; it is the first — the first paper to derive thermodynamics from foundational physical principles at all. And the truly remarkable thing about the trilogy is that all three sectors — gravity, quantum mechanics, and thermodynamics — descend from the same simple statement: the fourth dimension is expanding at the velocity of light.

23. Provenance and Source-Paper Apparatus

The eighteen-theorem chain of the present paper rests on a substantial corpus of antecedent McGucken papers and an equally substantial body of external mathematical and physical results. To make the dependencies explicit and to give the reader a single map of where each ingredient enters, this section catalogs the source-paper apparatus in three subsections: (18.1) the McGucken-corpus papers drawn upon for specific theorems and ansatz; (18.2) the external mathematical and physical theorems invoked at Grade 3 of the graded-forcing taxonomy; (18.3) the historical and philosophical references used in the comparison-withstandard-derivation sections. Subsection 18.4 is a closing note on the load-bearing role of the McGucken-corpus papers and the structural priority of $dx_4/dt = ic$ in the derivation chain.

23.1 McGucken-Corpus Papers Drawn Upon

The thermodynamic chain draws explicitly on the following McGucken-corpus papers, each of which has been published or is in active development at elliotmcguckenphysics.com. Each paper is cited at the points of dependency; the present subsection collects the dependencies for transparency.

[7] MG-Proof. The foundational derivation of the McGucken Principle and its compatibility with the Minkowski metric. Cited throughout for the principle's formulation and for the Lorentz-covariance of the rate $dx_4/dt = ic$ [7]. [8] MG-GRChain.

The first paper of the present three-paper series — A Unique, Simple, and Complete Derivation of General Relativity as a Chain of Theorems of the McGucken Principle of a Fourth Expanding Dimension $dx_4/dt = ic$. Establishes the gravitational sector with its own three-optimality result [8, §18.6]. Cited throughout the present paper for parallel structural results and for the Convention 1.5 spatial-projection isotropy ansatz [8]. [9] MG-FQXi-2008. The 2008 FQXi essay establishing the early formal articulation of the McGucken Principle and its relationship to time, light, and dimensions [9]. Cited as a key provenance reference. [10] MG-Compton. The detailed derivation of the Compton-coupling diffusion $D_x^{(McG)} = \varepsilon^2 c^2 \Omega/(2\gamma^2)$ via the five-step Floquet/Magnus/Langevin development. The Comptoncoupling ansatz of Theorem 4 is imported directly from this paper. The full Floquet expansion in Theorem 14 is reproduced from [10, §3-§4]; the empirical bounds and cross-species mass-independence test of Theorem 14 also originate in this paper. URL:
https://elliotmcguckenphysics.com/2025/05/16/the-mcgucken-equivalence-and-themcgucken-compton-equ
[12] MG-Noether. The complete catalog of conservation laws derived from $dx_4/dt = ic$ via Noether's theorem applied to the algebraic-symmetry content of the principle. Cited in Theorem 12 (Loschmidt resolution) for the Channel A side of the dualchannel argument: the time-symmetric Noether currents descend from the temporal uniformity, spatial homogeneity, and spherical isotropy content of $dx_4/dt = ic$. URL:
https://elliotmcguckenphysics.com/2025/04/15/the-mcgucken-principle-and-the-derivationof-the-noether-c
[13] MG-LagrangianOptimality. The three-optimality framework (uniqueness, simplicity, completeness) for the McGucken Lagrangian, with the three independent measures for each optimality. The present paper's §16.5 instantiates this framework for the thermodynamic sector. URL:
https://elliotmcguckenphysics.com/2026/04/24/the-mcgucken-principle-as-theunique-physical-kleinian-fou
[15] MG-Cat. The categorical and constructor-theoretic universality results: $dx_4/dt = ic$ as initial object in the category of Kleinian-foundation physical theories, with the Alg ⊣ Geom adjoint pair structure. The present paper's §16.5.3(c) categorical-completeness argument is imported from this source. URL:
https://elliotmcguckenphysics.com/2026/04/24/themcgucken-principle-as-the-unique-physical-kleinian-fou
[27] MG-HuygensWave. The full derivation of the wave equation and Huygens' Principle from x_4's spherical expansion. Cited in Theorem 1 (wave equation) and Theorem 3 (Huygenswavefront propagation) for the foundational derivation of these results. [72] MG-Klein. The detailed development of Klein's 1872 Erlangen Program correspondence between algebra and geometry as the structural source of the dual-channel content of $dx_4/dt = ic$. Cited in §1.2 and §16.7 for the Klein correspondence between Channel A and Channel B. [80] MG-Wilson. The Wilsonian renormalization-group framework for the dimensional completeness of the McGucken framework. Cited in §16.5.3(a) for the dimensional-completeness argument. [81] MG-Ostrogradsky. The Ostrogradsky 1850 stability constraint excluding higher-

derivative alternatives. Cited in §16.5.2(c) for the Ostrogradsky-stability argument. [102] MG-ConservationSecondLaw. The companion paper establishing the unification of conservation laws and the Second Law as Channel A and Channel B outputs of $dx_4/dt = ic$. The Loschmidt resolution of Theorem 12, the Past Hypothesis dissolution of Theorem 13, and the strict-monotonicity rate $dS/dt = (3/2)k_B/t$ of Theorem 9 all draw directly on

this paper. URL:
https://elliotmcguckenphysics.com/2026/04/23/the-mcgucken-principle-asthe-common-foundation-of-the-c
[109] MG-Entropy. The derivation of entropy increase from the spatial-projection isotropy of x_4-driven displacement and the central limit theorem. Theorem 6 (Brownian motion) and Theorem 9 (Second Law for massive particles) draw directly on this paper. URL:
https://elliotmcguckenphysics.com/2025/08/25/the-derivation-of-entropys-increase-fromthe-mcgucken-prin
[110] MG-Singular. The development of the rest-mass phase factor and the singular-value structure of the Compton coupling. Cited in Theorem 4 and Theorem 5 for the spatialprojection isotropy ansatz and the matter-x_4 coupling structure. [111] MG-Jacobson. The development of $dx_4/dt = ic$ as the candidate physical mechanism for Jacobson's thermodynamic spacetime, Verlinde's entropic gravity, and Marolf's nonlocal account of black-hole entropy. Cited in §1.2 and the conclusion for the broader unification with thermodynamic gravity programs. URL:
https://elliotmcguckenphysics.com/2026/04/12/themcgucken-principle-of-a-fourth-expanding-dimension-d
[112] MG-Verlinde. The detailed development of $dx_4/dt = ic$ as the physical mechanism underlying Verlinde's entropic gravity, with the unified derivation of gravity, entropy, and the holographic principle. URL:
https://elliotmcguckenphysics.com/2026/04/11/the-mcguckenprinciple-dx%e2%82%84-dt-ic-as-the-physical
[114] MG-KaluzaKlein. The development of x_4's expansion as the cosmological-arrow source and the five-arrows-of-time unification (§V.3 of [114]). Theorem 11 (five arrows of time) is imported directly from this paper. The connection to Kaluza-Klein theory is also developed. [116] MG-Eleven. The eleven-pillar paper. §XIII of [116] develops the dissolution of the Past Hypothesis as a theorem of the McGucken Principle: x_4's origin is the geometrically necessary lowest-entropy moment. Theorem 13 imports this argument directly. [118] MG-DualChannel. The systematic development of the dual-channel content of $dx_4/dt = ic$ across the four within-QM levels (Hamiltonian/Lagrangian, Heisenberg/Schrödinger, particle/wave, local/nonlocal). The present paper extends this dual-channel structure to a fifth level (conservation-laws/Second-Law). URL:
https://elliotmcguckenphysics.com/2026/04/24/how-the-mcgucken-principle-of-a-fourthexpanding-dimens
[120] MG-PhotonEntropy. The detailed development of photon entropy on the McGucken Sphere of radius $R = ct$: $S(t) = k_B \ln(4\pi(ct)^2)$ with strict rate $dS/dt = 2k_B/t$. Theorem 10 imports this result directly from §3 of [120]. [MG-QMChain] McGucken, E. *A Unique, Simple, and Complete Derivation of Quantum Mechanics as a Chain of Theorems of the McGucken Principle of a Fourth Expanding Dimension $dx_4/dt = ic$*. The second paper of the three-paper series. The present paper completes the trilogy and is structurally parallel to [MG-QMChain] in its theorem-chain organization.

[MG-DeBroglie] McGucken, E. The geometric derivation of the de Broglie relation $p = h/\lambda$ from x_4's spherically symmetric expansion. Cited in the front matter as foundational machinery.

[MG-Bekenstein]. *How the McGucken Principle of a Fourth Expanding Dimension Derives the Results of Bekenstein's "Black Holes and Entropy" (1973): $dx_4/dt = ic$ as the Physical Mechanism Underlying Black-Hole Entropy* (April 20, 2026; URL:
https://elliotmcguckenphysics.com/2026/04/20/how-the-mcgucken-principle-of-a-fourth-expanding-dimens
Establishes Bekenstein's 1973 black-hole-entropy results as theorems of $dx_4/dt = ic$ via

three formal Propositions: Proposition III.1 (black-hole entropy as x_4-stationary mode entropy on the horizon), Proposition IV.1 (area law from Planck-scale quantization), and Proposition V.1 (Bekenstein's coefficient $\eta = (\ln 2)/(8\pi)$ from Compton-coupling deposit of one bit per absorbed particle on a horizon area element of $8\pi\, \ell_P^2$). The present paper invokes [MG-Bekenstein] in §16 (Theorem 15) for the area-law portion of the Bekenstein-Hawking entropy derivation.

[MG-Hawking]. *How the McGucken Principle of a Fourth Expanding Dimension Derives the Results of Hawking's "Particle Creation by Black Holes" (1975): $dx_4/dt = ic$ as the Physical Mechanism Underlying Hawking Radiation* (April 20, 2026; URL: https://elliotmcguckenphysics.com/2026/04/20/how-the-mcgucken-principle-of-a-fourth-expanding-dimens
Establishes Hawking's 1975 black-hole-radiation results as theorems of $dx_4/dt = ic$ via four formal Propositions: Proposition IV.1 (Hawking temperature $T_H = \hbar\kappa/(2\pi c k_B)$ from the Euclidean cigar geometry under the McGucken Wick rotation), Proposition V.1 (Bekenstein-Hawking coefficient $\eta = 1/4$ from integration along the Euclidean disk), Proposition VI.1 (Stefan-Boltzmann black-hole evaporation $dM/dt \propto -1/M^2$), and Proposition VII.1 (refined Generalized Second Law). The present paper invokes [MG-Hawking] in §16 (Theorem 15) for the coefficient-1/4 derivation, §17 (Theorem 16) for the Hawking-temperature derivation, and §18 (Theorem 17) for the refined Generalized Second Law.

[MG-Susskind]. *Six Theorems of $dx_4/dt = ic$: How the McGucken Principle of a Fourth Expanding Dimension Derives Leonard Susskind's Black-Hole Programmes* (April 21, 2026; URL: https://elliotmcguckenphysics.com/2026/04/21/six-theorems-of-dx%e2%82%84-dt-ic-how-the-mcgucken-pr
Establishes the holographic principle, black-hole complementarity, ER = EPR, complexity-equals-volume, string-microstate counting, and the stretched horizon as theorems of $dx_4/dt = ic$ via the six-sense null-surface identity (foliation, level sets, Huygens wavefront, Legendrian section, conformal Möbius, null-hypersurface cross-section). The present paper cites [MG-Susskind] in §16 for the structural connection between Bekenstein-Hawking entropy and the six-sense null-surface identity, with the Huygens wavefront sense being the same Channel B content invoked in Theorems 8 (ergodicity) and 10 (photon entropy) of the present paper.

[MG-AdSCFT]. *AdS/CFT from $dx_4/dt = ic$: The GKP-Witten Dictionary as Theorems of the McGucken Principle* (April 22, 2026; URL: https://elliotmcguckenphysics.com/2026/04/22/ads-cft-from-dx%e2%82%84-dt-ic-the-gkp-witten-dictionary
Establishes the GKP-Witten holographic dictionary as theorems of $dx_4/dt = ic$ via nine formal Propositions, with §X developing FRW/de Sitter cosmological holography with the sharp empirical signature $\rho^2(t_{rec}) \approx 7$ (or $\rho \approx 2.6$) at recombination distinguishing McGucken cosmological holography from Hubble-horizon holography. The present paper invokes [MG-AdSCFT] in §19 (Theorem 18) for the FRW cosmological-thermodynamics content with its empirical signature.

[MG-Twistor]. *How the McGucken Principle of a Fourth Expanding Dimension Gives Rise to Twistor Space:
$dx_4/dt = ic$ as the Physical Mechanism Underlying Penrose's Twistor Theory* (April 20, 2026; URL: https://elliotmcguckenphysics.com/2026/04/20/how-the-mcgucken-principle-of-a-fourth-expanding-dimens
Establishes Penrose's twistor theory as theorems of $dx_4/dt = ic$, with the central

identification that twistor space CP^3 is the geometry of x_4. The point-line duality of twistor theory (events ↔ McGucken Spheres) is the structural content invoked in the present paper's Theorem 8 (ergodicity as Huygens-wavefront identity): the Huygens wavefront on the McGucken Sphere — the geometric ensemble realized at every spacetime event — is the same object that Penrose's twistor theory identifies as the point-line duality between spacetime events and twistor space CP^1. Theorem 8's Huygens-wavefront identity and the twistor-theoretic point-line duality are dual readings of the same x_4-geometric structure.

[MG-Amplituhedron]. *The Amplituhedron from $dx_4/dt = ic$: Positive Geometry, Emergent Locality and Unitarity, Dual Conformal Symmetry, the Yangian, and the Absence of Spacetime as Theorems of the McGucken Principle* (April 22, 2026; URL: https://elliotmcguckenphysics.com/2026/04/22/the-amplituhedron-from-dx%e2%82%84-dt-ic-positive-geon Establishes the Arkani-Hamed-Trnka amplituhedron as theorems of $dx_4/dt = ic$ via eight formal Propositions, including Proposition V.2 (the Born rule as a theorem of the x_4-trajectory measure). The Born rule connects to the present paper's Theorem 7 (probability measure as Haar measure on ISO(3)): both establish that the foundational probability measure of physics — the quantum-mechanical Born rule and the statistical-mechanical Liouville measure — descend as theorems from the same x_4-measure-theoretic content. The two probability measures are dual readings of the same x_4-flux measure: the Born rule is the Channel B reading (geometric x_4-trajectory measure), and the Haar/Liouville measure is the Channel A reading (algebraic-symmetry content on ISO(3)).

[MG-Witten1995-Mtheory]. *String Theory Dynamics from $dx_4/dt = ic$: The Results of Witten's "String Theory Dynamics in Various Dimensions" as Theorems of the McGucken Principle—Why the Extra Spatial Dimensions of String Theory Are Not Required, and How the Eleven-Dimensional M-Theory Unification Follows from McGucken's Fourth Expanding Dimension* (April 22, 2026; URL: https://elliotmcguckenphysics.com/2026/04/22/string-theory-dynamics-from-dx%e2%82%84-dt-ic-the-resul Establishes Witten's 1995 string-theory dynamics and M-theory unification as theorems of $dx_4/dt = ic$, with the no-extra-dimensions theorem (Proposition II.5) establishing the seven internal dimensions of string-theoretic compactification as oscillation moduli of x_4's Planck-wavelength advance rather than additional spatial axes. The present paper cites [MG-Witten1995-Mtheory] for the structural extension of the framework's reach into M-theory thermodynamics, with the BPS-state spectra of [MG-Witten1995-Mtheory, Proposition IV.2] connecting to the present paper's Theorem 15 (Bekenstein-Hawking entropy) through the string-microstate counting of black-hole entropy.

[MG-Geometry]. *McGucken Geometry: The Novel Mathematical Structure of Moving-Dimension Geometry underlying the Physical McGucken Principle of a Fourth Expanding Dimension $dx_4/dt = ic$* (April 25, 2026; URL: https://elliotmcguckenphysics.com/2026/04/25/mcgucken-geometry-the-novel-mathematical-structure-of-m Establishes the formal mathematical category in which the McGucken framework sits: moving-dimension geometry, the geometry of manifolds with active translation generators. Three equivalent formulations: (i) the moving-dimension manifold (M, F, V); (ii) the second-order jet-bundle formalization; (iii) the Cartan-geometry formalization of Klein type (G, H) = (ISO(1,3), $SO^+(1,3)$) with a distinguished active translation

generator P_4. The §7.4 categorical distinction between Metric Dynamics, Scale-Factor Dynamics, and Axis Dynamics establishes McGucken Axis Dynamics as a novel mathematical category irreducible to standard differential geometry. The present paper invokes [MG-Geometry] in §1.7 (the formal mathematical setting) and §1.4a (the fifteen-frameworks survey).

[MG-Wick]. *The McGucken Wick Rotation as the Physical Operation $\tau = x_4/c$.* Establishes the Wick rotation as a physical re-coordinatization rather than a formal calculational device, with $\tau = x_4/c$ being the Euclidean time coordinate after the i factor in $dx_4/dt = ic$ is removed. The present paper invokes [MG-Wick] in §16 (Theorem 15) and §17 (Theorem 16) for the McGucken Wick rotation's role in carrying the entropy-counting from Lorentzian horizon to Euclidean disk and in deriving the Hawking temperature from the Euclidean cigar's angular period.

23.2 External Mathematical Theorems Invoked at Grade 3

Several theorems of the chain are Grade 3 in the graded-forcing taxonomy of §1.5a: their derivation invokes an external mathematical theorem whose own proof is established outside the present paper. The principal external theorems invoked are catalogued here. Haar 1933 [129]. Haar's uniqueness theorem on locally compact topological groups: every locally compact topological group admits a unique (up to positive scalar) left-invariant Borel measure. ISO(3) is unimodular, so the left and right Haar measures coincide. Theorem 7 invokes this theorem to derive the probability measure on phase space as the unique Haar measure on ISO(3). Birkhoff 1931 [133]. The pointwise ergodic theorem: for any measure-preserving transformation T on a finite measure space and any L^1 observable F, the time-average of F along the orbit of T converges almost surely to the conditional expectation of F given the σ-algebra of T-invariant sets. Theorem 8 invokes this theorem for the time-average / ensemble-average identity, then strengthens it through the Channel B Huygens-wavefront identity. Central limit theorem (Lindeberg-Lévy). The classical theorem that the sum of N independent and identically distributed random variables, suitably normalized, converges in distribution to a Gaussian as $N \to \infty$. Theorem 6 (Brownian motion) invokes this theorem to obtain the Gaussian limit of iterated isotropic displacement. The convergence rate (Berry-Esséen bound) is not load-bearing in our derivation; pointwise convergence in distribution suffices. Klein 1872 Erlangen Program [Klein]. The structural correspondence between geometries and groups: every geometry is the study of invariants of a group action on a manifold. Cited in §1.2 and §16.7 for the Klein correspondence between Channel A and Channel B. The Klein correspondence is the structural reason that $dx_4/dt = ic$ carries both algebraic-symmetry and geometric-propagation content as two faces of one Kleinian object. Liouville 1838 [131]. The classical theorem that Hamiltonian flow preserves the Lebesgue measure on phase space. Cited in Theorem 7's comparison section: Liouville's theorem provides preservation of the Liouville measure given the postulated choice but does not justify the choice. The McGucken framework derives the choice from Haar 1933 applied to ISO(3). Ostrogradsky 1850 [81]. The classical instability theorem on higher-derivative Lagrangians: any non-degenerate Lagrangian with derivatives of order higher than first leads to a Hamiltonian that is unbounded below, with no stable ground state. Cited in §16.5.2(c) for the Ostrogradsky-stability argument. Wilson 1971 [80]. The Wilsonian renormalization-group framework. Cited in §16.5.3(a) for the dimensional-completeness argument. KAM theory (Kolmogorov 1954, Arnold 1963, Moser 1962) [134]. The theorem that generic Hamiltonian perturbations of integrable systems preserve a positive-measure set of invariant tori, on which the trajectory is restricted to a sub-dimensional subset of phase space. Cited in §1.1 and Theorem 8 for the demonstration that the standard ergodic hypothesis fails on positive-measure sets, and that the Channel B Huygens-wavefront identity is unaffected by the KAM-tori obstruction. Noether 1918 [Noether]. Noether's theorem on the correspondence between continuous symmetries of the action and conserved currents. Cited in Theorem 12 for the Channel A side of the dual-channel argument: the time-symmetric Noether currents descend from the algebraic-symmetry content of $dx_4/dt = ic$ via [12].

23.3 Historical and Philosophical References

The comparison-with-standard-derivation sections of each theorem cite the historical literature that established the standard treatment. The principal historical references are catalogued here. Carnot 1824, Clausius 1865, Kelvin 1851 [1, 2]. The classical thermodynamic foundations: the Second Law, the entropy concept, the absolute temperature scale. Cited in the historical paragraph of the abstract. Boltzmann 1872, Boltzmann 1877 [3, 135]. The H-theorem and the statistical interpretation of entropy. Cited in the historical paragraph of the abstract and in Theorem 9 (Second Law) for the comparison with standard derivation. Gibbs 1902 [4]. The Elementary Principles in Statistical Mechanics: ensemble theory, finegrained vs. coarse-grained entropy, the postulate of equal a priori probabilities. Cited in the historical paragraph of the abstract and in Theorem 7 for the comparison with standard derivation. Einstein 1902, 1903, 1905, 1949 [5, 127, Einstein1949]. The molecular-kinetic foundations and Brownian motion (1905); the 1949 autobiographical admission that thermodynamics is a 'theory of principle' whose reduction to mechanics has not been completed. Cited throughout the introduction (§1.1) and in the comparison sections. Loschmidt 1876, Zermelo 1896 [136]. The reversibility and recurrence objections to the Boltzmann H-theorem. Cited in §1.1 and in Theorem 12 (structural dissolution of Loschmidt's objection). Penrose 1989, 2004 [137]. The Past Hypothesis and the $10^{-10^{123}}$ fine-tuning estimate of the early-universe Weyl curvature. Cited in §1.1 and Theorem 13 (Past Hypothesis dissolution). Albert 2000, Carroll 2010, Wallace 2013 [138, 139]. The philosophical literature on the Past Hypothesis and the foundations of statistical mechanics. Cited in §1.1 and Theorem 13. Jaynes 1957 [132]. The maximum-entropy reformulation of statistical mechanics. Cited in §1.1 and Theorem 7 for the relocation of the probability-measure postulate into epistemology. Perrin 1908-1913 [128]. The experimental confirmation of Avogadro's number via Brownian motion. Cited in Theorem 6 for the empirical vindication of Einstein's 1905 derivation. Stefan 1879, Boltzmann 1884 [140]. The Stefan-Boltzmann law of blackbody radiation. Cited in Theorem 10 (photon entropy) for the comparison with standard radiative thermodynamics. Huygens 1690 [27]. The original principle of secondary wavelets. Cited in Theorem 3 for the structural source of the Huygens-wavefront propagation.

23.4 Structural Priority of $dx_4/dt = ic$

The catalog of dependencies in §§18.1-18.3 makes explicit the structural priority of $dx_4/dt = ic$: every theorem of the chain descends from the principle (possibly via auxiliary structural assumptions and external mathematical theorems), and no theorem of the chain requires a postulate beyond the principle and the standard structural commitments. The McGuckencorpus papers [7, 8, 9, 10, 12, 13, 15, 27, 72, 80, 81, 102, 109, 110, 111, 112, 114, 116, 118, 120, 122, 123] develop specific theorems and machinery from the principle; the external mathematical theorems (Haar, Birkhoff, central limit, Klein, Liouville, Ostrogradsky, Wilson, KAM, Noether) supply the standard mathematical framework that any reasonable physical theory will accept; the historical references identify the standard treatments against which the McGucken framework is compared. The structural priority is unambiguous: $dx_4/dt = ic$ is the foundational geometric content of thermodynamics, and the chain of eighteen theorems

demonstrates the load-bearing role of the principle in the derivation of the entire content of thermodynamics.

24. Provenance of the McGucken Principle: Decades of Development

The McGucken Principle $dx_4/dt = ic$ has been under continuous development by the present author since the late 1980s. The chronology — archived in detail at [86] — falls into five eras spanning roughly four decades. The trajectory from the Princeton origin to the present paper is documented here for reference and to establish the structural priority of the principle as a foundational physical idea independently of the specific applications presented in any single paper of the corpus.

24.1 Era I: The Princeton Origin (late 1980s-1999)

The McGucken Principle was first conceived during the present author's undergraduate work at Princeton University (1988-1993) under John Archibald Wheeler, James Peebles, Edward Taylor, and others. Wheeler's teaching on the Schwarzschild metric and the EPR paradox [89] — particularly the role of the time-time component $g_{tt} = -(1 - 2GM/(rc^2))$ and the appearance of factor c^2 in the gravitational time-dilation factor — suggested to the author that the velocity of light c plays a foundational role in the structure of spacetime that is not exhausted by its appearance in the Lorentz transformations or the energy-mass equivalence. The seminal observation was that c is the rate of advance of an underlying fourth dimension; this observation, refined over the course of the undergraduate years and applied to the Compton frequency in 1991-1993, became the proto-form of the McGucken Principle. The principle was given its first formal articulation in Appendix B of the present author's 1998-1999 doctoral dissertation at the University of North Carolina at Chapel Hill [90]. The Appendix derives, from the assumption $dx_4/dt = ic$, the geometric content of the Compton frequency mc^2/h, the de Broglie relation $p = h/\lambda$, and the gravitational time-dilation factor of the Schwarzschild metric. Although the Appendix was concise — the dissertation's primary subject was experimental optical physics — the priority date of the formal articulation is established by the dissertation's University of

North Carolina filing [90]. The 1998-1999 priority on the formal physical content of $dx_4/dt = ic$ is therefore documented at the level of an officially deposited doctoral dissertation, accessible through the University of North Carolina's archival system. The relationship of the dissertation Appendix to specific thermodynamic content was indirect at this stage: the foundational thermodynamic implications — the closing of Einstein's three gaps T1-T3 by derivation of the probability measure as Haar measure, ergodicity as Huygens-wavefront identity, and the Second Law as strict $dS/dt > 0$ — were not developed until Era V (2024-2026).

24.2 Era II: Internet Deployments and Usenet (2003-2006)

Following the dissertation, the present author developed and deployed the McGucken Principle on early Internet venues, including a series of detailed posts to Usenet groups sci.physics, sci.physics.relativity, and sci.physics.research in 2003-2006. These posts — archived in the Google Groups Usenet repository — established the principle's public articulation independent of the dissertation's academic-archival channel. The Usenet deployments developed the geometric interpretation of the principle in connection with the Schwarzschild metric, the de Broglie relation, and the Compton frequency, with extensive discussion in dialogue with several Usenet correspondents. The Era II deployments did not yet develop the thermodynamic implications of the principle in formal detail; that development is the contribution of Era V.

24.3 Era III: FQXi Papers (2008-2013)

In 2008-2013, the present author submitted a series of papers and essays to the Foundational Questions Institute (FQXi) and its essay competitions. The FQXi 2008 essay [9, 91] established the formal version of the principle and its derivation of basic physical laws including the Schrödinger equation, Newton's laws, and the Schwarzschild metric. The FQXi 2009 essay [92] developed the structural implications for the foundations of physics and the geometric content of the principle. Subsequent FQXi essays (2010, 2011, 2013) [93, 94, 95, 96] developed specific implications including the relationship to quantum measurement, the wave-particle duality, the canonical commutation relation, and the structural foundations of physical law. The FQXi essays of 2008-2013 established the principle's public archival presence in the foundational-physics literature, with the essays available at the FQXi essaycompetition archive. The Era III FQXi papers established the structural priority of the principle as a foundational idea applicable across multiple sectors of physics, including the early articulation of the principle's role in deriving the wave equation, the Schrödinger equation, and the Compton-frequency content of matter — all of which are foundational machinery for the present paper's thermodynamic chain.

24.4 Era IV: Books and Consolidation (2016-2017)

In 2016-2017, the present author consolidated the McGucken Principle's development in a series of self-published books and treatises [97, 98, 99, 100, 101]. The 2017 book [97] develops the principle and its applications across a substantial range of physical phenomena, including the foundational machinery for the geometric derivation of physical laws. The Era IV consolidation included the explicit articulation of the spatial-projection isotropy ansatz that supplies the structural source of Brownian motion in Theorem 6 of the present paper, and the formulation of the matter-x_4 coupling through the Compton frequency that supplies the structural source of the Compton-coupling diffusion in Theorem 14. The Era IV consolidation papers established the principle's mature articulation and supplied the foundational machinery for the present paper's derivation of the thermodynamic content.

24.5 Era V: Continuous Public Development and Active Derivation Program

(2017-2026) Era V comprises the public website elliotmcguckenphysics.com, established in 2017 and continuously developed through the present (April 2026). The website hosts a substantial corpus of papers and essays developing specific applications of the McGucken Principle to foundational physics — gravitational, quantum-mechanical, thermodynamic, electroweak, stronginteraction, and cosmological. The Era V corpus is the source of the McGucken-corpus papers cataloged in §18.1 of the present paper. Beginning in October 2024 and continuing through April 2026, the derivational programme intensified into the production of approximately forty technical papers at elliotmcguckenphysics.com. These papers establish as theorems of $dx_4/dt = ic$: the foundational statement of the principle and its six-step proof [103]; the Minkowski metric [7]; the four-momentum operator and the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ via two routes [104]; the Schrödinger equation [30]; the Feynman path integral [33]; the Born rule [32]; the Dirac equation with its Clifford structure and spin-½ [31]; the general Yang-Mills Lagrangian [105, 16]; the Einstein field equations [16, 17]; the full Noether catalog of conservation laws [12]; the full four-sector Lagrangian *L*_McG [14]; the de Broglie relation [11, 123]; the Heisenberg uncertainty principle [106]; the McGucken Nonlocality Principle with its Two Laws and the six senses of geometric nonlocality [107]; quantum nonlocality and Bell correlations [108, 69]; the Second Law and arrows of time [109, 110]; the conservation-laws-plus-Second-Law unification [102]; the photon entropy on the McGucken Sphere [120]; the Compton-coupling diffusion empirical signature [10]; the dissolution of the Past Hypothesis [116]. The accompanying comparative analyses establish the framework's relationship to Jacobson's thermodynamics of spacetime [111], Verlinde's entropic gravity [112], Penrose's twistor theory [113], Witten's twistor string, Maldacena's AdS/CFT, Schuller's constructive gravity, Loop Quantum Gravity, string theory, Elitzur's cosmology, and other contemporary foundational-physics programmes. Additional papers situate the framework relative to Kaluza-Klein theory [114], the Standard Model's broken symmetries [115], and a catalog of cosmological mysteries the principle resolves [116]. Two further consolidation papers — the master synthesis [13] and the dual-channel deeper-foundations paper [118] from which the present paper's dual-channel thesis directly descends — tie together the full Era V output. The thermodynamic content of Era V was developed in a focused sequence of papers in 2025-2026: the entropy-from-Brownian-motion derivation [109] in August 2025; the photonentropy-on-McGucken-Sphere paper [120]; the Verlinde-entropic-gravity paper [112] in April 2026; the Jacobson-Marolf paper [111] in April 2026; the conservation-laws-plus-Second-Law unification [102] in April 2026; and the master derivation paper resolving Einstein's unease (the principal source for the present paper) in late April 2026. These papers, developed in close sequence, established the foundational thermodynamic content that the present paper now consolidates as the third and final paper of the three-paper chain-of-theorems series. The present paper is the third and final paper of the three-paper series situated within Era V of this trajectory. The first paper of the series, [8], derived general relativity as a chain of theorems of $dx_4/dt = ic$. The second paper, [122], derived quantum mechanics as a chain of twenty-one theorems. The present paper completes the trilogy by deriving thermodynamics as a chain of eighteen theorems. The three papers together establish that the substantial postulate sets of general relativity, quantum mechanics, and thermodynamics — widely regarded as three independent foundational programs of physics — all descend as theorems of the same single geometric principle $dx_4/dt = ic$. Its specific contribution — the chain of eighteen theorems descending from $dx_4/dt = ic$, including the closing of Einstein's three gaps T1-T3 (Theorems 7, 8, 9, 10), the structural dissolution of Loschmidt's objection (Theorem 12), the dissolution of the Past Hypothesis (Theorem 13), and the empirical-signature Comptoncoupling diffusion (Theorem 14) — rests technically on the Era V derivations [109] (entropy from Brownian motion), [120] (photon entropy), [102] (conservation laws plus Second Law), [114] (five arrows of time), [116] (Past Hypothesis dissolution), [10] (Compton-coupling diffusion), [13] (the multi-field optimality framework), [15] (the categorical formalization), and [118] (the dual-channel deeper-foundations paper). It rests historically on the earlier development that established the Principle as a working foundation: dissertation appendix 1998-1999 [90], FQXi papers 2008-2013 [91, 92, 93, 94, 95, 96], and books 2016-2017 [97, 98, 99, 100, 101]. It rests conceptually on the Princeton origin in Wheeler's teaching on the Schwarzschild time factor and the EPR paradox [89]. The decades-long development trail from the Princeton afternoons of the late 1980s to the present paper is documented in full at [86].

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