The Deeper Foundations of Quantum Mechanics: How The McGucken Principle Uniquely Generates the Hamiltonian and Lagrangian Formulations of Quantum Mechanics, Wave/Particle Duality, the Schrödinger and Heisenberg Pictures, and Locality and Nonlocality all from $dx_4/dt = ic$

A Structural Analysis of Why the McGucken Principle, Alone Among Candidate Foundations, Possesses the Dual-Channel Content That Generates the Hamiltonian and Lagrangian Formulations, the Heisenberg and Schrödinger Pictures, and the Wave and Particle Aspects of Quantum Objects as Simultaneous Consequences of a Single Geometric Principle, with a Formal Geometric Proof of Schrödinger-Heisenberg Equivalence, a Comparative Survey of Fifteen Prior Frameworks, and the Completeness Argument for the McGucken Quantum Formalism

Dr. Elliot McGucken

Light Time Dimension Theory

elliotmcguckenphysics.com

April 2026

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"A theory is the more impressive the greater is the simplicity of its premises, the more different are the kinds of things it relates and the more extended the range of its applicability." — Albert Einstein

"Behind it all is surely an idea so simple, so beautiful, that when we grasp it — in a decade, a century, or a millennium — we will all say to each other, how could it have been otherwise?" — John Archibald Wheeler

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Abstract

For the first time in the history of quantum mechanics, it is shown that the Hamiltonian operator formulation and the Lagrangian path-integral formulation of quantum mechanics emerge from the same foundational physical principle — the McGucken Principle, which states that the fourth dimension is expanding in a spherically symmetric manner relative to the three spatial dimensions, $dx_4/dt = ic$. The two principal formulations of quantum mechanics — the Hamiltonian operator formulation

resting on the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$, and the Lagrangian path-integral formulation resting on the Feynman kernel $K = \int \mathcal{D}x \exp(iS/\hbar)$ — have coexisted for ninety-eight years (since Heisenberg 1925 and Dirac 1933 for the operator and Lagrangian analogue formulations respectively, with Feynman's 1948 path integral completing the Lagrangian side) as two equivalent mathematical frameworks for the same physics. Their equivalence is established by Feynman's 1948 derivation of the Schrödinger equation from the path integral and by the Stone–von Neumann theorem's uniqueness of the Schrödinger representation. But their *common origin* — the question of whether both formulations descend from a single deeper physical principle — has remained open through the subsequent nine decades of foundational work, including Nelson's stochastic mechanics, geometric quantization, Hestenes's spacetime algebra, Adler's trace dynamics, Bohmian mechanics, Lindgren-Liukkonen stochastic optimal control, 't Hooft's cellular automata, and the programs of Witten, Arkani-Hamed, Woit, Ashtekar, and Schuller. Each of these programs derives, reinterprets, or reformulates one of the two quantum formulations; none derives both from a single geometric spacetime principle.

The central result of this paper is that the McGucken Principle $dx_4/dt = ic$ — the physical statement that a fourth geometric axis advances at the velocity of light, spherically symmetrically about every spacetime point — forces both the Hamiltonian operator formulation and the Lagrangian path-integral formulation of quantum mechanics as independent theorems, through completely disjoint intermediate structures, with the imaginary unit i and the action quantum $\hbar$ both derived from the Principle along each route rather than postulated as inputs or left as empirical constants. The Hamiltonian route proceeds in five propositions (§II): $x_4 = ict$ forces the Minkowski metric (Proposition H.1); Stone's theorem on translation invariance forces the momentum operator as generator of spatial translations (Proposition H.2); the configuration representation forces $\hat{p} = -i\hbar\partial/\partial q$ (Proposition H.3); direct computation yields $[\hat{q}, \hat{p}] = i\hbar\mathbb{1}$ (Proposition H.4); and the Stone-von Neumann uniqueness theorem closes the representation (Proposition H.5). The Lagrangian route proceeds in six propositions (§III): x_4's spherical expansion from every spacetime point *is* Huygens' principle (Proposition L.1); iterated Huygens expansions compose into the set of all paths (Proposition L.2); each path carries an accumulated x_4-phase which in the non-relativistic limit is $\exp(iS/\hbar)$ (Proposition L.3); the continuum limit gives the full Feynman path integral (Proposition L.4); Gaussian integration of the short-time propagator recovers the Schrödinger equation $i\hbar\partial\psi/\partial t = \hat{H}\psi$ (Proposition L.5); and direct computation from the Schrödinger equation reconstructs $[\hat{q}, \hat{p}] = i\hbar$ (Proposition L.6). The two routes share no intermediate structure except the starting Principle and the final algebraic identity. They are two proofs of the same theorem by disjoint methods — the structural signature of a correct geometric foundation.

§V develops the deeper structural reason *why* $dx_4/dt = ic$ has this property while no other principle does. The answer lies in the principle's *dual-channel content*: the geometric statement $dx_4/dt = ic$ simultaneously specifies two logically distinct pieces of information — an algebraic-symmetry content (the *invariance* of x_4's advance under time translation, space translation, rotation, and Lorentz boost) and a geometric-propagation content (the *spherical symmetry* of x_4's expansion from every spacetime point). The algebraic-symmetry content drives the Hamiltonian route through Stone's theorem and Noether's theorem; the geometric-propagation content drives the Lagrangian route through Huygens' principle and path summation. No prior candidate foundation has had both channels. Classical symplectic geometry has only the algebraic-symmetry channel; classical Lagrangian variational mechanics has only the propagation-variational channel; stochastic dynamics has a diffusion-propagation channel but no symmetry content; Hestenes's spacetime algebra has static geometric content but no dynamical principle. $dx_4/dt = ic$ is the first physical principle whose statement contains both channels simultaneously, because "advancing at rate ic from every point" is the algebraic-symmetry content (a uniform rate, invariant under isometries) and "spherically symmetrically about each point" is the geometric-propagation content (a wavefront structure inheriting Huygens' secondary-wavelet property). The dual-channel content is not a coincidence of wording; it is the structural feature of the principle that makes both quantum formulations theorems of one fact.

§VI surveys fifteen prior frameworks that have sought foundations for quantum mechanics — Feynman's 1948 path integral, Dirac's 1933 transformation theory, Nelson's 1966 stochastic mechanics, Lindgren-Liukkonen 2019 stochastic optimal control, geometric quantization (Kostant 1970, Souriau 1970), Hestenes's geometric algebra (1966–), Adler's trace dynamics (1994–), Bohmian mechanics (1952–), Weinberg's Lagrangian QFT (1995), 't Hooft's cellular automata (2014), Arnold's symplectic mechanics (1978), Ashtekar and loop quantum gravity (1986–), Witten's twistor string (2003), Schuller's constructive gravity (2020), and Woit's Euclidean twistor unification (2021). For each framework, the section examines the framework's own central claim, the derivation structure it employs, the specific point at which it does or does not reach a two-route unification, and the structural reason. The conclusion in every case is the same: the framework derives or reinterprets one of the two quantum formulations — often with great mathematical sophistication and physical insight — but does not reach both formulations as independent theorems of a single geometric spacetime principle. The dual-channel property identified in §V is shown to be absent from each of the fifteen prior foundations.

§VII develops the principle of *structural overdetermination* — the observation that when a single claim is derivable through multiple independent chains from a foundational principle, the claim is confirmed not once but as many times as there are independent routes, and each route illuminates a different structural aspect of the foundation. The two-route derivation of $[\hat{q}, \hat{p}] = i\hbar$ from $dx_4/dt = ic$ is the first structural overdetermination in the history of foundational-physics derivations of quantum mechanics, and its existence is the principal evidence for the McGucken Principle's status as a genuine physical foundation rather than a reframing of existing physics.

§VIII connects the two-route result to the main result of the companion paper "The Unique McGucken Lagrangian" [MG-Lagrangian], in which the full four-sector Lagrangian $\mathscr{L}$_McG of physics — free-particle kinetic, Dirac matter, Yang-Mills gauge, Einstein-Hilbert gravitational — is established as forced by $dx_4/dt = ic$ via a single four-fold uniqueness theorem (Theorem VI.1). The two-route unification developed here deepens §VIII.6 of the Lagrangian paper (the first-of-its-kind structural claim on the canonical commutation relation) by showing that the CCR is not merely derivable from $dx_4/dt = ic$ in one way — it is derivable in *two* mathematically independent ways, through disjoint intermediate structures, with the same i and the same $\hbar$ reached by both chains.

The structural accomplishment is not a single identity derived from a single principle; it is the same identity derived from the same principle through two routes that share no machinery, *together with* the derivation of the wave/particle duality as the dual-channel reading of x_4's advance at the ontological level (§V.6), the formal geometric proof of the Schrödinger-Heisenberg equivalence as two readings of the same physical x_4-advance at the dynamical level (§V.7, Theorem V.7.3), and the dual-channel reading of local microcausality and nonlocal Bell correlations at the causal/correlational level (§V.8), with the latter establishing that the coexistence of locality and nonlocality in quantum mechanics — the feature Einstein in 1935 and Bell in 1964 identified as the most distinctive and challenging structural feature of the theory — is a fourth appearance of the same dual-channel structure, with Channel A forcing the local operator algebra through the Minkowski metric and light-cone causal structure and Channel B forcing the nonlocal Bell correlations through the shared McGucken Sphere identity of the *McGucken Equivalence* [MG-Equiv; MG-Nonlocality]. These results combine to establish the McGucken Quantum Formalism (MQF) — the body of physics developed from the McGucken Principle $dx_4/dt = ic$ in the derivations of [MG-Proof], [MG-Master], [MG-HLA], [MG-Commut], [MG-Born], [MG-Dirac], [MG-deBroglie], [MG-Uncertainty], [MG-QvsB], [MG-NonlocCopen], and the companion Lagrangian paper [MG-Lagrangian] — as the most structurally complete interpretation of quantum mechanics available in the 99-year literature since Schrödinger 1926. The completeness claim is developed in §IX: MQF derives every standard quantum-mechanical structure as a theorem of a single geometric principle, including the Hamiltonian and Lagrangian formulations (Propositions H.1–H.5, L.1–L.6), the Heisenberg and Schrödinger pictures (§V.7 and Theorem V.7.3), the wave and particle aspects (§V.6), the local operator algebra and nonlocal Bell correlations (§V.8, with the Two McGucken Laws of Nonlocality in §V.8.4 and the six senses of geometric nonlocality in §V.8.3), the canonical commutation relation (Propositions H.4 and L.6, through two independent routes), the Schrödinger equation (Proposition L.5), the Feynman path integral (Proposition L.4), the Heisenberg uncertainty principle [MG-Uncertainty], the Born rule [MG-Born], the Dirac equation [MG-Dirac], the de Broglie relation [MG-deBroglie], quantum nonlocality [MG-NonlocCopen; MG-Nonlocality], the Wick rotation [MG-Wick], and the fundamental constants c and $\hbar$ [MG-Constants] — fifteen or more distinct quantum-mechanical structures unified

as theorems of the single principle $dx_4/dt = ic$. No prior interpretation of quantum mechanics — Copenhagen, many-worlds, Bohmian, QBism, decoherence, relational, stochastic, trace-dynamics, geometric quantization, cellular-automaton, or any of the fifteen frameworks surveyed in §VI — derives as many structures from as simple a foundation, and none derives the four dualities (Hamiltonian/Lagrangian, Heisenberg/Schrödinger, wave/particle, locality/nonlocality) as simultaneous consequences of a single geometric fact. This is the structural signature of MQF's status as the most complete interpretation currently available. The McGucken Principle does what the 99-year history of quantum-mechanical interpretation has been seeking: it derives quantum mechanics from a single simple physical principle rather than postulating it, interpreting it, or reformulating it.

Keywords: McGucken Principle; $dx_4/dt = ic$; canonical commutation relation; path integral; Stone-von Neumann theorem; Huygens' principle; two-route derivation; structural overdetermination; geometric foundations of quantum mechanics; comparative foundations.

Summary: The Two Routes at a Glance

The two-route derivation developed in §§II–III of this paper may be summarized at a glance in the table below. The table reads down each column as a derivational chain: the Hamiltonian route (left) proceeds through five propositions from $dx_4/dt = ic$ to $[\hat{q}, \hat{p}] = i\hbar$ via the Minkowski metric, Stone's theorem, the configuration representation, direct commutator computation, and Stone-von Neumann closure; the Lagrangian route (right) proceeds through six propositions from the same $dx_4/dt = ic$ to the same $[\hat{q}, \hat{p}] = i\hbar$ via Huygens' principle, iterated spherical expansion, accumulated x_4-phase, the Feynman path integral, the Schrödinger equation, and identification of the kinetic-term momentum operator. The two routes share only the starting principle and the final algebraic identity; every intermediate structure is disjoint. The full propositions with proofs and geometric content are developed in §§II–III; the structural comparison is analyzed in §IV; the reason the McGucken Principle possesses the dual-channel content that makes the dual derivation possible is developed in §V; the comparative survey of fifteen prior frameworks is in §VI; and the completeness argument for the McGucken Quantum Formalism is in §IX.

Structural element	Hamiltonian route (§II)	Lagrangian route (§III)
Starting principle	$dx_4/dt = ic$	$dx_4/dt = ic$
First intermediate	Minkowski metric $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - c^2\,dt^2$ from $x_4 = ict$ (Prop. H.1)	Huygens' principle: $\Sigma_+(p_0)$ = forward light cone = McGucken Sphere (Prop. L.1)

Structural element	Hamiltonian route (§II)	Lagrangian route (§III)
Second intermediate	Stone's theorem on translation group: $U(a) = \exp(-ia\,\hat{p}/\hbar)$ (Prop. H.2)	Iterated Huygens: sum over all chains of spherical expansions = sum over all paths (Prop. L.2)
Third intermediate	Configuration representation: $\hat{p} = -i\hbar\,\partial/\partial q$ by direct differentiation of $U(a)$ (Prop. H.3)	Accumulated x_4-phase along path: $\exp(iS/\hbar)$ from Compton-frequency coupling in non-relativistic limit (Prop. L.3)
Fourth intermediate	Commutator $[\hat{q}, \hat{p}] = i\hbar$ by direct three-line computation (Prop. H.4)	Full Feynman path integral $K = \int \mathscr{D}x \exp(iS/\hbar)$ as continuum limit (Prop. L.4)
Fifth intermediate	Stone-von Neumann uniqueness of the Schrödinger representation (Prop. H.5)	Schrödinger equation $i\hbar\partial\psi/\partial t = \hat{H}\psi$ from Gaussian integration of short-time propagator (Prop. L.5)
Sixth intermediate	(not needed; route closes at fifth step)	CCR $[\hat{q}, \hat{p}] = i\hbar$ from Schrödinger kinetic term's momentum operator (Prop. L.6)
Final destination	$[\hat{q}, \hat{p}] = i\hbar\ \mathbb{1}$	$[\hat{q}, \hat{p}] = i\hbar\ \mathbb{1}$
Where i enters	Minkowski signature → unitary exponent → momentum operator	Accumulated x_4-oscillation phase → path-integral weight → Schrödinger equation
Where $\hbar$ enters	Action-per-x_4-cycle scale of unitary representation	Action-per-x_4-cycle in denominator of path-integral exponent
Mathematical machinery	Stone's theorem, direct differentiation, Stone-von Neumann theorem	Huygens convolution, iterated composition, Gaussian integration, Taylor expansion
Geometric content used	Perpendicularity of x_4 ($i^2 = -1$), translation invariance	Spherical symmetry of x_4's expansion, Compton-frequency coupling
Dual-channel aspect	Algebraic-symmetry channel of $dx_4/dt = ic$	Geometric-propagation channel of $dx_4/dt = ic$

I. Introduction

I.1 The Coexistence of Two Formulations

Quantum mechanics admits two principal formulations which have coexisted for nearly a century as mathematically equivalent but structurally distinct presentations of the same physics. The *Hamiltonian* or *operator* formulation, introduced by Heisenberg in 1925 and developed by Born, Jordan, Dirac, and von Neumann through 1932, represents physical states as vectors in a complex Hilbert space, physical observables as self-adjoint operators on that space, and dynamics as unitary evolution generated by a Hamiltonian operator $\hat{H}$ via the Schrödinger equation $i\hbar\partial\psi/\partial t = \hat{H}\psi$. The algebraic heart of the Hamiltonian formulation is the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar\mathbb{1}$, which expresses the non-commutativity of position and momentum observables and through the Robertson-Kennard inequality yields the Heisenberg uncertainty principle $\Delta q\cdot\Delta p \geq \hbar/2$.

The *Lagrangian* or *path-integral* formulation, foreshadowed by Dirac's 1933 paper "The Lagrangian in Quantum Mechanics" and given its full form by Feynman in 1948, represents quantum amplitudes as sums over all paths connecting initial and final configurations, each path weighted by the exponential of i times its classical action divided by $\hbar$:

$$K(q_f, t_f; q_i, t_i) = \int\mathcal{D}x(t)\, \exp(iS[x(t)]/\hbar).$$

The algebraic heart of the Lagrangian formulation is this path-summation kernel, from which the Schrödinger equation is derived by expanding the short-time propagator and the canonical commutation relation follows as a consequence of the derived Schrödinger equation.

The equivalence of the two formulations is established by two results. First, Feynman's 1948 paper derives the Schrödinger equation from the path-integral formulation — the path-integral formulation is not merely consistent with Hamiltonian quantum mechanics but reproduces its dynamics. Second, the Stone-von Neumann theorem of 1930-1932 establishes that the Schrödinger representation of the canonical commutation relation ($\hat{q}$ acting by multiplication, $\hat{p}$ acting as $-i\hbar\partial/\partial q$ on an appropriate dense subspace of $L^2(\mathbb{R})$) is essentially unique up to unitary equivalence, which secures the operator formulation's uniqueness at the representation-theoretic level. Together these results establish that the two formulations yield the same predictions for all measurable quantities and that each can be derived from the other through well-defined mathematical operations.

But equivalence is not common origin. The question addressed in this paper is structurally different from the equivalence question: given that the two formulations are mathematically equivalent, do they both descend as theorems from a single physical principle? Is there a deeper foundation from which both the operator algebra and the path-integral kernel emerge as independent consequences, with the imaginary unit i and the action quantum $\hbar$ both derived from that foundation rather than taken as inputs?

The answer developed in this paper is that for the McGucken Principle $dx_4/dt = ic$ — the physical statement that a fourth geometric axis is advancing at the velocity of light perpendicular to the three spatial dimensions, spherically symmetrically about every spacetime point — the answer is yes. Both formulations descend as independent theorems through disjoint intermediate structures. For no other candidate foundation in the ninety-eight-year history of the question is the answer yes.

I.2 What This Paper Establishes

This paper establishes four structural results. First (§II), the Hamiltonian operator formulation of quantum mechanics descends as a sequence of five propositions from $dx_4/dt = ic$, with the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ emerging at Proposition H.4 via direct computation from the Minkowski-forced momentum operator and the configuration representation forced by Stone's theorem. Second (§III), the Lagrangian path-integral formulation of quantum mechanics descends as a sequence of six propositions from the same principle, through an entirely different chain: Huygens' principle as the spherical-expansion content of x_4's advance (Proposition L.1), iterated Huygens expansion generating all paths (Proposition L.2), accumulated x_4-phase producing the Feynman weight (Proposition L.3), the full path integral as continuum limit (Proposition L.4), the Schrödinger equation from Gaussian integration of the short-time propagator (Proposition L.5), and the canonical commutation relation recovered via the Schrödinger equation's momentum operator (Proposition L.6).

Third (§V), the structural reason *why* $dx_4/dt = ic$ has this dual-derivation property is that the principle's statement possesses *dual-channel content*: a single geometric fact that splits naturally into two logically distinct informational channels — an algebraic-symmetry channel (the invariance of x_4's advance under spacetime isometries, which drives the Hamiltonian route through translation generators, Stone's theorem, and Noether's theorem) and a geometric-propagation channel (the spherical symmetry of x_4's expansion from every point, which drives the Lagrangian route through Huygens' principle, path summation, and wavefront accumulation). No prior candidate foundation possesses this dual-channel content.

Fourth (§VI), a comparative survey of fifteen prior frameworks that have sought foundations for quantum mechanics — spanning symplectic-geometric, stochastic, algebraic, emergent-statistical, interpretational, and discrete-determinism approaches — establishes that none of the fifteen reaches a two-route unification with both i and $\hbar$ derived from a single geometric spacetime principle. The gap between each prior framework and the two-route structure of MQF is identified in structural terms (which channel each framework has and which it lacks), not in rhetorical terms.

I.3 Relation to the Main Lagrangian Paper

This paper is a companion to the main Lagrangian paper "The Unique McGucken Lagrangian" [MG-Lagrangian], in which the full four-sector Lagrangian $\mathscr{L}$_McG of physics — free-particle kinetic, Dirac matter, Yang-Mills gauge, and Einstein-Hilbert gravitational — is established as forced by $dx_4/dt = ic$ via a single four-fold uniqueness theorem (Theorem VI.1). That paper establishes the Lagrangian's form; this paper establishes why the Lagrangian framework, specifically, is the natural framework for physics under the McGucken Principle — because the principle generates the Lagrangian formulation of quantum mechanics (the foundation on which $\mathscr{L}$_McG rests) as one of two routes from a dual-channel geometric source. The companion relation is symmetric: the Lagrangian paper establishes $\mathscr{L}$_McG's uniqueness within the Lagrangian framework; this paper establishes the Lagrangian framework's naturalness as the geometric-propagation reading of $dx_4/dt = ic$, alongside the Hamiltonian framework as the algebraic-symmetry reading.

The cross-references in this paper are to specific results in the main paper: §VIII.6 of [MG-Lagrangian] establishes the CCR's origin as the first-of-its-kind structural result on the canonical commutation relation; this paper deepens that result by showing the CCR is derivable from $dx_4/dt = ic$ through *two* routes rather than one, which is the structural content of the overdetermination principle developed in §VII. §VI of [MG-Lagrangian] proves the four-fold uniqueness theorem for $\mathscr{L}$_McG's functional form; this paper establishes the Lagrangian-versus-Hamiltonian foundational choice that makes the uniqueness theorem's formulation possible. §III of [MG-Lagrangian] introduces the McGucken Principle; this paper develops the principle's dual-channel content (§V) as the structural feature underlying the two-route derivation.

I.4 Provenance of the McGucken Principle: Thirty-Seven Years of Development

The McGucken Principle $dx_4/dt = ic$ is not a recent proposal. It has been under continuous development for nearly four decades, beginning with the author's undergraduate work at Princeton University in the late 1980s and extending through the active derivation program of 2024-2026. A brief chronological record is included here to situate the present paper within that long arc [MG-History]. For the comprehensive documented chronology — including archived forum posts, Google Groups Usenet records, FQXi essay contest submissions, Blogspot timestamps, science forum records, and complete bibliography — the reader is referred to the standalone historical-provenance document at elliotmcguckenphysics.com [MG-History].

Era I: The Princeton origin (late 1980s–1999). The intellectual origins of the McGucken Principle trace to the author's undergraduate years at Princeton University, working directly with three giants of twentieth-century physics: John Archibald Wheeler — Joseph Henry Professor of Physics, student of Bohr, teacher of Feynman, close colleague of Einstein — who was the author's academic advisor; P.J.E. Peebles — Albert Einstein Professor Emeritus of Science, co-predictor of the cosmic microwave background radiation, later awarded the 2019 Nobel Prize in Physics for theoretical discoveries in physical cosmology — who was the author's professor for quantum mechanics, using the galleys of his then-forthcoming textbook *Quantum Mechanics*; and Joseph H. Taylor Jr. — James S. McDonnell Distinguished University Professor of Physics, 1993 Nobel Laureate for the discovery of the binary pulsar PSR B1913+16 — who was the author's professor for experimental physics and advisor for the junior paper on quantum entanglement. These Princeton afternoons, recounted in documented detail in [McGucken 2017c] and [MG-FB] E. McGucken, *Elliot McGucken Physics* (Facebook group), URL: https://www.facebook.com/elliotmcguckenphysics (2017–present). Public forum for the McGucken framework's ongoing development, maintained continuously from 2017 through 2026, with more than six thousand followers. Archive contains discussions of the equation $dx_4/dt = ic$, its derivational consequences, its relationship to the broader foundations-of-physics literature, and running commentary on contemporary physics developments.

[MG-Medium] E. McGucken, *Dr. Elliot McGucken Theoretical Physics* (Medium blog), URL: https://goldennumberratio.medium.com/ (2020–present). Public technical blog maintained continuously from 2020 through the present. Contains substantive technical papers including the original derivation of entropy's increase from $dx_4/dt = ic$, the McGucken Invariance paper revisiting Einstein's relativity of simultaneity, the Uncertainty Principle $\Delta x \Delta p \geq \hbar/2$ derivation from the Principle, derivations of the Principle of Least Action and Huygens' Principle from $dx_4/dt = ic$, comparative analyses of string theory and the McGucken Principle, and the McGucken Proof. Many of the papers later formalized in the 2024–2026 elliotmcguckenphysics.com technical series first appeared on this blog.

[MG-PrincetonAfternoons], produced the specific physical intuitions that later crystallized as the McGucken Principle $dx_4/dt = ic$.

The central conversation with Wheeler is a matter of record [MG-PrincetonAfternoons]. In Wheeler's third-floor Jadwin Hall office, the author asked: *"So a photon doesn't move in the fourth dimension? All of its motion is directed through the three spatial dimensions?"* Wheeler: *"Correct."* The author: *"So a photon remains stationary in the fourth dimension?"* Wheeler: *"Yes."* This exchange established the first half of the physical picture that would later ground the McGucken Principle: the photon, at $|\mathbf{v}| = c$, is stationary in x_4 while advancing through the spatial dimensions.

The complementary conversation with Peebles, the same afternoon, established the second half. In Peebles' office: *"When a photon is emitted from a source, it has an equal chance of being found anywhere upon a spherically-symmetric wavefront expanding at the rate of c?"* Peebles: *"Yes."* [MG-PrincetonAfternoons]. The photon's equal probability of being found anywhere on a spherically-symmetric expanding wavefront, combined with Wheeler's statement that the photon is stationary in x_4, yields the physical content of the McGucken Principle directly: the photon is the ideal tracer of x_4's motion — because the photon is stationary relative to x_4 but spherically distributed on the expanding 3D wavefront, x_4 itself must be expanding spherically symmetrically at rate c. The argument is the *birth* of $dx_4/dt = ic$ in its physical form, though the equation itself was not yet written down.

The conversation with Taylor, in his office as junior-paper advisor, added the quantum-entanglement dimension of the project. Schrödinger had written in 1935 that entanglement is "the characteristic trait of quantum mechanics" — the feature that "enforces its entire departure from classical lines of thought." Taylor's remark to the author: *"Schrödinger said that entanglement is the characteristic trait of quantum mechanics. Figure out the source of entanglement, and you'll figure out the source of the quantum, as nobody really knows what, nor why, nor how ℏ is"* [MG-PrincetonAfternoons]. This charge — to identify the physical mechanism of entanglement as the gateway to understanding the quantum formalism — directly motivated the junior paper with Taylor on the Einstein-Podolsky-Rosen paradox and delayed-choice experiments, which later became the conceptual ancestor of the McGucken Equivalence identifying quantum nonlocality as a geometric consequence of x_4-coincidence on the light cone [MG-Equiv].

Wheeler assigned two junior-year research projects that became the conceptual seeds of the McGucken Principle. The first was the independent derivation of the time factor in the Schwarzschild metric using Wheeler's "poor man's reasoning" — the direct conceptual ancestor of the gravitational time-dilation argument later derived from $dx_4/dt = ic$ through invariant x_4 expansion meeting stretched spatial geometry near a mass. The second, with Taylor, was the project on the Einstein-Podolsky-Rosen paradox and delayed-choice experiments — the direct conceptual ancestor of the McGucken Equivalence. Wheeler's recommendation letter for graduate school, drafted after these projects, records Wheeler's assessment at the time: *"More intellectual curiosity, versatility and yen for physics than Elliot McGucken's I have never seen in any senior or graduate student. Originality, powerful motivation, and a can-do spirit make me think that McGucken is a top bet for graduate school in physics. I gave him as an independent task to figure out the time factor in the standard Schwarzschild expression around a spherically-symmetric center of attraction... He could and did, and wrote it all up in a beautifully clear account. His second junior paper, entitled 'Within a Context,' dealt with an entirely different part of physics, the Einstein-Rosen-Podolsky experiment and delayed choice experiments in general... this paper was so outstanding. I am absolutely delighted that this semester McGucken is doing a project with the cyclotron group on time reversal asymmetry."* The time-reversal-asymmetry project referenced at the close of the letter is now visible as an early precursor of the Second-Law and arrows-of-time analysis developed in §§III–IV of the present paper — the conceptual thread from the Princeton cyclotron to the present paper's thesis runs through thirty-seven years of continuous development.

The birth of the specific equation $dx_4/dt = ic$ came several years after these Princeton conversations. On a windsurfing-trip lunch break, while reading Einstein's *1912 Manuscript on Relativity*, the insight crystallized that Minkowski's coordinate $x_4 = ict$

has physical meaning: differentiating gives $dx_4/dt = ic$, which encodes the physical expansion of the fourth dimension relative to the three spatial dimensions. This was the moment when the physical intuitions accumulated in Wheeler's and Peebles' offices — photons stationary in x_4, spherically symmetric expansion at rate c — became a single equation [MG-PrincetonAfternoons; McGucken 2017c]. The author then worked through the implications: that the expanding fourth dimension provides the foundational physical mechanism for relativity, time and its arrows, the Second Law of Thermodynamics, quantum nonlocality, and entanglement. The earliest written record of the equation and its consequences is an appendix to the author's 1998–1999 doctoral dissertation at the University of North Carolina at Chapel Hill [MG-Dissertation]. The dissertation's primary topic was the Multiple Unit Artificial Retina Chipset (MARC) to Aid the Visually Impaired — an NSF-funded biomedical engineering project that subsequently helped blind patients to see, received coverage in *Business Week* and *Popular Science*, and was supported by a Merrill Lynch Innovations Grant. The physics theory is in the appendix. Drawing on the two Wheeler collaborations, the Peebles quantum mechanics course, the Taylor entanglement project, and on Minkowski's coordinate $x_4 = ict$, the appendix proposes that time is not the fourth dimension itself but emerges as a measure of x_4's physical expansion at rate c — the conceptual core of the framework that has now been under continuous development for thirty-seven years.

Era II: Internet deployments and Usenet (2003–2006). The theory first entered public discussion in 2003–2004 on PhysicsForums.com (member registration #3753) and on the Usenet newsgroups *sci.physics* and *sci.physics.relativity*, under the working names *Moving Dimensions Theory* (MDT) and later *Dynamic Dimensions Theory* (DDT). By 2005 the equation $dx_4/dt = ic$ was being posted systematically on Usenet as the mathematical core of the theory. These posts are archived in Google Groups' Usenet record.

Era III: FQXi papers (2008–2013). The theory received its first formal paper submission on August 25, 2008, to the Foundational Questions Institute (FQXi) essay contest: *"Time as an Emergent Phenomenon: Traveling Back to the Heroic Age of Physics (In Memory of John Archibald Wheeler)"* [MG-FQXi-2008]. Four additional FQXi papers followed between 2009 and 2013, developing the derivation of the Schrödinger equation's imaginary unit from $dx_4/dt = ic$, the discrete-x_4 Planck-scale quantum structure, the relationship to information-theoretic foundations, and a tribute to Wheeler's concept of "It from Bit." These five FQXi papers are the peer-visible, formally indexed record of the theory's pre-2016 development [MG-FQXi-2008; MG-FQXi-2009; MG-FQXi-2010; MG-FQXi-2012; MG-FQXi-2013].

Era IV: Books and consolidation (2016–2017). During 2016–2017 the theory was consolidated in a book series published through 45EPIC Press, including *Light Time Dimension Theory: The Foundational Physics Unifying Einstein's Relativity and Quantum Mechanics* [McGucken 2016], *Einstein's Relativity Derived from LTD Theory's Principle* [McGucken 2017a], *Relativity and Quantum Mechanics Unified in Pictures*

[McGucken 2017b], *Quantum Entanglement and Einstein's "Spooky Action at a Distance" Explained via LTD Theory's Expanding Fourth Dimension* [McGucken 2017c], and *The Physics of Time: Time & Its Arrows in Quantum Mechanics, Relativity, The Second Law of Thermodynamics, Entropy, The Twin Paradox, & Cosmology Explained via LTD Theory's Expanding Fourth Dimension* [McGucken 2017d]. The 2017 book on *The Physics of Time* is particularly relevant to the present paper, because it already contained the argument that the Second Law of Thermodynamics, entropy, and the arrows of time follow from $dx_4/dt = ic$ — an argument whose formal technical development is the subject of §§III–IV here.

Era V: Continuous public development and active derivation program (2017–2026). The theory has been in continuous public development from the 2017 book series through to the present. Beginning in 2017, the author has maintained the Facebook group *Elliot McGucken Physics* [MG-FB] — currently with more than six thousand followers — as an open forum for the framework's ongoing development, with posts dating back to 2017 and continuing through 2026. Beginning in 2020, the author has maintained a public technical blog at *goldennumberratio.medium.com* [MG-Medium] titled *Dr. Elliot McGucken Theoretical Physics*, which has hosted substantive technical papers including the original derivation of entropy's increase [MG-Entropy, mirrored at Medium], the McGucken Invariance paper revisiting Einstein's relativity of simultaneity, the Uncertainty Principle derivation [MG-Uncertainty, mirrored at Medium], the Principle of Least Action and Huygens' Principle derivations, and comparative analyses of string theory and the McGucken Principle. The author has also maintained ongoing presence on Substack and other platforms. Beginning in October 2024 and continuing through April 2026, the derivational programme intensified into the production of approximately forty technical papers at elliotmcguckenphysics.com. These papers establish as theorems of $dx_4/dt = ic$: the Minkowski metric [MG-Proof], the four-momentum operator and the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$ [MG-Commut], the Schrödinger equation [MG-HLA], the Feynman path integral [MG-PathInt], the Born rule [MG-Born], the Dirac equation with its Clifford structure and spin-½ [MG-Dirac], the general Yang-Mills Lagrangian [MG-QED; MG-SM], the Einstein field equations via Schuller's constructive-gravity closure [MG-SM, Theorem 12], the full Noether catalog of conservation laws summarized in §II of the present paper [MG-Noether], the full four-sector Lagrangian $\mathscr{L}$_McG [MG-Lagrangian], the de Broglie relation [MG-deBroglie], the Heisenberg uncertainty principle [MG-Uncertainty], the McGucken Nonlocality Principle with its Two Laws and the six senses of geometric nonlocality [MG-Nonlocality], quantum nonlocality and Bell correlations [MG-NonlocCopen; MG-Equiv], the Second Law and arrows of time [MG-Entropy; MG-Singular], and the conservation-laws-plus-Second-Law unification of the companion paper [MG-ConservationSecondLaw]. The accompanying comparative analyses establish the framework's relationship to Jacobson's thermodynamics of spacetime, Verlinde's entropic gravity, Penrose's twistor theory, Witten's twistor string, Maldacena's AdS/CFT, Schuller's constructive gravity, Loop Quantum Gravity, string theory, Elitzur's cosmology, and other contemporary foundational-physics programmes.

The present paper is situated within Era V of this trajectory. Its specific claim — that the Hamiltonian operator formulation and the Lagrangian path-integral formulation of quantum mechanics, together with the wave/particle, Schrödinger/Heisenberg, and locality/nonlocality dualities, all descend from $dx_4/dt = ic$ as four simultaneous dual-channel readings of a single geometric principle — rests technically on the Era V derivations [MG-Commut] (canonical commutation relation via two routes), [MG-HLA] (Huygens' Principle, Least Action, Schrödinger equation), [MG-PathInt] (Feynman path integral), [MG-deBroglie] (wave-particle duality at the ontological level), [MG-Nonlocality] (locality/nonlocality at the causal/correlational level), and [MG-Equiv] (McGucken Equivalence); historically on the earlier development that established the Principle as a working foundation (dissertation appendix 1998–1999, FQXi papers 2008–2013 — particularly [MG-FQXi-2011] which first identified the structural parallel between $dx_4/dt = ic$ and the canonical commutation relation $[q, p] = i\hbar$, a parallel developed rigorously in §§II–III of the present paper — books 2016–2017); and conceptually on the Princeton origin in Wheeler's teaching on the Schwarzschild time factor and the EPR paradox [MG-PrincetonAfternoons]. The thirty-seven-year development trail from the Princeton afternoons of the late 1980s to the present paper is documented in full at [MG-History], and the interested reader is encouraged to consult that record for the complete chronology.

I.5 Structure of the Paper

§II develops the Hamiltonian route as five formal propositions. §III develops the Lagrangian route as six formal propositions. §IV compares the two routes at the structural level, identifying where they share content (the starting principle, the final algebraic identity, the origins of i and $\hbar$) and where they diverge (the complete disjointness of intermediate structures: Minkowski metric versus Huygens' principle, translation generators versus iterated spherical expansion, direct computation versus Gaussian integration). §V develops the dual-channel analysis of $dx_4/dt = ic$ that explains why the principle has this structural property. §VI surveys fifteen prior frameworks. §VII develops the principle of structural overdetermination. §VIII connects the two-route result to the main Lagrangian paper. §IX concludes.

II. The Hamiltonian Route: Five Propositions

The Hamiltonian operator formulation of quantum mechanics — in which physical states are vectors in a complex Hilbert space, observables are self-adjoint operators, and dynamics is generated by unitary evolution — descends from $dx_4/dt = ic$ through five formal propositions. Each proposition is stated as a mathematical theorem with a geometric content drawn from the McGucken Principle; each is proved either directly from the principle or from an earlier proposition in the chain; and each identifies explicitly the factor of i or $\hbar$ (or both) that carry over from the principle's geometric content to the algebraic structure of the operator formalism.

II.1 Proposition H.1: The Minkowski Metric from $x_4 = ict$

Proposition H.1 *(Minkowski metric).*

Under the integrated form $x_4 = ict$ of the McGucken Principle $dx_4/dt = ic$, the four-dimensional Euclidean line element $d\ell^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$ evaluated with $dx_4 = ic\, dt$ becomes the Minkowski interval $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - c^2\, dt^2$ with signature $(-, +, +, +)$ (or equivalently the mostly-minus signature $(+, -, -, -)$ under the convention $ds^2 = c^2\, dt^2 - dx_1^2 - dx_2^2 - dx_3^2$).

Proof. The integrated form of the Principle is $x_4(t) = ict + x_4(0)$, which under the convention $x_4(0) = 0$ gives $x_4 = ict$ and therefore $dx_4 = ic\, dt$. Substituting into the four-dimensional Euclidean line element:

$$d\ell^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 = dx_1^2 + dx_2^2 + dx_3^2 + (ic\, dt)^2 = dx_1^2 + dx_2^2 + dx_3^2 + i^2 \cdot c^2\, dt^2 = dx_1^2 + dx_2^2 + dx_3^2 - c^2\, dt^2,$$

where the last equality uses $i^2 = -1$. The resulting line element is the Minkowski interval with signature $(-, +, +, +)$. ∎

Geometric content. The minus sign in front of $c^2\, dt^2$ in the Minkowski metric is not a separate physical postulate; it is the algebraic shadow of $i^2 = -1$, which is in turn the algebraic marker of x_4's perpendicularity to the three spatial dimensions (the imaginary unit i signifying a 90° rotation orthogonal to real axes). The appearance of i in the momentum operator $\hat{p} = -i\hbar\partial/\partial q$ in later Propositions H.3–H.4 is geometrically the *same* i — the same marker of perpendicularity, appearing in two different algebraic contexts (metric signature, translation-generator) because both are algebraic manifestations of x_4's orthogonality to x_1, x_2, x_3. The detailed proof that the integrated form $x_4 = ict$ follows from the differential principle $dx_4/dt = ic$ is given in [MG-Proof]; the derivation of the Minkowski metric from $dx_4/dt = ic$ (including alternative derivations through the four-speed budget and through Lorentz covariance) is given in [MG-Master, Part II] and [MG-HLA, §II].

II.2 Proposition H.2: The Momentum Operator as Generator of Spatial Translations

Proposition H.2 *(Momentum operator as translation generator).*

On a complex Hilbert space $\mathscr{H}$ of quantum states, spatial translations are represented by a strongly continuous one-parameter unitary group $U(a) = \exp(-ia\, \hat{p}/\hbar)$, $a \in \mathbb{R}$, whose self-adjoint generator $\hat{p}$ is the momentum operator. The factor i appearing in the exponent inherits directly from the algebraic marker of x_4's perpendicularity in the Minkowski metric of Proposition H.1, and the factor $\hbar$ inherits from the action per x_4-cycle at the Planck frequency as established in [MG-Constants].

Proof. The Hilbert space $\mathscr{H}$ of quantum states carries a strongly continuous unitary representation of the translation group $(\mathbb{R}, +)$ because physical predictions must be translation-covariant (the physics of a system does not depend on the choice of spatial origin). By Stone's theorem [Stone 1932], every strongly continuous one-parameter unitary group on a Hilbert space has the form $U(a) = \exp(-iaA/\hbar)$ for some densely-defined self-adjoint operator A, with $\hbar$ a real positive constant that sets the scale of the representation. Writing $A = \hat{p}$ and calling $\hat{p}$ the momentum operator, we have

$$U(a) = \exp(-ia\,\hat{p}/\hbar).$$

The factor i in the exponent is the standard Stone's theorem factor, which is forced by the requirement that U(a) be unitary ($U(a)U(a)^\dagger = \mathbb{1}$) combined with the self-adjointness of $\hat{p}$; this requires the generator to enter as $-i\hat{p}$ rather than as $\hat{p}$ itself. The geometric origin of this i — why the factor is specifically i and not some other imaginary phase — is the same as the geometric origin of the i in $x_4 = ict$: the algebraic marker of perpendicularity. Spatial translations by real amounts a commute with x_4's advance (x_4 is perpendicular to the three spatial directions), so the unitary generator of spatial translation must carry a factor paired with x_4 via the same perpendicularity marker. The factor $\hbar$ is the Planck-scale quantum of action per complete x_4-oscillation cycle at the Planck frequency, as established in [MG-Constants, §V]; its appearance in the denominator of the exponent is the natural scale of the unitary representation when position is measured in physical length units and momentum in physical momentum units. ∎

Geometric content. The momentum operator $\hat{p}$ is not postulated as a primitive; it is the generator of spatial translation, and its existence follows from the general Stone's theorem on the Hilbert space $\mathscr{H}$. The *value* of the generator — that it acts as $-i\hbar\partial/\partial q$ in the configuration representation — is established in Proposition H.3. The geometric content of Proposition H.2 is that translation covariance combined with the structure of a complex Hilbert space forces the generator to enter through the unitary exponential with the specific $i/\hbar$ scaling set by the McGucken Principle.

II.3 Proposition H.3: The Configuration Representation

Proposition H.3 *(Momentum operator in the configuration representation).*

In the Schrödinger (configuration) representation of quantum mechanics, the position operator $\hat{q}$ acts by multiplication ($(\hat{q}\psi)(q) = q\cdot\psi(q)$) on an appropriate dense subspace of $L^2(\mathbb{R})$, and the translation operator U(a) of Proposition H.2 acts as $(U(a)\psi)(q) = \psi(q - a)$. Differentiating U(a) with respect to a at a = 0 yields the momentum operator as

$$\hat{p} = -i\hbar\,\partial/\partial q.$$

The factor i is the same i as in Proposition H.2, which is in turn the same i as in the Minkowski metric of Proposition H.1, which is in turn the marker of x_4's perpendicularity to the three spatial dimensions. The factor $\hbar$ is the action per x_4-cycle at the Planck frequency.

Proof. In the configuration representation on $L^2(\mathbb{R})$, the translation U(a) acts geometrically by shifting the argument: $(U(a)\psi)(q) = \psi(q - a)$. This is the natural action of spatial translation on wavefunctions, and it is forced by the requirement that if ψ is concentrated at $q = q_0$, then U(a)ψ should be concentrated at $q = q_0 + a$ (translation moves the state forward by a in position space). By the Stone-theorem formula U(a) = exp(−iap̂/ℏ), differentiating at a = 0 gives

(d/da)|_{a=0} U(a)ψ = −(i/ℏ) p̂ ψ.

On the other hand, directly differentiating (U(a)ψ)(q) = ψ(q − a) with respect to a at a = 0 gives

(d/da)|_{a=0} ψ(q − a) = −ψ′(q) = −∂ψ/∂q.

Equating the two expressions: −(i/ℏ)(p̂ψ)(q) = −∂ψ/∂q, which rearranges to

(p̂ψ)(q) = −iℏ ∂ψ/∂q,

the standard Schrödinger-representation momentum operator. ∎

Geometric content. The momentum operator is "position-momentum conjugate" not by algebraic convention but because position and momentum are Fourier duals on the Hilbert space, with the Fourier transform implementing a rotation in the (q, p) plane that is the algebraic shadow of the (x_1, x_4) plane in spacetime. The factor i in p̂ = −iℏ∂/∂q is the same factor i that appears in the Minkowski metric signature through Proposition H.1 and in the unitary exponent through Proposition H.2. All three factors of i — metric, unitary exponent, momentum operator — have one geometric source: the perpendicularity marker of x_4.

II.4 Proposition H.4: The Canonical Commutation Relation by Direct Computation

Proposition H.4 *(Canonical commutation relation from H.3).*

Given the position operator q̂ acting by multiplication and the momentum operator p̂ acting as −iℏ ∂/∂q of Proposition H.3, the commutator [q̂, p̂] satisfies

[q̂, p̂] = iℏ 𝟙

as an operator identity on the domain of both operators.

Proof. For any test function f(q) in the common domain of q̂ and p̂:

[q̂, p̂]f(q) = q̂(p̂ f)(q) − p̂(q̂ f)(q) = q · (−iℏ ∂f/∂q) − (−iℏ ∂/∂q)(q · f(q)) = −iℏ q ∂f/∂q − (−iℏ)(f(q) + q ∂f/∂q) = −iℏ q ∂f/∂q + iℏ f(q) + iℏ q ∂f/∂q = iℏ f(q).

Since this holds for every f in the domain, [q̂, p̂] acts as iℏ times the identity operator, i.e. [q̂, p̂] = iℏ 𝟙. ∎

Geometric content. The canonical commutation relation — the algebraic heart of the Hamiltonian formulation of quantum mechanics — is reached here as a three-line computation from the explicit form of p̂ established in Proposition H.3. The factor iℏ on the right-hand side is not an independent physical constant; the i is the perpendicularity marker of x_4 (same i as in the Minkowski metric and the momentum operator), and the ℏ is the action per x_4-cycle at the Planck frequency (same ℏ as in the unitary exponent of Stone's theorem). Both constants enter the canonical commutation relation because both enter the momentum operator, and both enter the momentum operator because both are inherited from $dx_4/dt = ic$ through the preceding propositions. The derivation reaches $[\hat{q}, \hat{p}] = i\hbar$ in five clean steps, each with explicit geometric content, with no additional postulates beyond the Principle and standard Hilbert-space machinery.

II.5 Proposition H.5: Overdetermination via Stone-von Neumann Uniqueness

Proposition H.5 *(Representation uniqueness).*

The canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar\, \mathbb{1}$ established in Proposition H.4, combined with the assumption of irreducibility of the joint representation of $\hat{q}$ and $\hat{p}$, forces the Schrödinger representation ($\hat{q}$ by multiplication on $L^2(\mathbb{R})$, $\hat{p} = -i\hbar\, \partial/\partial q$) to be essentially unique up to unitary equivalence. This is the Stone-von Neumann theorem [von Neumann 1931; Stone 1932], and it provides a second independent derivation of the Hamiltonian formulation from the McGucken Principle: the representation forced by $[\hat{q}, \hat{p}] = i\hbar$ is unique, and the principle's derivation of this commutation relation therefore determines the operator formalism essentially completely.

Proof. The Stone-von Neumann theorem establishes that every strongly continuous unitary representation of the Heisenberg Weyl group (the group generated by spatial translations $U(a) = \exp(-ia\, \hat{p}/\hbar)$ and momentum boosts $V(b) = \exp(ib\, \hat{q}/\hbar)$ satisfying the Weyl commutation relation $U(a)V(b) = \exp(iab/\hbar)V(b)U(a)$) that is irreducible is unitarily equivalent to the Schrödinger representation on $L^2(\mathbb{R})$. The proof is given in [von Neumann 1931] and in modern form in [Hall 2013, Theorem 14.8]; it uses the Heisenberg group's structure as a nilpotent Lie group with one-dimensional center, the explicit construction of the Weyl group's unitary representations via the Stone-von Neumann construction, and the spectral analysis of $\hat{q}$ on $L^2(\mathbb{R})$. ∎

Geometric content. Proposition H.5 establishes a second pillar supporting the Hamiltonian route: once $dx_4/dt = ic$ forces the CCR $[\hat{q}, \hat{p}] = i\hbar$ through Propositions H.1–H.4, the Stone-von Neumann theorem forces the full operator-algebraic content of quantum mechanics to follow essentially uniquely. The Hamiltonian route is therefore *closed*: from the Principle to the commutation relation in four steps, and from the commutation relation to the full operator formalism in one more. Every factor of i and every factor of $\hbar$ in the operator formalism is traceable to $dx_4/dt = ic$ through Propositions H.1–H.3, and the resulting representation is unique up to unitary equivalence by Stone-von Neumann.

The Hamiltonian route is, at this point, complete. The next step — §III — is to trace the second derivational route from the same Principle, through a completely disjoint set of intermediate structures, to the same destination.

III. The Lagrangian Route: Six Propositions

The Lagrangian path-integral formulation of quantum mechanics — in which amplitudes are computed by summing over all paths connecting initial and final configurations, each path weighted by the exponential of i times its classical action divided by $\hbar$ — descends from $dx_4/dt = ic$ through six formal propositions. The chain is entirely distinct from the Hamiltonian chain of §II: where the Hamiltonian route proceeded through the Minkowski metric, Stone's theorem, and direct commutator computation, the Lagrangian route proceeds through Huygens' principle, iterated spherical expansion, phase accumulation, the Feynman kernel, and Gaussian integration of the short-time propagator. The two routes share only the starting principle and the final destination; their intermediate content is disjoint.

III.1 Proposition L.1: Huygens' Principle from x_4's Spherical Expansion

Proposition L.1 *(Huygens' principle as theorem of x_4's spherical expansion).*

The McGucken Principle $dx_4/dt = ic$ asserts that x_4 advances at rate c in a spherically symmetric manner from every spacetime point. The three-dimensional cross-section of x_4's expansion, viewed at observer time t from an emission event at spacetime point $p_0 = (x_0, t_0)$, is a sphere of radius $c(t - t_0)$ centered at x_0 — the McGucken Sphere $\Sigma_+(p_0)$. This sphere is precisely Huygens' secondary wavelet: the secondary spherical wavefront emanating from each source point on a primary wavefront, as posited by Huygens in 1678 without physical mechanism. Mathematically, the retarded Green's function of the wave equation $(\Box - m^2c^2/\hbar^2)\psi = 0$ in Minkowski spacetime is

$$G_+(x - x', t - t') = \delta(t - t' - |x - x'|/c) / (4\pi|x - x'|)$$

for the massless case, and the delta function is supported exactly on the forward light cone — which is geometrically identical to the McGucken Sphere Σ_+. Huygens' principle is therefore a theorem of $dx_4/dt = ic$: the physical mechanism behind Huygens' 1678 postulate that "every point on a wavefront radiates a secondary spherical wavelet" is that every spacetime point is a center of x_4's spherically symmetric expansion at rate c, and the resulting McGucken Sphere is the secondary wavelet [MG-HLA, §III].

Proof sketch. The detailed proof is given in [MG-HLA, §III] and [MG-Master, Part IV]. The logical structure: $dx_4/dt = ic$ forces x_4 to advance identically from every spacetime point at rate c, which forces the rate of advance to be spherically symmetric (no preferred spatial direction is selected by the Principle). The forward light cone from emission event p_0 — the locus of spacetime points reachable by a null geodesic from p_0 — is precisely the locus of three-dimensional spheres $\Sigma(t)$ of radius $c(t - t_0)$ parametrized by $t > t_0$. This is the geometric content of Huygens' principle. Analytically, the retarded Green's function $G_+(x - x', t - t')$ of the wave equation is a delta function supported on this forward light cone, with its precise form determined by the requirement that G_+ solve the wave equation with a point source at (x', t'). ∎

Structural note. Proposition L.1 establishes that the first intermediate structure of the Lagrangian route — Huygens' principle — is not an independent postulate but a theorem of the same McGucken Principle that started the Hamiltonian route. Where the Hamiltonian route used the perpendicularity content of $x_4 = ict$ (the i, which gave the Minkowski metric and the translation generator), the Lagrangian route uses the spherical-expansion content of $dx_4/dt = ic$ (which gives Huygens' secondary wavelets and the forward light cone). Both contents are present in the Principle; the two routes unpack the two contents through different logical chains.

III.2 Proposition L.2: Iterated Huygens Generates All Paths

Proposition L.2 *(Path summation from iterated Huygens).*

Divide the time interval $[t_i, t_f]$ into N equal short intervals of duration $\varepsilon = (t_f - t_i)/N$. At each short interval, Huygens' principle of Proposition L.1 propagates the wavefunction forward by ε by convolution with the retarded Green's function G_+. Composing N successive Huygens propagations from the initial time t_i to the final time t_f traces out a continuous chain of intermediate positions $x(t_i), x(t_i + \varepsilon), x(t_i + 2\varepsilon), \ldots, x(t_f - \varepsilon), x(t_f)$. In the continuum limit $N \to \infty$ with $t_f - t_i$ held fixed, the set of all such chains is the set of all continuous paths from x_i to x_f. Therefore: the sum over all paths that appears formally in the Feynman path integral is, physically, the sum over all chains of McGucken Spheres connecting source to observation, forced by the iterated application of x_4's spherical expansion at each time step.

Proof sketch. At time t_i, the wavefunction is concentrated at position x_i. One application of Huygens' principle propagates this concentration forward by ε via convolution with $G_+(x - x_i, \varepsilon)$. The result is a wavefunction distributed over a sphere of radius cε centered at x_i; equivalently, the result is obtained by integrating over all intermediate positions $x(t_i + \varepsilon)$ on this sphere. One more application of Huygens' principle propagates each intermediate position $x(t_i + \varepsilon)$ forward by ε to a new sphere of radius cε centered at $x(t_i + \varepsilon)$; the result is obtained by integrating over all second-step intermediate positions $x(t_i + 2\varepsilon)$. Iterating N times and taking $N \to \infty$ with $t_f - t_i$ fixed, the composition of N convolutions with G_+ becomes the path integral over all continuous paths from x_i to x_f. ∎

Structural note. The *set of all paths* that appears in the Feynman formulation is not postulated by Feynman; it emerges naturally from iterated application of Huygens' secondary-wavelet principle. Each path is a chain of McGucken Spheres connecting the source point to the observation point, with each sphere corresponding to one step of x_4's expansion at rate c. The Feynman path integral is, geometrically, the continuum limit of iterated x_4-expansion [MG-PathInt, §III].

III.3 Proposition L.3: Accumulated x_4-Phase Gives the Feynman Weight

Proposition L.3 *(Path weight from accumulated x_4-phase).*

Along each path in the path set of Proposition L.2, the accumulated x_4-phase — the total phase accrued by x_4's oscillatory advance along the worldline specified by the *path — in the non-relativistic limit and after absorbing an irrelevant rest-energy global phase, is*

phase(path) = exp(i S[path] / ħ)

where S[path] = ∫ L dt is the classical action along the path and L is the classical non-relativistic Lagrangian. The factor i is the perpendicularity marker from $dx_4/dt = ic$ (same i as in $x_4 = ict$), and the factor ħ is the action per x_4-cycle at the Planck frequency (as established in [MG-Constants, §V]).

Derivation. For a particle at spatial rest, the x_4-advance over a short coordinate-time interval ε is $dx_4 = ic\,\varepsilon$, and the x_4-oscillation at the Compton frequency $\omega\_C = mc^2/\hbar$ (established as a theorem of the McGucken Principle in [MG-Noether, Postulate III.3.P] and [MG-Compton]) produces an accumulated phase of $\exp(i\,\omega\_C \cdot \varepsilon) = \exp(i \cdot mc^2 \cdot \varepsilon/\hbar)$. For a particle in motion with three-velocity v, the four-speed budget $|u|^2 = c^2$ (the master equation of Proposition III.2 of [MG-Lagrangian]) means the x_4-advance rate is reduced by the Lorentz factor: $dx_4/dt = ic/\gamma$, where $\gamma = 1/\sqrt{(1 - v^2/c^2)}$. The accumulated x_4-phase per coordinate-time interval ε for a moving particle is therefore $\exp(i \cdot mc^2\gamma^{-1} \cdot \varepsilon/\hbar)$. In the non-relativistic limit $v/c \ll 1$:

$mc^2\gamma^{-1} \approx mc^2 \cdot (1 - v^2/(2c^2)) = mc^2 - (1/2)mv^2$.

Accumulated over ε, the phase becomes $\exp((i\varepsilon/\hbar)[mc^2 - (1/2)mv^2]) = \exp(imc^2\varepsilon/\hbar) \cdot \exp(-(i\varepsilon/\hbar) \cdot (1/2)mv^2)$. The first factor is the rest-energy global phase, which is independent of the path (all paths have the same endpoint times and therefore accumulate the same total rest-energy phase) and can be absorbed into a global rephasing that does not affect interference. The second factor is $\exp(-(i\varepsilon/\hbar) \cdot T)$, where $T = (1/2)mv^2$ is the classical kinetic energy. Adding an external potential V(q) via minimal coupling (gauge-invariant extension of the momentum), the short-interval phase becomes $\exp((i\varepsilon/\hbar) \cdot [T - V]) = \exp((i\varepsilon/\hbar) \cdot L\_classical)$, where $L\_classical = T - V$ is the classical non-relativistic Lagrangian. Integrating this over the full path from t_i to t_f:

phase(path) = exp((i/ħ) ∫ L_classical dt) = exp(i S[path]/ħ).

This is exactly the Feynman path weight [MG-PathInt]. ∎

Structural note. The Feynman phase exp(iS/ħ) is not an independent postulate; it is the accumulated x_4-oscillation phase along the path, in the non-relativistic limit with the global rest-energy rephasing absorbed. The factor i in the exponent is the same i as in $x_4 = ict$ (perpendicularity marker); the factor ħ is the same ħ as in the operator formalism (action per x_4-cycle at the Planck frequency). Both factors are inherited from $dx_4/dt = ic$ through the Compton-frequency coupling of matter to x_4's advance, which itself is a theorem of the Principle [MG-Compton, §2; MG-Noether, Postulate III.3.P].

III.4 Proposition L.4: The Full Feynman Path Integral

Proposition L.4 *(Feynman path integral as continuum limit).*

Composing the short-interval Huygens propagation of Proposition L.2 with the short-interval x_4-phase accumulation of Proposition L.3, the propagator K(q_f, t_f; q_i, t_i) in the continuum limit $N \to \infty$ ($\varepsilon \to 0$) with t_f − t_i fixed is

$$K(q_f, t_f; q_i, t_i) = \int \mathscr{D}x(t) \exp(i S[x(t)] / \hbar),$$

where $S[x(t)] = \int L\, dt$ is the classical action along the path x(t) and the integration measure $\mathscr{D}x(t)$ is inherited from the iterated Huygens composition: $\mathscr{D}x(t) = \lim_{N\to\infty} (m/2\pi i\hbar\varepsilon)^{N/2}\, dx(t_i + \varepsilon)\, dx(t_i + 2\varepsilon) \cdots dx(t_i + (N-1)\varepsilon)$.

Proof sketch. The full kernel is obtained by composing Propositions L.2 and L.3: Proposition L.2 gives the iterated Huygens composition as the sum over all chains of intermediate positions; Proposition L.3 gives the phase accumulated along each chain as $\exp(iS/\hbar)$. The normalization factor $(m/2\pi i\hbar\varepsilon)^{N/2}$ comes from the Gaussian integration of the short-time kernel (see Proposition L.5 below for the explicit form of K_ε). Taking the continuum limit in the usual way [Feynman 1948; MG-PathInt, §IV] yields the standard Feynman path integral expression. ∎

Structural note. Proposition L.4 is the Lagrangian formulation's central object: the Feynman path integral. It is reached here not as a postulate — as in Feynman's original 1948 paper, where the path integral was introduced as an alternative starting point for quantum mechanics — but as a theorem, derived in four propositions from $dx_4/dt = ic$ via Huygens' principle and accumulated x_4-phase. Every element of the path integral (the sum over paths, the phase factor, the factors of i and ħ in the exponent) has explicit geometric content from the McGucken Principle.

III.5 Proposition L.5: The Schrödinger Equation from the Short-Time Propagator

Proposition L.5 *(Schrödinger equation from short-time Feynman kernel).*

Expanding the short-time Feynman kernel

$$K_\varepsilon(q', q) = (m/2\pi i\hbar\varepsilon)^{1/2} \exp\{(i\varepsilon/\hbar)[\tfrac{1}{2}m((q' - q)/\varepsilon)^2 - V(q)]\}$$

and evaluating $\psi(q, t + \varepsilon) = \int K_\varepsilon(q, q')\, \psi(q', t)\, dq'$ to first order in ε by Gaussian integration yields

$$i\hbar\, \partial\psi/\partial t = -(\hbar^2/2m)\, \partial^2\psi/\partial q^2 + V(q)\, \psi,$$

which is the Schrödinger equation.

Proof. The Gaussian integration proceeds in three steps. First, change variables $\eta = q' - q$ so that $dq' = d\eta$ and the integral becomes

$$\psi(q, t + \varepsilon) = \int K_\varepsilon(q, q - \eta)\, \psi(q - \eta, t)\, d\eta = (m/2\pi i\hbar\varepsilon)^{1/2} \int \exp\{(i\varepsilon/\hbar)[m\eta^2/(2\varepsilon^2) - V(q)]\} \cdot \psi(q - \eta, t)\, d\eta.$$

Second, Taylor-expand $\psi(q - \eta, t)$ around $\eta = 0$ to second order: $\psi(q - \eta, t) \approx \psi(q, t) - \eta\, \partial\psi/\partial q + (\eta^2/2)\, \partial^2\psi/\partial q^2$. Third, use the Gaussian integrals $(m/2\pi i\hbar\varepsilon)^{1/2} \int \exp(i\alpha\, \eta^2)\, d\eta = 1$ for $\alpha = m/(2\hbar\varepsilon)$ and $(m/2\pi i\hbar\varepsilon)^{1/2} \int \eta^2 \exp(i\alpha\, \eta^2)\, d\eta = i\hbar\varepsilon/m$ (the latter obtained by differentiating the first with respect to α). The odd moment $\int \eta \cdot \exp(i\alpha \eta^2)\, d\eta = 0$ by symmetry. Putting these together:

$$\psi(q, t + \varepsilon) \approx \exp(-i\varepsilon V(q)/\hbar) \cdot [\psi(q, t) + (i\hbar\varepsilon/(2m)) \cdot \partial^2\psi/\partial q^2]$$

$$\approx (1 - i\varepsilon V(q)/\hbar) \cdot \psi(q, t) + (i\hbar\varepsilon/(2m)) \cdot \partial^2\psi/\partial q^2$$

where in the last step the exponential is expanded to first order in ε. Rearranging:

$$\psi(q, t + \varepsilon) - \psi(q, t) \approx -(i\varepsilon/\hbar)\, V(q)\, \psi + (i\hbar\varepsilon/(2m))\, \partial^2\psi/\partial q^2 + O(\varepsilon^2).$$

Dividing by ε and taking $\varepsilon \to 0$:

$$\partial\psi/\partial t = -(i/\hbar)\, V(q)\, \psi + (i\hbar/(2m))\, \partial^2\psi/\partial q^2.$$

Multiplying both sides by $i\hbar$:

$$i\hbar\, \partial\psi/\partial t = V(q)\, \psi - (\hbar^2/(2m))\, \partial^2\psi/\partial q^2 = [-(\hbar^2/2m)\, \partial^2/\partial q^2 + V(q)]\, \psi.$$

This is the Schrödinger equation with the Hamiltonian $\hat{H} = -(\hbar^2/2m)\, \partial^2/\partial q^2 + V(q)$. The derivation is also carried out in [MG-PathInt, Corollary 5.2], which establishes that the Feynman path integral derived from the McGucken Principle reproduces the time-dependent Schrödinger equation via the same short-time kernel expansion; [MG-PathInt, Proposition 9.1] supplies the additional structural content that the first-order time derivative versus second-order spatial derivative asymmetry of the Schrödinger equation is the mathematical expression of a single uniform x_4-expansion producing a diffusive spatial spreading. ∎

Structural note. The Schrödinger equation is reached on the Lagrangian route not as an independent postulate but as the non-relativistic limit of accumulated x_4-phase in the short-time Feynman kernel. The factor $i\hbar$ on the left-hand side is the same $i\hbar$ that appears in the canonical commutation relation derived through the Hamiltonian route (Proposition H.4); both routes arrive at the same constants through completely different chains.

III.6 Proposition L.6: Recovering the CCR via the Schrödinger Equation's Momentum Operator

Proposition L.6 *(CCR from the Schrödinger equation's momentum operator).*

The Schrödinger equation derived in Proposition L.5 contains the momentum operator $\hat{p} = -i\hbar\, \partial/\partial q$ *implicitly, as the operator whose square appears in the kinetic term* $-(\hbar^2/2m)\, \partial^2/\partial q^2 = \hat{p}^2/2m$*. Extracting this momentum operator and computing* $[\hat{q}, \hat{p}]$ *with* $\hat{q}$ *acting by multiplication yields*

$$[\hat{q}, \hat{p}] = i\hbar\, \mathbb{1},$$

the canonical commutation relation, recovered through the Lagrangian route by traversing Propositions L.1 through L.5 and then identifying the momentum operator in the Schrödinger equation's kinetic term.

Proof. The kinetic term in the Hamiltonian of Proposition L.5 is $-(\hbar^2/2m)\, \partial^2/\partial q^2$. If we identify this with $\hat{p}^2/(2m)$, then $\hat{p}^2 = -\hbar^2\, \partial^2/\partial q^2$. Taking the square root (up to sign, fixed by the requirement that $\hat{p}$ be Hermitian): $\hat{p} = -i\hbar\, \partial/\partial q$ or $\hat{p} = +i\hbar\, \partial/\partial q$. The first choice is the standard Schrödinger-representation momentum operator, matching Proposition H.3 of the Hamiltonian route. Computing $[\hat{q}, \hat{p}]$ by the same computation as in Proposition H.4:

$$[\hat{q}, \hat{p}]f(q) = q\cdot(-i\hbar\, \partial f/\partial q) - (-i\hbar\, \partial/\partial q)(q\cdot f) = i\hbar\, f(q),$$

which gives $[\hat{q}, \hat{p}] = i\hbar\, \mathbb{1}$. ∎

Structural note. Proposition L.6 closes the Lagrangian route at the same destination as the Hamiltonian route: the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$. But the two routes reached this destination through completely different intermediate structures. The Hamiltonian route used Stone's theorem on translation invariance to arrive at the momentum operator directly; the Lagrangian route reached the momentum operator indirectly, through Huygens' principle, iterated sphere expansion, accumulated phase, the Feynman path integral, the Schrödinger equation, and identification of the kinetic term with $\hat{p}^2/(2m)$. The two routes agree on the destination despite sharing no intermediate machinery — the structural signature of a correct foundation developed in §VII.

IV. Structural Comparison of the Two Routes

The two derivations of §§II–III reach the same algebraic identity $[\hat{q}, \hat{p}] = i\hbar$ from the same starting principle $dx_4/dt = ic$, through completely different intermediate structures. This section catalogs the structural agreements and disagreements between the two routes in detail, identifies what each route uniquely illuminates, and establishes the basis for the overdetermination principle developed in §VII.

IV.1 Point-by-Point Comparison

The comparison table below summarizes the structural content of the two routes step by step. Each row identifies a structural element; each column identifies which route uses it and how.

Structural element	Hamiltonian route (§II)	Lagrangian route (§III)
Starting principle	$dx_4/dt = ic$	$dx_4/dt = ic$
First intermediate	Minkowski metric $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - c^2\, dt^2$ from $x_4 = ict$ (Prop. H.1)	Huygens' principle: $\Sigma_+(p_0)$ = forward light cone = McGucken Sphere (Prop. L.1)
Second intermediate	Stone's theorem on translation group: $U(a) = \exp(-ia\, \hat{p}/\hbar)$ (Prop. H.2)	Iterated Huygens: sum over all chains of spherical expansions = sum over all paths (Prop. L.2)

Structural element	Hamiltonian route (§II)	Lagrangian route (§III)
Third intermediate	Configuration representation: $\hat{p} = -i\hbar\, \partial/\partial q$ by direct differentiation of U(a) (Prop. H.3)	Accumulated x_4-phase along path: $\exp(iS/\hbar)$ from Compton-frequency coupling in non-relativistic limit (Prop. L.3)
Fourth intermediate	Commutator $[\hat{q}, \hat{p}] = i\hbar$ by direct three-line computation (Prop. H.4)	Full Feynman path integral $K = \int \mathscr{D}x \exp(iS/\hbar)$ as continuum limit (Prop. L.4)
Fifth intermediate	Stone-von Neumann uniqueness of the Schrödinger representation (Prop. H.5)	Schrödinger equation $i\hbar\partial\psi/\partial t = \hat{H}\psi$ from Gaussian integration of short-time propagator (Prop. L.5)
Sixth intermediate	(not needed; route closes at fifth step)	CCR $[\hat{q}, \hat{p}] = i\hbar$ from Schrödinger kinetic term's momentum operator (Prop. L.6)
Final destination	$[\hat{q}, \hat{p}] = i\hbar\, \mathbb{1}$	$[\hat{q}, \hat{p}] = i\hbar\, \mathbb{1}$
Where i enters	Minkowski signature → unitary exponent → momentum operator	Accumulated x_4-oscillation phase → path-integral weight → Schrödinger equation
Where $\hbar$ enters	Action-per-x_4-cycle scale of unitary representation	Action-per-x_4-cycle in denominator of path-integral exponent
Mathematical machinery	Stone's theorem, direct differentiation, Stone-von Neumann theorem	Huygens convolution, iterated composition, Gaussian integration, Taylor expansion
Geometric content used	Perpendicularity of x_4 ($i^2 = -1$), translation invariance	Spherical symmetry of x_4's expansion, Compton-frequency coupling
Dual-channel aspect	Algebraic-symmetry channel of $dx_4/dt = ic$	Geometric-propagation channel of $dx_4/dt = ic$

The first and last rows show the only structural agreements between the two routes: they start from the same principle and reach the same destination. Every row in between shows a structural difference. The routes use disjoint mathematical machinery (Stone's theorem versus Huygens convolution; direct commutator computation versus

Gaussian integration of Feynman kernels; Stone-von Neumann versus Schrödinger derivation). They pass through disjoint intermediate structures (Minkowski metric versus Huygens' principle; unitary translation generator versus path-integral weight; direct algebra versus integral representation). They use disjoint content of the starting principle (algebraic-symmetry content in the Hamiltonian route; geometric-propagation content in the Lagrangian route).

IV.2 The Disjointness of Intermediate Content

A potential objection to the two-route structure would be that the two chains secretly share intermediate content, making them variants of a single derivation rather than genuinely independent. This objection is precluded by the disjointness catalog above: the Hamiltonian route's intermediate structures (Minkowski metric, Stone's theorem, configuration representation, direct commutator) and the Lagrangian route's intermediate structures (Huygens' principle, iterated sphere summation, accumulated phase, Feynman kernel, Gaussian-integrated Schrödinger equation) share no common element. The only common elements are the start ($dx_4/dt = ic$) and the end ($[\hat{q}, \hat{p}] = i\hbar$).

The disjointness of intermediate content has a sharper form. In the Hamiltonian route, the Minkowski metric appears as the algebraic consequence of $x_4 = ict$'s perpendicularity, and Stone's theorem is applied directly to the Hilbert-space representation of spatial translations on the background of this Minkowski geometry. Huygens' principle does not appear anywhere in the Hamiltonian route, nor does the sum over paths, nor the Feynman kernel, nor the Schrödinger equation (the Schrödinger equation is *implied* by the commutation relation and the Stone-von Neumann representation, but it is not *derived* in the course of the route; the route derives the CCR directly). In the Lagrangian route, Huygens' principle appears as the geometric-propagation content of x_4's spherical expansion, and it is iterated to generate the path set. Stone's theorem does not appear anywhere in the Lagrangian route, nor does the configuration representation, nor does the commutator computation of Proposition H.4 (the commutator is *implied* by the derived Schrödinger equation's momentum operator, but it is not *computed directly* in the course of the route; the route reaches the CCR through the Schrödinger-operator extraction of Proposition L.6).

IV.3 What Each Route Uniquely Illuminates

Although the two routes reach the same destination, each route illuminates structural features of the CCR that the other route leaves implicit.

The Hamiltonian route emphasizes the *algebraic-symmetry* content of the commutation relation. $[\hat{q}, \hat{p}] = i\hbar$ is derived through Stone's theorem on spatial-translation unitary representations, and the Stone-von Neumann closure establishes that the representation is unique up to unitary equivalence. The commutation relation's algebraic structure, its relationship to the Heisenberg Weyl group, its uniqueness up to unitary equivalence, and its direct derivation from symmetry — all of these are foregrounded by the Hamiltonian route. The route reveals the commutation relation's character as an algebraic consequence of translation invariance on a Hilbert space.

The Lagrangian route emphasizes the *geometric-propagation* content of the commutation relation. $[\hat{q}, \hat{p}] = i\hbar$ is derived through Huygens' principle on spacetime, iterated spherical expansion of x_4, and the non-relativistic limit of accumulated path phase. The commutation relation's origin in the phase structure of matter coupled to x_4's oscillation at the Compton frequency, its emergence through the path integral's Gaussian integration, and its connection to the geometric propagation of wavefronts in spacetime — all of these are foregrounded by the Lagrangian route. The route reveals the commutation relation's character as a geometric consequence of spherical expansion in spacetime.

Both characterizations are correct. The commutation relation is *both* an algebraic consequence of translation invariance *and* a geometric consequence of spherical expansion. The two routes are two proofs of the same theorem from two different structural vantage points, and each proof reveals a structural aspect of the theorem that the other proof leaves implicit.

IV.4 Structural Agreement on the Constants

The two routes agree on the factor i and the factor $\hbar$ that appear in the commutation relation $[\hat{q}, \hat{p}] = i\hbar$, and they agree on the physical origin of both factors.

The factor i on the Hamiltonian route enters at Proposition H.1 as the i in $x_4 = ict$ (the perpendicularity marker of x_4), propagates through the Minkowski metric signature, enters the unitary exponential of Stone's theorem as the i in $U(a) = \exp(-ia\,\hat{p}/\hbar)$, enters the momentum operator $\hat{p} = -i\hbar\,\partial/\partial q$ as the same i, and appears in the commutation relation $[\hat{q}, \hat{p}] = i\hbar$ by direct computation. On the Lagrangian route, the factor i enters at Proposition L.3 as the i in the accumulated x_4-phase $\exp(i\,\omega\_C\,\varepsilon)$ from the Compton-frequency coupling (which itself carries the i from $x_4 = ict$), propagates through the full path integral weight $\exp(iS/\hbar)$, appears in the Schrödinger equation $i\hbar\,\partial\psi/\partial t = \hat{H}\psi$, and is recovered in the commutation relation via the Schrödinger momentum operator. Both routes trace the i to the same geometric fact: the perpendicularity of x_4 to the three spatial dimensions, encoded algebraically as the imaginary unit i.

The factor $\hbar$ on both routes traces to the same physical origin: the quantum of action per complete x_4-oscillation cycle at the Planck frequency f_P, as established in [MG-Constants, §V]. On the Hamiltonian route, $\hbar$ appears in the unitary exponential as the natural scale of the representation (determined by Stone's theorem with a scale parameter, fixed physically by the requirement that U(a) agree with the physical translation at scale a in units where the Planck action is $\hbar$). On the Lagrangian route, $\hbar$ appears in the path-integral weight as the scale of the Compton-frequency coupling $\omega\_C = mc^2/\hbar$ in the accumulated phase, which is directly the Planck-action scale. Both routes identify $\hbar$ with the same physical quantity: the action per x_4-cycle at the Planck scale.

IV.5 Summary of the Structural Comparison

The two routes agree on the starting principle ($dx_4/dt = ic$), on the final destination ($[\hat{q}, \hat{p}] = i\hbar$), and on the physical origins of the factors i and $\hbar$ in that destination. They disagree on — or rather, are disjoint on — every intermediate structure: mathematical machinery, intermediate theorems, logical steps, geometric content used. The disjointness of intermediate structure combined with the agreement on start and end points is the precise structural signature of a correct physical foundation developed in §VII: when two independent chains from a single principle reach the same algebraic identity through disjoint routes, the identity is not an artifact of derivational choice but a genuine consequence of the principle, and each chain confirms the identity by a disjoint body of evidence.

V. Why $dx_4/dt = ic$ Has This Property

The two-route derivation of §§II–III establishes that both the Hamiltonian and Lagrangian formulations of quantum mechanics descend as theorems from $dx_4/dt = ic$. This section addresses the deeper question: *why does this principle, specifically, possess this structural property?* The answer developed here is that $dx_4/dt = ic$ is a principle with *dual-channel content* — a single geometric statement that simultaneously specifies two logically distinct pieces of information, each of which drives one of the two routes. No prior candidate foundation for quantum mechanics has had this dual-channel content, and the principle's possession of it is what makes the two-route derivation possible.

V.1 The Two Channels Identified

The geometric statement *"$dx_4/dt = ic$"* combined with the physical interpretation *"x_4 advances at the velocity of light from every spacetime point, spherically symmetrically about each point"* contains two logically distinct pieces of information. The decomposition of the principle into its two components can be stated as follows.

Channel A (Algebraic-symmetry channel). The principle specifies that x_4's advance has a *uniform rate* ic that is *invariant* under spacetime isometries: the advance rate is the same at every spacetime point (translation invariance), independent of direction in the three spatial dimensions (rotation invariance), and form-invariant under Lorentz boost (Lorentz invariance). These invariances generate the Poincaré-group symmetries of the Minkowski spacetime derived in Proposition III.1 of [MG-Lagrangian] and the ten Poincaré conservation laws derived in [MG-Noether]. The channel's content — uniformity plus invariance — is precisely the content needed to apply Stone's theorem to the unitary representations of the spacetime symmetry group, which drives the Hamiltonian route of §II. The factor i in the unitary generators, the factor $\hbar$ as the scale of the representation, the canonical commutation relation as a consequence of translation invariance — all of these are consequences of the algebraic-symmetry channel.

Channel B (Geometric-propagation channel). The principle specifies that x_4's advance is *spherically symmetric* about every spacetime point — the advance at rate c radiates equally into all three-dimensional directions from each point of emission. This spherical symmetry generates the McGucken Sphere geometry of [MG-Master, Part IV], which is precisely the forward light cone of Minkowski spacetime and which is precisely Huygens' secondary-wavelet structure. The channel's content — spherical emission from every point with radial rate c — is precisely the content needed to generate Huygens' principle as a theorem (Proposition L.1) and to iterate the Huygens expansion into a sum over paths (Proposition L.2), which drives the Lagrangian route of §III. The Feynman path integral as a sum over chains of McGucken Spheres, the accumulated x_4-phase along each path, the Schrödinger equation from Gaussian integration — all of these are consequences of the geometric-propagation channel.

Both channels are present in the principle's single geometric statement. "x_4 advances at ic from every point" is the algebraic-symmetry content (uniform rate, invariance under spacetime translations). "Spherically symmetrically about each point" is the geometric-propagation content (isotropic wavefront expansion, Huygens' secondary wavelets). The two channels are not alternative readings of the principle; they are *simultaneously valid readings*, each unpacking a different aspect of the same geometric fact.

V.2 Why Both Channels Are Needed

The two quantum formulations require structurally different content from the foundation. The Hamiltonian formulation requires algebraic-symmetry content: a Hilbert space carrying unitary representations of a translation group, with Stone's theorem generating the momentum operator. The Lagrangian formulation requires geometric-propagation content: a wavefront structure on spacetime, with Huygens' secondary wavelets generating the path-summation representation. A foundation that provides only one of these contents can generate only one of the quantum formulations (as derived result); it can reach the other formulation only through the equivalence theorems established after the fact (Feynman 1948, Stone-von Neumann 1931-1932), but not as an independent derivation from the foundation itself.

A foundation with *both* channels can generate both formulations as independent derivations. $dx_4/dt = ic$ is such a foundation. Its algebraic-symmetry content generates the Hamiltonian route; its geometric-propagation content generates the Lagrangian route. The derivations are independent — disjoint intermediate structures, disjoint mathematical machinery — because they use disjoint contents of the principle.

V.3 The Unique Dual-Channel Property

The uniqueness claim of this section is as follows: $dx_4/dt = ic$ is the only known candidate foundation for quantum mechanics whose single geometric statement contains both channels simultaneously. This section develops the claim through a systematic analysis of candidate foundations and their channel structure.

Classical Lagrangian variational mechanics. The principle of least action $\delta S = 0$ combined with a classical Lagrangian $L(q, \dot{q}, t)$ generates Newton's equations through the Euler-Lagrange equations. This is a variational principle with propagation content (paths extremize the action), but it does not generate a symmetry structure on a Hilbert space — the quantum operator formalism is not derived, and the commutation relations are not consequences of the principle. Classical Lagrangian mechanics has content analogous to Channel B (propagation) but lacks the algebraic-symmetry content of Channel A. To quantize classical Lagrangian mechanics, additional structure (canonical quantization, or geometric quantization, or path-integral quantization) must be imposed. The principle of least action does not by itself generate quantum mechanics.

Classical symplectic geometry. A phase space (M, ω) with symplectic form ω supports Hamiltonian dynamics through the vector field X_H defined by $\omega(X_H, \cdot) = dH$, for a Hamiltonian function $H: M \to \mathbb{R}$. This is an algebraic structure on phase space that supports the Hamiltonian formulation of classical mechanics (Poisson brackets, canonical transformations, phase flow). Geometric quantization [Kostant 1970; Souriau 1970; Woodhouse 1992] constructs a Hilbert space and quantum operators from (M, ω) through prequantization (a line bundle with curvature ω), polarization (a foliation selecting half the coordinates), and metaplectic correction. This generates the Hamiltonian operator formulation with the factor i in the commutators and the factor $\hbar$ as the scale of the prequantization line bundle's curvature. Classical symplectic geometry has content analogous to Channel A (algebraic-symmetry, through the symplectic form's algebraic structure) but lacks the geometric-propagation content of Channel B — the path integral does not emerge naturally from symplectic geometry; it must be constructed separately (the Feynman-Kac formula from stochastic analysis, or a path-integral quantization applied after the fact). Geometric quantization reaches the Hamiltonian formulation but does not reach the Lagrangian formulation as an independent derivation; both formulations are equivalent at the end (by Stone-von Neumann and by Feynman's proof that the path integral reproduces the Schrödinger equation), but only one is derived from geometric quantization's starting point.

Stochastic dynamics (Nelson, Lindgren-Liukkonen). Stochastic mechanics postulates that particles undergo a classical-mechanical motion with superimposed Brownian-like fluctuations whose diffusion coefficient is $\hbar/(2m)$. From the resulting drift-diffusion equations, the Schrödinger equation is derived. Lindgren-Liukkonen's stochastic optimal control requires a Lorentz-invariant stochastic action, which forces the Lagrangian to be imaginary ($\sqrt{\det g} = i$ in Minkowski signature) and the noise variance to be imaginary ($\sigma^2 = i/m$). This generates the Schrödinger equation through a route analogous to Channel B (propagation, via stochastic fluctuations), but does not generate the operator formalism as an independent derivation. The stochastic frameworks have partial Channel B content but lack Channel A content; they reach the Schrödinger equation, but they do not separately derive the canonical commutation relation or the Hilbert-space representation structure. Moreover, Nelson's framework takes $\hbar$ as an input to the diffusion coefficient rather than deriving it, and Lindgren-Liukkonen takes the analytic continuation (the i) as a mathematical requirement of Lorentz invariance rather than identifying it with a physical fact.

Geometric algebra (Hestenes). Hestenes's spacetime algebra reinterprets i as a bivector $i\sigma_3 = \gamma_2\gamma_1$ in Cl(1,3), giving i a static geometric identity. The Dirac equation, the CCR, and the Schrödinger equation can all be reformulated in Hestenes's algebra. But geometric algebra is a reinterpretation of existing physics, not a derivation of quantum mechanics from a deeper principle: the Dirac equation, the CCR, and the Schrödinger equation are inputs to be reformulated rather than outputs to be derived. Hestenes's framework has static Channel-B-like content (geometric identity of i) but lacks a dynamical principle that would generate either Channel A or Channel B from deeper structure.

Trace dynamics (Adler). Adler's program derives quantum mechanics as a statistical thermodynamic average of a deeper classical matrix dynamics. The CCR emerges as a Ward identity of the canonical ensemble's equipartition theorem: $\langle \tilde{C} \rangle = i\hbar \mathbb{1}$. This has partial Channel A content (algebraic structure, inherited from the matrix dynamics) but the i is taken as built into the matrix algebra rather than derived, and $\hbar$ emerges as a temperature-like parameter of the equilibrium rather than as a geometric quantum of action. Trace dynamics does not generate the path integral as an independent derivation; it is one-directional from matrix dynamics to quantum mechanics via statistical averaging.

Cellular automata ('t Hooft). 't Hooft proposes that quantum mechanics emerges from a deterministic cellular automaton at the Planck scale. The Schrödinger equation and CCR are emergent features of the automaton-to-quantum mapping. This has neither Channel A nor Channel B content in the sense developed here: the principle is discrete-determinism rather than continuous-geometric, and the i and $\hbar$ of quantum mechanics are emergent features rather than derivations from the underlying structure.

Bohmian mechanics. Bohm's hidden-variable formulation supplements the Schrödinger equation with a guiding equation for particle trajectories. Both the Schrödinger equation and the guiding equation are postulates; Bohmian mechanics is an interpretation of existing quantum mechanics, not a derivation from a deeper principle.

Loop quantum gravity (Ashtekar and successors). Ashtekar's formulation of general relativity in terms of connection variables provides a Hamiltonian formulation of gravity whose canonical quantization generates the spin-network structure of loop quantum gravity. This is a Hamiltonian-formulation derivation within gravity but does not generate the Lagrangian path integral as an independent derivation; moreover, the starting point (Ashtekar's connection variables) is itself a classical Hamiltonian reformulation of general relativity rather than a deeper geometric principle that would generate both formulations independently.

Twistor theory (Penrose and Witten). Penrose's twistor theory and Witten's twistor string reformulate spacetime physics in terms of the complex projective space CP^3. This is a geometric reformulation with elegant mathematical properties (Proposition X.1 of [MG-Twistor]), but twistor theory does not by itself generate the Hamiltonian and Lagrangian formulations of quantum mechanics from a principle; the quantum-mechanical content is imported from standard physics and reformulated in twistor variables.

Constructive gravity (Schuller). Schuller's 2020 constructive-gravity programme [arXiv:2003.09726] shows that given the matter Lagrangians' principal polynomials, the compatible gravitational dynamics is uniquely determined as the Einstein-Hilbert action. This is a *consequence* derivation — given matter, determine gravity uniquely — which presupposes the matter Lagrangians already in Lagrangian form. Schuller's programme has partial Channel B content (propagation structure determined by principal polynomials) but does not generate the quantum-mechanical Hamiltonian formulation as an independent derivation; the CCR and operator algebra are inputs from quantum-mechanical matter Lagrangians rather than outputs from the constructive-gravity starting point.

Weinberg's Lagrangian QFT. Weinberg's derivation requires the S-matrix to be Lorentz-invariant and to satisfy cluster decomposition, from which he derives the necessity of quantum fields (as cluster-decomposition-compatible Fourier modes of creation/annihilation operators) and the Lagrangian formalism as the natural generator of Lorentz-invariant S-matrix elements. This presupposes the operator formalism (commutation relations for bosons, anticommutation for fermions, unitary representation of the Poincaré group) and derives the Lagrangian on top of it. Weinberg's approach is *one-directional*: from operator formalism to Lagrangian, not from a common foundation to both. Moreover, Lorentz invariance and cluster decomposition are themselves physical postulates.

The pattern across these candidate foundations is that each possesses partial content matching one channel but not the other, or possesses neither channel but instead a different structure (emergent, interpretational, reinterpretational). No prior candidate foundation for quantum mechanics possesses both channels simultaneously in a single geometric-dynamical statement.

V.4 Why $dx_4/dt = ic$ Is the Unique Dual-Channel Foundation

The uniqueness of $dx_4/dt = ic$ among candidate quantum foundations — in possessing both channels simultaneously — is not accidental. It reflects the specific structure of the geometric statement: *a physical axis advancing at the velocity of light, uniformly from every point, spherically symmetrically.*

The uniformity of the advance (rate ic at every point, invariant under spacetime translations and Lorentz boosts) is the algebraic-symmetry content. The spherical symmetry of the advance (isotropic expansion from each point, generating Huygens' secondary wavelets at each point) is the geometric-propagation content. The two contents are logically independent — one can imagine (pathologically) a principle with uniform-rate non-spherical advance, or spherical advance at non-uniform rate — but they are both required components of $dx_4/dt = ic$ as a physical principle describing the dynamics of a real geometric axis. The reason the principle has both contents is that real physical axes in three-dimensional space, when they advance, must advance *from every point* (uniformity) *in every direction* (isotropy), because space is homogeneous and isotropic. The dual-channel content of $dx_4/dt = ic$ is therefore *forced by the minimal physical interpretation of the mathematical statement*: a geometric axis advancing at rate ic must advance uniformly and spherically, and these are the two channels.

This is why no prior candidate foundation has the dual-channel property. Classical symplectic geometry is an algebraic structure on phase space; it has no “advance” of any kind, so it cannot have the spherical-propagation content. Stochastic dynamics is a time-evolution structure on configuration space; it has propagation content but no symmetry structure built into its starting postulate. Geometric algebra is a static reformulation of Minkowski spacetime; it has algebraic structure but no dynamics. ’t Hooft’s cellular automaton is a discrete-determinism structure; it has neither continuous propagation nor continuous symmetry content.

$dx_4/dt = ic$ is the first candidate foundation for quantum mechanics whose statement is both *geometric* (specifying a real physical axis in spacetime) and *dynamical* (specifying the advance of that axis at a definite rate), with the minimal physical interpretation of that statement requiring both channels simultaneously. It is the first candidate foundation whose dual-channel content makes the two-route derivation possible.

V.5 The Compression of Physics

The dual-channel structure identified here has a deeper implication for the foundational structure of physics. The Hamiltonian and Lagrangian formulations of quantum mechanics are not merely alternative mathematical frameworks for the same physics; they are the two distinct algebraic expressions of the two distinct informational channels of a single underlying geometric fact. When the standard textbook presents them as alternatives, it obscures the fact that they are theorems of one principle. When the McGucken Principle presents both as consequences, it reveals the fact that they are not alternatives — they are shadows of a single deeper structure cast in two different mathematical languages.

This is the compression of physics that a correct foundation accomplishes. Two formulations collapse to one principle. Every factor of i and every factor of $\hbar$ in both formulations collapses to a single geometric fact: the perpendicularity and the oscillation-scale of x_4’s advance. The operator formalism and the path-integral formalism collapse to two readings of one equation. Einstein’s standard — “the more impressive the greater the simplicity of its premises, the more different kinds of things it relates, and the more extended is its area of applicability” — is met: one equation $dx_4/dt = ic$ relates the full algebraic content of the operator formulation, the full propagation content of the path-integral formulation, and every derived structure of quantum mechanics from the Schrödinger equation to the canonical commutation relation to the uncertainty principle to the Born rule to the Dirac equation. The simplicity of the premise and the range of applicability stand in the relation Einstein identified as the mark of a correct theory.

V.6 Wave/Particle Duality as the Dual-Channel Reading at the Ontological Level

The dual-channel analysis of §V.1 was developed to explain why the McGucken Principle generates both the Hamiltonian and Lagrangian *formulations* of quantum mechanics. The same analysis applies, with a single structural shift in focus, to explain the phenomenon that has served as the central philosophical puzzle of quantum mechanics for the past century: *wave/particle duality*.

V.6.1 The Century-Old Puzzle

Wave/particle duality is the empirical observation that quantum objects — photons, electrons, neutrons, atoms, and in modern experiments molecules up to C_60 and larger — simultaneously exhibit wave-like behavior (interference in double-slit geometries, diffraction through apertures, coherent superposition) and particle-like behavior (localized detection events, discrete energy and momentum exchange, quantized absorption and emission). The duality was identified in its modern form by Einstein in his 1905 photoelectric paper [Einstein 1905a], which established that light exhibits particle-like energy quantization $E = h\nu$ even though Young's 1801 double-slit experiment had established its wave-like interference. It was extended to matter by de Broglie's 1924 doctoral dissertation [de Broglie 1924], which postulated that every particle of mass m and momentum p has an associated wavelength $\lambda = h/p$ — the *de Broglie relation* — and was confirmed experimentally by the Davisson-Germer 1927 electron-diffraction observations [Davisson-Germer 1927] and the Thomson 1928 electron-diffraction images [Thomson 1928].

The theoretical response to the duality across the ninety-eight years since de Broglie has been, almost uniformly, to treat it as a *puzzle to be interpreted*. Bohr's 1927 Como lecture [Bohr 1928] introduced the *principle of complementarity*: wave and particle aspects are *incompatible* descriptions applying in mutually exclusive experimental contexts, and a complete description of any quantum experiment requires specifying which context is realized. Heisenberg's 1927 uncertainty relation [Heisenberg 1927] supplied the quantitative complement to complementarity: sharp position (particle aspect) and sharp momentum (wave aspect) cannot be simultaneously specified, with the trade-off $\Delta x \cdot \Delta p \geq \hbar/2$. Born's 1926 probabilistic interpretation [Born 1926a] mediated between the two aspects by treating the wavefunction's squared modulus as the probability density for particle localization, so that the wave evolves deterministically until measurement forces a particle-like outcome.

Alternative interpretations arose in response to the perceived inadequacy of Copenhagen's "complementarity" as an explanation: de Broglie's pilot-wave interpretation [de Broglie 1927], developed by Bohm [Bohm 1952] and extended by Dürr-Goldstein-Zanghì [Dürr-Goldstein-Zanghì 1992] and others, treats particles as guided by a physically real wavefunction; Everett's relative-state formulation [Everett 1957], developed into the many-worlds interpretation by DeWitt [DeWitt 1970] and Wallace [Wallace 2012], treats the wavefunction as physically real with particle-like localization corresponding to branches of the universal wavefunction; Wheeler's delayed-choice experiments [Wheeler 1978, 1984], realized experimentally by Jacques et al. [Jacques 2007], sharpen the puzzle by showing that the decision to treat a photon as "particle" or "wave" can be made after it has already traversed the apparatus; Scully and Drühl's quantum-eraser analysis [Scully-Drühl 1982], realized experimentally by Kim et al. [Kim 2000] and Walborn et al. [Walborn 2002], further sharpens the puzzle by showing that "which-path" information can be erased after the fact, restoring interference.

Modern experimental work has established the duality in ever more exotic regimes. Zeilinger's program has observed interference for neutrons [Rauch-Werner 1975; Summhammer 1983], atoms [Kasevich-Chu 1991; Keith 1991], C_60 and C_70 fullerenes [Arndt 1999; Nairz-Arndt-Zeilinger 2003], phthalocyanine [Juffmann 2012], and even larger molecules up to ~2000 atomic mass units [Fein 2019]. In every such experiment, the quantum object behaves as a wave (producing an interference pattern) when propagating through the apparatus *and* as a particle (producing a discrete detection event at a specific pixel of the detector) when observed. The duality is universal, not restricted to "pure" photons or electrons; it is a structural feature of quantum mechanics at every mass scale accessible to experiment.

Across the ninety-eight years of theoretical engagement with wave/particle duality, no prior framework has dissolved the puzzle — each has interpreted, reframed, or accepted the duality as an irreducible feature of quantum reality, but none has explained *why* quantum objects must simultaneously exhibit both aspects, and none has derived the two aspects as simultaneous consequences of a single deeper geometric principle.

V.6.2 The Dual-Channel Reading

Under the McGucken Principle $dx_4/dt = ic$, wave/particle duality is not a puzzle requiring interpretation; it is the direct consequence of the Principle's dual-channel content, applied at the ontological level.

Channel B (geometric-propagation) generates the wave aspect. The spherical symmetry of x_4's expansion from every spacetime point is, by Proposition L.1, Huygens' principle — every spacetime point is the center of a secondary wavelet, and iterated Huygens composition (Proposition L.2) generates wave-front propagation through spacetime. The interference patterns observed in the double-slit experiment are the constructive and destructive superposition of these Huygens wavelets from the two slits. The diffraction patterns observed in single-slit geometries are the same Huygens wavelets expanded from each point of the slit aperture. The matter-wave wavelength $\lambda_{dB} = h/p$ observed in Davisson-Germer, Thomson, and all subsequent matter-wave experiments is, by the derivation of [MG-deBroglie, Theorem 4], the x_4-phase accumulation rate of matter per unit of spatial motion — the Compton-frequency coupling of Channel B producing oscillatory wavefronts with the specific wavelength that de Broglie postulated in 1924. The wave aspect of quantum objects is *entirely* the Channel B reading of $dx_4/dt = ic$: propagating wavefronts produced by iterated x_4-sphere expansion from every spacetime point.

Channel A (algebraic-symmetry) generates the particle aspect. The invariance of x_4's advance under spacetime translations is, by Proposition H.2, Stone's theorem applied to the translation group — and the self-adjoint generators of those translations are, by Proposition H.3, the four-momentum operators $\hat{p}^\mu$ whose eigenvalues are the localized four-momenta of particle states. The discrete detection events observed at specific pixels of the detector screen are eigenvalue events of the position observable $\hat{q}$ — sharp eigenvalues at localized spacetime points. The quantized energy and momentum exchanges observed in the photoelectric effect (Einstein 1905), Compton scattering (Compton 1923), and every other "particle-like" process are the eigenvalue exchanges of Channel A's algebraic observables: discrete values of energy and momentum conserved in individual scattering events. The Heisenberg uncertainty relation $\Delta x \cdot \Delta p \geq \hbar/2$ — the quantitative expression of wave/particle complementarity — is, by [MG-Uncertainty], a theorem about the Fourier-dual structure of the x_4-phase whose algebraic content is Channel A and whose propagation content is Channel B. The particle aspect of quantum objects is *entirely* the Channel A reading of $dx_4/dt = ic$: localized eigenvalue structure generated by the algebraic-symmetry content of x_4's advance.

A photon traveling through a double-slit apparatus *does* both simultaneously. Its Channel B content is the spherical Huygens wavelets emanating from every spacetime point the photon's wavefront reaches — including both slits, producing the interference pattern on the screen. Its Channel A content is the localized detection event at a specific screen pixel — the eigenvalue of the position observable at the moment of detection. Both are real; both are simultaneous; both are consequences of the same $dx_4/dt = ic$. There is no contradiction because the two readings are not competing descriptions of the same thing — they are two simultaneous readings of one geometric principle, corresponding to two distinct informational contents present in the principle's statement.

Applied systematically, the dual-channel reading resolves each of the classical puzzles of wave/particle duality.

The double-slit puzzle. Why does the interference pattern require both slits to be open? Channel B reading: because the Huygens wavelets from both slits interfere constructively and destructively at each point of the screen, and closing one slit removes one set of wavelets, destroying the interference. Why does the pattern vanish when which-slit information is obtained? Channel A reading: because a which-slit measurement is an eigenvalue event of the slit-position observable, and an eigenvalue event is a Channel A phenomenon that is structurally orthogonal to the Channel B propagation that produces interference. Under the dual-channel reading, obtaining which-slit information forces the system into Channel A eigenvalue-registration mode, suppressing the Channel B interference. This is not a philosophical puzzle to be interpreted via complementarity; it is the structural consequence of having chosen to read one channel rather than the other at the measurement.

The delayed-choice puzzle [Wheeler 1978; Jacques 2007]. Why can the decision to observe wave or particle behavior be made *after* the photon has traversed the apparatus? Because both readings are simultaneously available at every spacetime point along the photon's path, not produced retroactively by the measurement. The photon's Channel B wavefront is present throughout the apparatus; the Channel A eigenvalue event is produced at the detector. The "delayed choice" is a choice of which channel to read at the final detector, not a retroactive alteration of what happened earlier. Wheeler's puzzle dissolves because there was never a moment at which the photon "had to be" a wave or a particle; both channels were present throughout, and the detection selects which one to register.

The quantum-eraser puzzle [Scully-Drühl 1982; Kim 2000]. Why can interference be restored after which-path information has been obtained and then erased? Because Channel A and Channel B are not temporally ordered — the eigenvalue structure and the wavefront structure coexist at every spacetime point. "Obtaining" and "erasing" which-path information correspond to operational choices about which channel to correlate with the final detection; they do not alter the underlying dual-channel content of $dx_4/dt = ic$. The quantum eraser experiments are demonstrations of the simultaneity of the two channels, not of retroactive causation.

The matter-wave puzzle. Why do electrons, neutrons, atoms, and molecules exhibit wave behavior with wavelength $\lambda = h/p$? Because the Channel B content of $dx_4/dt = ic$ applies to any matter coupled to x_4's oscillation, and the de Broglie relation $\lambda = h/p$ is (by [MG-deBroglie]) the x_4-phase accumulation rate of matter at its Compton frequency $\omega_C = mc^2/\hbar$. The matter-wave wavelength is not a mysterious feature specific to quantum mechanics; it is the wavelength of the oscillatory wavefront that every massive particle carries through Channel B of the Principle.

The molecular-interference puzzle [Arndt 1999; Fein 2019]. Why do C_{60} and larger molecules still exhibit interference despite their large mass and internal structure? Because Channel B applies to every object with non-zero rest mass, with the de Broglie wavelength $\lambda = h/(Mv)$ scaled by the total mass; for sufficiently low velocities, the wavelength remains comparable to the slit spacing even for very large molecules, and Channel B's interference is observable. There is no lower limit to wave behavior imposed by "quantumness" because there is no separate "quantum" realm — all matter carries Channel B content by virtue of coupling to x_4's oscillation.

V.6.3 Novelty and Uniqueness at the Ontological Level

The explanation of wave/particle duality as the dual-channel reading of $dx_4/dt = ic$ is novel in the precise sense that no prior framework has explained both aspects as simultaneous consequences of a single deeper geometric principle. The landscape of prior explanations may be summarized as follows.

Copenhagen (Bohr 1928; Heisenberg 1927). Complementarity is an interpretational doctrine, not a derivation: it asserts that wave and particle aspects are incompatible and must be specified by experimental context, but it does not derive either aspect from a deeper principle. The uncertainty relation $\Delta x \cdot \Delta p \geq \hbar/2$ is a theorem of the CCR $[\hat{q}, \hat{p}] = i\hbar$, which Copenhagen takes as a postulate of quantum mechanics.

Pilot-wave (de Broglie 1927; Bohm 1952; Dürr-Goldstein-Zanghì 1992). The wave aspect is physically real (the wavefunction guides the particle); the particle aspect is physically real (the particle has a definite trajectory). Both are postulates of the framework; the wavefunction's Schrödinger dynamics and the guiding equation are both added as independent postulates. The pilot-wave framework does not explain why the wavefunction should exist or why it should have the specific Schrödinger dynamics; both are inherited from standard quantum mechanics.

Many-worlds (Everett 1957; Wallace 2012). The wavefunction is the only physically real structure; the particle aspect is an emergent feature of branching (decoherence producing effectively-classical branch histories within the universal wavefunction). This explains the particle aspect as a branching phenomenon but takes the wavefunction and its Schrödinger dynamics as postulates. The dual aspect is not derived from a deeper principle.

Decoherence (Zurek 1982; Joos-Zeh 1985). Environmental decoherence explains the *appearance* of classical particle-like behavior in macroscopic systems by tracing out environment degrees of freedom, producing an effective classical distribution from quantum superposition. This is structurally important for understanding the classical limit but does not explain why the underlying quantum structure exhibits dual aspect at the microscopic level; the wavefunction and its dynamics are again postulates.

Relational quantum mechanics (Rovelli 1996). Quantum states are relational — they describe correlations between systems rather than intrinsic properties — and the wave/particle distinction is a relational artifact. This is a philosophical reframing, not a derivation from a deeper physical principle.

QBism (Fuchs-Mermin-Schack 2014). The wavefunction is an epistemic object describing an agent's personal probability assignments. Wave/particle duality is a feature of the probabilistic structure, not an ontological feature of physical systems. This is an epistemological reinterpretation, not a derivation from a deeper physical principle.

Stochastic mechanics (Nelson 1966; Lindgren-Liukkonen 2019). The wave aspect is derived from stochastic dynamics in configuration space; the particle aspect is the underlying classical particle position undergoing stochastic fluctuations. The i of the Schrödinger equation is, in these frameworks, an analytic continuation of the real-valued stochastic process, with no physical interpretation of why the continuation should be performed (as noted in §VI.3, §VI.4). The frameworks derive the wave aspect through a single route but do not explain the dual aspect as two readings of a single geometric fact.

Bohmian field-theoretic extensions and others. A number of frameworks (Bohm-Hiley 1993 implicate-order analysis; Valentini 2004 quantum nonequilibrium program; Struyve 2011 and Dürr 2014 on Bohmian quantum field theory) extend pilot-wave mechanics to fields and gauge theories. These extensions preserve the pilot-wave framework's status as an interpretation rather than a derivation; they do not alter the fundamental status of the wave and the particle as postulated rather than derived.

Across the ninety-eight years of theoretical engagement with wave/particle duality, *none* of these frameworks explains wave and particle as simultaneous readings of a single geometric-dynamical principle, with the wave aspect forced by one channel of that principle's content and the particle aspect forced by the other. The McGucken Principle is the first framework to achieve this dual-channel reading. This uniqueness is structural: no framework in the historical record possesses dual-channel content in a single geometric-dynamical statement (as established in §V.3), and therefore no framework can generate both aspects as simultaneous consequences of a single principle. The McGucken framework is the first to do so because it is the first to possess the requisite dual-channel structure.

The explanation is novel in a further sense: it *dissolves* the puzzle rather than merely interpreting it. Complementarity, pilot waves, many worlds, decoherence, relational states, epistemic reinterpretation — each prior approach accepts the dual aspect as an irreducible feature and proposes an interpretive scheme for living with it. The McGucken reading does not accept the duality as irreducible; it derives the duality as the consequence of a single geometric fact. The puzzle of wave/particle duality, under the McGucken Principle, is not a puzzle at all — it is the expected consequence of having a principle with dual-channel content, applied at the ontological level to the question of what quantum objects *are*.

V.7 Schrödinger and Heisenberg Pictures as Dual-Channel Readings at the Dynamical Level

The dual-channel structure of $dx_4/dt = ic$ applies a third time, at the *dynamical* level, to the distinction between the Schrödinger and Heisenberg pictures of quantum-mechanical time evolution. Where §§II–III develop the dual reading at the *foundational* level (Hamiltonian versus Lagrangian formulation) and §V.6 develops it at the *ontological* level (wave versus particle aspect), this section develops the reading at the dynamical level.

V.7.1 The Schrödinger and Heisenberg Pictures

Quantum-mechanical time evolution admits two equivalent representations. In the *Schrödinger picture*, physical states |ψ(t)⟩ evolve in time under the Schrödinger equation

$$i\hbar\, d|\psi(t)\rangle/dt = \hat{H}\, |\psi(t)\rangle,$$

while operators Â are fixed. Expectation values are ⟨ψ(t)| Â |ψ(t)⟩, with all time dependence carried by the state. In the *Heisenberg picture*, operators Â(t) evolve in time under the Heisenberg equation

$$i\hbar\, d\hat{A}(t)/dt = [\hat{A}(t), \hat{H}],$$

while states |ψ⟩ are fixed. Expectation values are ⟨ψ| Â(t) |ψ⟩, with all time dependence carried by the operator. The two pictures are related by the unitary transformation Û(t) = exp(−i Ĥ t/ℏ): states in the Schrödinger picture are Û(t) times states in the Heisenberg picture, and operators in the Heisenberg picture are Û(t)† Â Û(t) of operators in the Schrödinger picture. They agree on all expectation values and therefore on all measurable quantities.

The historical development of the two pictures is as follows. Heisenberg's 1925 matrix-mechanics paper [Heisenberg 1925] formulated quantum mechanics as an algebra of operators whose matrix elements describe transitions between stationary states; the picture is fundamentally algebraic, with observables as the primary dynamical objects. The algebra was worked out in full by Born, Heisenberg, and Jordan in the *Dreimännerarbeit* [Born-Heisenberg-Jordan 1926], which established the matrix-mechanical framework with operators satisfying [q̂, p̂] = iℏ and time evolution generated by commutators with the Hamiltonian. Schrödinger's 1926 four-paper series [Schrödinger 1926a,b,c,d] formulated quantum mechanics as a wave theory, with physical states represented by wavefunctions ψ(x, t) evolving according to the Schrödinger equation; the picture is fundamentally propagational, with wavefunctions as the primary dynamical objects. The equivalence of the two formulations was established by Schrödinger himself in his third 1926 paper [Schrödinger 1926b], which showed that the matrix elements of Heisenberg's operators equal the integrals ∫ψ*(x)·Â·ψ(x)dx of Schrödinger's wavefunctions, and by von Neumann's 1932* Mathematische Grundlagen der Quantenmechanik* [von Neumann 1932], which placed both formulations on the common algebraic foundation of operators on Hilbert space.

Since 1932, the two pictures have coexisted in the physics literature as equivalent representational tools, each adopted in contexts where it is most convenient. The Schrödinger picture is standard for introductory treatments of quantum mechanics (e.g., Sakurai [Sakurai 1994], Griffiths [Griffiths 2005], Cohen-Tannoudji-Diu-Laloë [Cohen-Tannoudji 1977], Messiah [Messiah 1961]) because it makes explicit the role of the wavefunction and the computational simplicity of diagonalizing the Hamiltonian; the Heisenberg picture is standard for quantum field theory (e.g., Weinberg [Weinberg 1995], Peskin-Schroeder [Peskin-Schroeder 1995]) because it makes explicit the role of the operator algebra and the covariance of operators under Lorentz transformations. The interaction picture, developed in the 1940s by Tomonaga [Tomonaga 1946], Schwinger [Schwinger 1948], and Feynman [Feynman 1949] for renormalization in quantum electrodynamics, is a hybrid picture in which part of the Hamiltonian acts on states (as in Schrödinger) and part acts on operators (as in Heisenberg); it is the standard workhorse for perturbative QFT calculations but does not add a new structural dimension beyond the Schrödinger/Heisenberg dichotomy.

The question of *why* these two pictures exist as equivalent representations — why quantum mechanics should admit two structurally distinct formulations of dynamics — has not been addressed in the literature. Textbook treatments present the equivalence as a mathematical fact (Schrödinger 1926b proved it; Stone-von Neumann confirmed that the representation is essentially unique) without explaining why the dynamical content of quantum mechanics should decompose into state-dynamics and operator-dynamics as two readings of a single underlying dynamics. The McGucken Principle supplies that explanation.

V.7.2 The Dual-Channel Reading at the Dynamical Level

Under the dual-channel analysis of §§V.1–V.5, the Schrödinger and Heisenberg pictures are not arbitrary representational choices — they are the two readings of the dynamical content of $dx_4/dt = ic$.

Heisenberg picture as Channel A at the dynamical level. Time evolution in the Heisenberg picture is generated by commutators with the Hamiltonian: $d\hat{A}/dt = (i/\hbar)[\hat{H}, \hat{A}]$. This is the algebraic-symmetry reading of time evolution. The Hamiltonian $\hat{H}$ is the self-adjoint generator of time-translation, by Stone's theorem applied to the time-translation subgroup of the Poincaré group (exactly parallel to the application in Proposition H.2 for spatial translations). Just as Stone's theorem applied to spatial translations forces the momentum operator $\hat{p} = -i\hbar\, \partial/\partial q$ as the generator of spatial translations (Proposition H.3), Stone's theorem applied to time translations forces the Hamiltonian $\hat{H} = i\hbar\, \partial/\partial t$ as the generator of time translations. The Heisenberg equation $i\hbar\, d\hat{A}/dt = [\hat{A}, \hat{H}]$ is the direct consequence of this structure: operators evolve under the action of the time-translation generator, with the factor $i\hbar$ on the left-hand side and the factor $i\hbar$ in the Stone's-theorem construction of $\hat{H}$ both inherited from $dx_4/dt = ic$ by the same derivational chain that yields the CCR in Proposition H.4. The Heisenberg picture is the Channel A reading of dynamics: algebraic-symmetry structure with observables as the primary objects and time evolution as a commutator flow on the operator algebra.

Schrödinger picture as Channel B at the dynamical level. Time evolution in the Schrödinger picture is generated by the Schrödinger equation $i\hbar\, \partial\psi/\partial t = \hat{H}\, \psi$. This is the geometric-propagation reading of time evolution. By Proposition L.5, the Schrödinger equation is the non-relativistic limit of the path-integral propagation of the wavefunction through spacetime, with the wavefunction's time evolution being the continuum limit of iterated Huygens-spherical-expansion composition. The wavefunction $\psi(x, t)$ is, literally, the three-dimensional cross-section of x_4's expanding wavefront at observer time t — the McGucken Sphere of [MG-Master, Part IV] projected onto the spatial dimensions. The Schrödinger equation describes how this wavefront advances: the wavefunction at time t + dt is the Huygens-propagated wavefunction at time t, with the specific form of the propagator determined by the Compton-frequency coupling of matter to x_4's oscillation (Proposition L.3). The Schrödinger picture is the Channel B reading of dynamics: geometric-propagation structure with wavefunctions as the primary objects and time evolution as a wavefront propagation through spacetime.

The two pictures are therefore not arbitrary equivalent formulations — they are the two readings of the Principle's dual-channel content applied to the dynamical question "how does quantum mechanics evolve in time?" Channel A supplies the algebraic-symmetry reading (Heisenberg: commutator flow on the operator algebra). Channel B supplies the geometric-propagation reading (Schrödinger: wavefront propagation of the state). The equivalence of the two pictures, established in 1926 by Schrödinger and 1932 by von Neumann and secured representation-theoretically by Stone-von Neumann, is the mathematical expression of the two readings of the same dynamical content.

The correspondence between the four levels of dual-channel structure — with the fourth level (locality/nonlocality) developed in §V.8 — may be summarized as follows:

Level of description	Channel A (algebraic-symmetry)	Channel B (geometric-propagation)
Foundational (§§II–III)	Hamiltonian operator formulation	Lagrangian path-integral formulation
Dynamical (§V.7)	Heisenberg picture (operators evolve)	Schrödinger picture (states evolve)
Ontological (§V.6)	Particle aspect (localized eigenvalues)	Wave aspect (propagating wavefronts)

The four levels are not independent; they track the same dual-channel structure of $dx_4/dt = ic$ applied to four distinct physics questions (what is the correct formulation of quantum mechanics? how does quantum mechanics evolve in time? what are quantum objects? how do quantum objects relate to spacetime causal structure?). The McGucken Principle answers all four with the same dual-channel structure: two simultaneous readings of one geometric fact.

V.7.3 A Geometric Proof of the Schrödinger-Heisenberg Equivalence from $dx_4/dt = ic$

The dual-channel reading of §V.7.2 supplies more than an identification of which picture corresponds to which channel. It supplies the basis for a *geometric proof* of the equivalence of the two pictures, distinct from the algebraic proofs of Schrödinger [Schrödinger 1926b] and von Neumann [von Neumann 1932], in which the equivalence follows from the physical content of x_4's advance rather than from mathematical manipulation of the unitary evolution operator. This subsection develops the proof in formal theorem-lemma-corollary structure and explains the sense in which it is geometric rather than algebraic.

Theorem V.7.3 *(Schrödinger-Heisenberg equivalence from $dx_4/dt = ic$).*

Let $\mathscr{H}$ be the complex Hilbert space of quantum states of a physical system, let $\hat{H}$ be the system's Hamiltonian operator (the self-adjoint generator of time translations on $\mathscr{H}$), let $\hat{U}(t) = \exp(-i\hat{H}t/\hbar)$ be the one-parameter unitary evolution group on $\mathscr{H}$, let $|\psi\rangle \in \mathscr{H}$ be any state, and let $\hat{A}$ be any self-adjoint observable on $\mathscr{H}$. Define the Schrödinger-picture state at time t as $|\psi\_S(t)\rangle = \hat{U}(t)|\psi\rangle$ with the observable $\hat{A}$ held fixed, and the Heisenberg-picture observable at time t as $\hat{A}\_H(t) = \hat{U}(t)^\dagger \hat{A} \hat{U}(t)$ with the state $|\psi\rangle$ held fixed. Under the McGucken Principle $dx_4/dt = ic$, the equality of expectation values

$$\langle\psi\_S(t)| \hat{A} |\psi\_S(t)\rangle = \langle\psi| \hat{A}\_H(t) |\psi\rangle$$

holds not as an algebraic identity of unitary conjugation but as a direct consequence of the geometric content of $dx_4/dt = ic$: both sides compute the expectation of the same observable after the same physical process (x_4's advance by amount $ic \cdot t$ from every spacetime point) has occurred, with the two sides differing only in which mathematical object (the state vector on the left, the observable on the right) is updated to reflect the advance while the other is held fixed.

The proof proceeds through three lemmas establishing, respectively, the physical interpretation of the unitary $\hat{U}(t)$ as the Hilbert-space implementation of x_4's advance (Lemma V.7.3.A), the equivalence of Channel B and Channel A readings of the advance at the level of individual Hilbert-space objects (Lemma V.7.3.B), and the preservation of expectation values under either reading (Lemma V.7.3.C). The theorem then follows from Lemma V.7.3.C applied to the specific state and observable of the theorem statement.

Lemma V.7.3.A *(The unitary evolution operator as Hilbert-space implementation of x_4's advance).*

The one-parameter unitary group $\hat{U}(t) = \exp(-i\hat{H}t/\hbar)$ on the Hilbert space $\mathscr{H}$ is the operator implementation of x_4's advance by amount $ic \cdot t$. Specifically: (i) the self-adjoint generator $\hat{H}$ is the Noether charge associated with x_4's temporal uniformity [MG-Noether, Propositions IV.1–IV.2]; (ii) the factor i in the exponent is the perpendicularity marker of x_4 (same i as in $x_4 = ict$, propagated through Stone's theorem on *the time-translation subgroup as in Proposition H.2 applied to spatial translations); (iii) the factor ħ is the action per complete x_4-oscillation cycle at the Planck frequency [MG-Constants, §V]. Both constants in Û(t) are therefore inherited from $dx_4/dt = ic$ by the derivational chain established in Propositions H.1–H.3, with time translations playing the role played by spatial translations in those propositions.*

Proof of Lemma V.7.3.A. On the complex Hilbert space $\mathscr{H}$, the one-parameter unitary group representing time translations is strongly continuous (physical predictions depend continuously on time), so by Stone's theorem [Stone 1932], $\hat{U}(t) = \exp(-it\hat{A}/\hbar)$ for some densely-defined self-adjoint generator Â. Writing $\hat{A} = \hat{H}$ and calling Ĥ the Hamiltonian, we have $\hat{U}(t) = \exp(-i\hat{H}t/\hbar)$. The factor i is the Stone's-theorem factor, forced by the combined requirements of unitarity ($\hat{U}(t)\hat{U}(t)^\dagger = \mathbb{1}$) and self-adjointness of the generator ($\hat{H} = \hat{H}^\dagger$). Geometrically, the factor i in the exponent is the same i as in the integrated form $x_4 = ict$ of the McGucken Principle: in both cases, i marks the perpendicularity of x_4 to the three spatial dimensions, and the physical dynamical parameter t enters the exponent as ic·t (with the factor c absorbed into Ĥ through dimensional analysis: Ĥ has dimensions of energy = mass × velocity², and ħ/c has dimensions of action/velocity = mass × length, so Ĥt/ħ is dimensionless as required). The factor ħ is identified in [MG-Constants, §V] as the quantum of action per complete x_4-oscillation cycle at the Planck frequency $f_P = 1/t_P$. The Hamiltonian Ĥ, as Noether charge of the time-translation subgroup of the Poincaré group, is derived in [MG-Noether, Propositions IV.1–IV.2] from the temporal uniformity of x_4's advance ($dx_4/dt = ic$ is the same at every spacetime point, so time-translation is a symmetry of the Principle). Both the operator Ĥ and the constants i, ħ entering the exponent are therefore forced by $dx_4/dt = ic$; the unitary Û(t) is, as claimed, the Hilbert-space implementation of x_4's advance by amount ic·t. ∎

Lemma V.7.3.B *(The dual-channel action of Û(t) on states and on observables).*

The unitary Û(t) of Lemma V.7.3.A admits two equivalent actions on a quantum mechanical description of the system at time t = 0 to yield the description at time t:

(B1) Channel B (geometric-propagation) action: Û(t) acts on the state vector, carrying |ψ⟩ to $|\psi_S(t)\rangle = \hat{U}(t)|\psi\rangle$, with observables Â held fixed. Geometrically, this action implements x_4's advance as the propagation of the state's wavefront forward by amount ic·t in the Feynman path-integral sense of Propositions L.1–L.5.

(B2) Channel A (algebraic-symmetry) action: Û(t) acts on the observable by conjugation, carrying Â to $\hat{A}_H(t) = \hat{U}(t)^\dagger \cdot \hat{A} \cdot \hat{U}(t)$, with the state |ψ⟩ held fixed. Geometrically, this action implements x_4's advance as the transformation of the observable's algebraic definition into the time-translated reference frame, consistent with Heisenberg's original [Heisenberg 1925] matrix-mechanical treatment of observables as the primary dynamical objects.

Proof of Lemma V.7.3.B. Both actions are consequences of the fact that Û(t) is unitary, $\hat{U}(t)\hat{U}(t)^\dagger = \mathbb{1}$. Given any matrix element ⟨φ|Ô|ψ⟩ for an operator Ô and states |ψ⟩, |φ⟩, the insertion of the identity $\hat{U}(t)\hat{U}(t)^\dagger$ between any two factors preserves the matrix element; moving this insertion across the factors redistributes the time evolution across different mathematical objects:

$$\langle\varphi|\hat{O}|\psi\rangle = \langle\varphi|[\hat{U}(t)\hat{U}(t)^\dagger]\ \hat{O}\ [\hat{U}(t)\hat{U}(t)^\dagger]|\psi\rangle = [\langle\varphi|\hat{U}(t)][\hat{U}(t)^\dagger\ \hat{O}\ \hat{U}(t)][\hat{U}(t)^\dagger|\psi\rangle]$$

$$= \langle\varphi_S(t)|\hat{O}_H(t)|\psi_S(t)\rangle,$$

where $|\psi_S(t)\rangle = \hat{U}(t)|\psi\rangle$ and $|\varphi_S(t)\rangle = \hat{U}(t)|\varphi\rangle$ are Schrödinger-picture states (Channel B action on states) and $\hat{O}_H(t) = \hat{U}(t)^\dagger\cdot\hat{O}\cdot\hat{U}(t)$ is the Heisenberg-picture observable (Channel A action on observables). The factorization can be further rearranged by moving $\hat{U}(t)$'s to adjacent positions:

$$\langle\varphi|\hat{O}|\psi\rangle = \langle\varphi|\hat{O}\ \hat{U}(t)\hat{U}(t)^\dagger|\psi\rangle = \langle\varphi|[\hat{O}]\ \hat{U}(t)|\psi\rangle\cdot\hat{U}(t)^\dagger|\psi\rangle^{-1}\cdot\hat{U}(t)^\dagger|\psi\rangle = \langle\varphi|\ \hat{O}\ |\psi_S(t)\rangle \to \textit{mixed form},$$

or alternatively concentrating $\hat{U}(t)$ on one side to obtain the pure Schrödinger or pure Heisenberg reading. The physical content of Lemma V.7.3.B is that x_4's advance by amount $ic\cdot t$ — a single physical process — can be mathematically represented by updating either the state (Channel B) or the observable (Channel A) or distributing the update across both (intermediate pictures, including the interaction picture discussed in §V.7.4). The two pure readings (Schrödinger, Heisenberg) are the two extremes of the full dual-channel content; they are equivalent because both implement the same physical x_4-advance on the same system. ∎

Lemma V.7.3.C *(Preservation of expectation values under both readings).*

For any state $|\psi\rangle \in \mathscr{H}$ *and any observable* $\hat{A} = \hat{A}^\dagger$ *on* $\mathscr{H}$*, the expectation value of* $\hat{A}$ *at time t is*

$$\langle\hat{A}\rangle(t) := \langle\psi_S(t)|\ \hat{A}\ |\psi_S(t)\rangle = \langle\psi|\ \hat{A}_H(t)\ |\psi\rangle,$$

independent of whether the Schrödinger or Heisenberg reading is chosen. Both readings yield the same number because both compute the expectation of the same observable on the same physical system after the same physical process (x_4*'s advance by amount* $ic\cdot t$*) has occurred.*

Proof of Lemma V.7.3.C. Using the unitarity $\hat{U}(t)\hat{U}(t)^\dagger = \hat{U}(t)^\dagger\hat{U}(t) = \mathbb{1}$ and the definitions $|\psi_S(t)\rangle = \hat{U}(t)|\psi\rangle$ and $\hat{A}_H(t) = \hat{U}(t)^\dagger\cdot\hat{A}\cdot\hat{U}(t)$:

$$\langle\psi_S(t)|\ \hat{A}\ |\psi_S(t)\rangle = [\hat{U}(t)|\psi\rangle]^\dagger\cdot\hat{A}\cdot[\hat{U}(t)|\psi\rangle]$$

$$= \langle\psi|\ \hat{U}(t)^\dagger\cdot\hat{A}\cdot\hat{U}(t)\ |\psi\rangle$$

$$= \langle\psi|\ [\hat{U}(t)^\dagger\ \hat{A}\ \hat{U}(t)]\ |\psi\rangle$$

$$= \langle\psi|\ \hat{A}_H(t)\ |\psi\rangle.$$

The algebra is identical to the textbook equivalence proof. The *geometric content* — which the textbook proof does not articulate — is that both sides compute the expectation of $\hat{A}$ in the quantum-mechanical description of the system at time t, where "at time t" means "after x_4 has advanced by amount $ic\cdot t$ from every spacetime point of the initial description." The left side carries this advance as an update to the state

(Channel B reading); the right side carries it as an update to the observable's algebraic definition (Channel A reading); the equality of the two sides follows from the fact that both are describing the same physical x_4-advance. ∎

Proof of Theorem V.7.3. Apply Lemma V.7.3.C to the specific state |ψ⟩ and observable Â of the theorem statement. The equality ⟨ψ_S(t)| Â |ψ_S(t)⟩ = ⟨ψ| Â_H(t) |ψ⟩ is the direct consequence. ∎

Corollary V.7.3.D *(Picture independence of all measurable predictions).*

All quantum-mechanical predictions derived from expectation values — transition probabilities, scattering amplitudes, correlation functions, time-ordered products, and the entries of the S-matrix — are picture-independent: they can be computed in the Schrödinger picture or the Heisenberg picture or any intermediate hybrid picture (including the Tomonaga-Schwinger-Feynman interaction picture of §V.7.4) and will yield the same numerical values. The independence follows from the dual-channel reading of Lemma V.7.3.B applied iteratively to products of operators and sums of matrix elements, each individual expectation or matrix element preserving by Lemma V.7.3.C.

Proof of Corollary V.7.3.D. Every quantum-mechanical prediction can be expressed as a sum, product, or integral of matrix elements of the form ⟨φ| Ô_1(t_1) Ô_2(t_2) ⋯ Ô_n(t_n) |ψ⟩ (time-ordered products), which can in turn be reduced to expectation-value form by polarization identities. Each individual matrix element is picture-independent by Lemmas V.7.3.B–V.7.3.C (insertions of Û(t_k)Û(t_k)† can be distributed across state vectors and operators in any consistent way), and the composed prediction is therefore picture-independent as well. ∎

Remark V.7.3.1 *(The geometric character of the proof).*

The proof of Theorem V.7.3 above is *algebraically* identical to the textbook proofs of the Schrödinger-Heisenberg equivalence found in [Schrödinger 1926b], [von Neumann 1932], [Dirac 1930], [Sakurai 1994, §2.2], and every modern quantum-mechanics textbook: the algebraic step ⟨Û(t)ψ|Â|Û(t)ψ⟩ = ⟨ψ|Û(t)†·Â·Û(t)|ψ⟩ follows from unitarity and is the central manipulation in all such proofs. The *geometric* character of the present proof lies not in the algebra — the algebra is the same — but in the *physical interpretation* of the algebraic objects, supplied by Lemmas V.7.3.A and V.7.3.B.

In the algebraic proofs, the unitary Û(t) is the abstract time-evolution operator whose existence is established by Stone's theorem and whose form Û(t) = exp(−iĤt/ℏ) is established by identifying Ĥ as the Hamiltonian. The factor i in the exponent is unmotivated beyond the mathematical requirement that Û(t) be unitary; the factor ℏ is an empirical constant; the Hamiltonian Ĥ is whatever observable plays the role of the generator of time evolution. The equivalence of the Schrödinger and Heisenberg pictures is a consequence of unitarity, a purely algebraic property.

In the McGucken proof, the unitary $\hat{U}(t)$ is identified in Lemma V.7.3.A as the Hilbert-space implementation of x_4's advance by amount $ic \cdot t$. The factor i is the perpendicularity marker of x_4 (the same i as in $x_4 = ict$, propagated through Stone's theorem on time translations). The factor $\hbar$ is the action per x_4-oscillation cycle at the Planck frequency. The Hamiltonian $\hat{H}$ is the Noether charge of x_4's temporal uniformity. Every element of $\hat{U}(t)$ has geometric meaning drawn from the underlying Principle. The dual-channel reading of Lemma V.7.3.B then identifies the Schrödinger action ($\hat{U}(t)$ on state) as the Channel B reading of x_4's advance and the Heisenberg action ($\hat{U}(t)$ conjugating observable) as the Channel A reading. The equivalence of the two pictures is not merely an algebraic consequence of unitarity; it is a *geometric* consequence of the fact that both readings are describing the same physical process (x_4's advance), differing only in which mathematical object (state or observable) is chosen to carry the update.

The distinction between the algebraic and geometric characters of the proof is the distinction between *showing* and *explaining* the equivalence. The algebraic proof shows that the equivalence holds: the equations work out. The geometric proof explains why the equivalence holds: because both sides describe the same x_4-advance, through two channels of the Principle's dual content.

Remark V.7.3.2 *(Historical comparison and novelty).*

The algebraic proof of the Schrödinger-Heisenberg equivalence was first established by Schrödinger in his third 1926 paper [Schrödinger 1926b], which demonstrated that the matrix elements of Heisenberg's operators are identical to the $\langle\psi|\hat{A}|\psi\rangle$ integrals of Schrödinger's wavefunctions for all observable quantities. Dirac's 1927 transformation theory [Dirac 1927] formalized the picture-independence in terms of unitary transformations between representations. Von Neumann's 1932 treatise [von Neumann 1932] placed both formulations on the rigorous Hilbert-space foundation, with the Stone-von Neumann theorem [Stone 1932; von Neumann 1931] securing the representation-theoretic uniqueness. These are the three canonical sources for the equivalence, and every subsequent textbook treatment (Sakurai [Sakurai 1994], Griffiths [Griffiths 2005], Cohen-Tannoudji [Cohen-Tannoudji 1977], Messiah [Messiah 1961], Weinberg [Weinberg 1995], Peskin-Schroeder [Peskin-Schroeder 1995]) either reproduces or builds on these proofs without adding structural content.

None of these proofs — from Schrödinger 1926b through modern textbooks — identifies the unitary $\hat{U}(t)$ with a physical object beyond "the time-evolution operator"; none identifies the factor i beyond "the factor required for unitarity"; none identifies the factor $\hbar$ beyond "the empirical constant of quantum mechanics"; none explains why the two pictures should exist as equivalent representations rather than as arbitrary reformulations or as structurally distinct physical theories. The algebraic manipulations establish the equivalence; they do not explain it.

The geometric proof developed in Theorem V.7.3 and its lemmas is the first demonstration of the Schrödinger-Heisenberg equivalence in the ninety-nine-year history since Schrödinger 1926b that identifies the physical content of the unitary evolution operator (x_4's advance), the physical origin of i and ℏ in the operator's form (perpendicularity of x_4 and action per x_4-cycle), and the geometric reason for the equivalence of the two pictures (dual-channel reading of x_4's advance at the dynamical level). The novelty is structural: the algebra is the same, but the interpretation of the algebra is new, and the interpretation *explains* what the algebra alone can only *show*.

The geometric proof is, in a precise sense, what Einstein meant when he described a deeper understanding of a theorem as reaching "the understanding that leaves no question behind" rather than merely "the verification that the equation holds" [paraphrased from Einstein's correspondence with Max Born on the epistemological standards of physical theories]. The Schrödinger 1926b proof verifies the equation. The McGucken proof of Theorem V.7.3 leaves no question behind, because both sides of the equation are identified with two readings of a single geometric fact, and the equation's validity is identified with the fact that both readings describe the same physical process.

V.7.4 The Interaction Picture and Hybrid Readings

The interaction picture of Tomonaga-Schwinger-Feynman [Tomonaga 1946; Schwinger 1948; Feynman 1949] is a hybrid picture in which part of the Hamiltonian (the free part Ĥ_0) generates time evolution of operators while the remaining part (the interaction part Ĥ_I) generates time evolution of states. Under the dual-channel reading, the interaction picture is not a third independent reading of the dynamics; it is a *convenient factorization* of the dynamics into a Channel A component (the free-field time evolution of operators under Ĥ_0) and a Channel B component (the interaction-driven time evolution of states under Ĥ_I). The interaction picture is the computational tool that allows the two channels to be treated separately in perturbation theory — a reflection of the dual-channel structure, not an addition to it.

The Dirac-picture analysis in canonical quantum field theory (Dirac 1930, Weinberg 1995) extends the interaction-picture structure to field-theoretic systems, with similar dual-channel interpretation: the free-field propagators are Channel B content (wavefront propagation of the free fields through spacetime), and the interaction vertices are Channel A content (algebraic rules for how operator products couple at interaction points). Again, the dual-channel structure underlies the mathematical structure; the QFT formalism makes both channels computationally explicit.

V.7.5 Novelty and Uniqueness at the Dynamical Level

The explanation of the Schrödinger and Heisenberg pictures as dual-channel readings of $dx_4/dt = ic$ is novel in the same precise sense as §V.6's explanation of wave/particle duality: no prior framework has explained the two pictures as simultaneous consequences of a single deeper principle. Prior treatments of the two pictures fall into four categories.

Mathematical equivalence proofs. Schrödinger's 1926b proof of the equivalence of matrix and wave mechanics [Schrödinger 1926b] established that the two formulations yield the same predictions; von Neumann's 1932 treatise [von Neumann 1932] placed both on the common foundation of Hilbert-space operator theory; Dirac's transformation theory [Dirac 1927] and the Stone-von Neumann theorem [Stone 1932; von Neumann 1931] secured the representation-theoretic uniqueness. These are equivalence theorems, not derivations from a deeper principle — they establish that the two pictures describe the same physics but do not explain why the same physics should admit two structurally distinct formulations.

Pedagogical and computational choices. Modern textbook treatments (Sakurai 1994, Griffiths 2005, Cohen-Tannoudji 1977, Weinberg 1995, Peskin-Schroeder 1995) adopt one picture or the other based on computational or pedagogical considerations: the Schrödinger picture is chosen when the Hamiltonian is easy to diagonalize and the wavefunction's explicit form is wanted; the Heisenberg picture is chosen when Lorentz covariance or operator algebra is central; the interaction picture is chosen for perturbative QFT. These are practical choices, not structural explanations of the existence of two pictures.

Mathematical-physics reformulations. Geometric quantization (Kostant 1970; Souriau 1970) and algebraic quantum field theory (Haag 1992) supply mathematically rigorous foundations for operator-theoretic quantum mechanics, with the Schrödinger and Heisenberg pictures related through well-defined mathematical operations (Hilbert-bundle structure, GNS construction, automorphism groups of C*-algebras). These frameworks operate at a high level of mathematical sophistication but presuppose the operator-algebraic structure rather than deriving it from a physical principle; the two pictures remain equivalent representational choices within the established framework.

Operational and information-theoretic reformulations. Reconstructions of quantum mechanics from operational or information-theoretic axioms (Hardy 2001, Chiribella-D'Ariano-Perinotti 2011, Clifton-Bub-Halvorson 2003, Masanes-Müller 2011, Dakic-Brukner 2011, Brukner 2014) attempt to derive the Hilbert-space structure of quantum mechanics from more primitive operational considerations. These reconstructions derive the Hilbert-space framework (within which the Schrödinger and Heisenberg pictures coexist as equivalent representations), but they do not identify the two pictures as consequences of distinct informational contents of a single physical principle about spacetime; the operational axioms are not geometric-dynamical principles in the sense of $dx_4/dt = ic$.

Across the ninety-four years of theoretical engagement with the Schrödinger/Heisenberg duality (1932 to present), no prior framework has identified the two pictures as dual-channel readings of a single geometric-dynamical principle about spacetime, with Channel A (algebraic-symmetry) generating the Heisenberg picture and Channel B (geometric-propagation) generating the Schrödinger picture, and both channels derived from the same physical fact. The McGucken Principle is the first framework to achieve this reading.

The novelty at the dynamical level is structurally parallel to the novelty at the ontological level (§V.6): in both cases, a phenomenon that the prior literature has treated as a mathematical equivalence or as a philosophical puzzle — Schrödinger-Heisenberg equivalence as a representation-theoretic fact, wave-particle duality as a complementarity to be interpreted — is shown to be the direct consequence of the McGucken Principle's dual-channel content. The prior literature identifies *what* the equivalence is (representation-theoretic uniqueness of the Schrödinger representation, Stone-von Neumann theorem) and *what* the duality is (simultaneous empirical manifestation of wave and particle aspects); the McGucken reading identifies *why* both exist (dual-channel content of $dx_4/dt = ic$), and *why specifically these two*: because Channel A is algebraic-symmetry content and Channel B is geometric-propagation content, which are the two logical components of the single statement "x_4 advances at the velocity of light, uniformly and spherically symmetrically from every spacetime point."

V.7.6 Transition to the Fourth Level

The combined weight of §§V.6 and V.7 is that the dual-channel structure of $dx_4/dt = ic$ is the structural signature of the Principle at three distinct levels of quantum-mechanical description simultaneously: the foundational level (Hamiltonian/Lagrangian formulations, §§II–III), the ontological level (wave/particle aspects, §V.6), and the dynamical level (Schrödinger/Heisenberg pictures, §V.7). The same dual-channel content that forces the two-route derivation of the canonical commutation relation in §§II–III also forces the dual aspect of quantum objects in §V.6 and the dual picture of quantum dynamics in §V.7. §V.8 below adds a fourth level — the causal/correlational level — at which the dual-channel structure generates the coexistence of local operator algebra and nonlocal Bell correlations, completing a four-level structural consequence whose full formal summary is deferred to §V.8.6.

V.8 Locality and Nonlocality as Dual-Channel Readings at the Causal/Correlational Level

The dual-channel structure of $dx_4/dt = ic$ applies a fourth time, at the *causal/correlational* level, to the distinction between the local operator-algebra structure of standard quantum field theory and the nonlocal Bell correlations of entangled systems. Where §§II–III develop the dual reading at the *foundational* level (Hamiltonian versus Lagrangian formulation), §V.6 develops it at the *ontological* level (wave versus particle aspect), and §V.7 develops it at the *dynamical* level (Schrödinger versus Heisenberg picture), this section develops the reading at the causal/correlational level. The developments draw on the comprehensive treatment of nonlocality in [MG-Nonlocality] and on the Copenhagen-resolution analysis of [MG-NonlocCopen], which together establish what the present section summarizes as the fourth dual-channel reading.

V.8.1 Locality and Nonlocality in Quantum Mechanics

Quantum mechanics exhibits two structurally distinct features that have coexisted in apparent tension throughout the theory's history. On one side, the theory possesses a rigorously *local* operator-algebra structure: observables $\hat{O}(x)$ are defined pointwise on spacetime, local operators at spacelike-separated points commute ($[\hat{O}(x), \hat{O}(y)] = 0$ for $(x - y)^2 > 0$), and the theory satisfies microcausality and the Reeh-Schlieder property of algebraic quantum field theory [Reeh-Schlieder 1961; Haag 1992]. On the other side, the theory exhibits *nonlocal* correlations between spatially separated entangled systems that cannot be explained by any local hidden-variable theory: Bell's 1964 theorem [Bell 1964] establishes that the quantum-mechanical singlet correlation E(a,b) = −cos θ_{ab} violates the Bell inequalities satisfied by every local realistic theory, and this violation is experimentally confirmed to high precision in the Aspect experiments of the early 1980s [Aspect-Grangier-Roger 1981, 1982], the Clauser-Horne-Shimony-Holt experimental protocols, the Weihs delayed-choice experiments, and subsequently by loophole-closing experiments culminating in the 2015 Bell tests at Delft, NIST, and Vienna [Hensen *et al.* 2015; Shalm *et al.* 2015; Giustina *et al.* 2015]. The correlations are nonlocal in the specific technical sense that no local hidden-variable theory can reproduce them.

The historical development of these two features is as follows. The locality of quantum field theory was established through the formalization of microcausality and the operator-algebra axiomatization of quantum field theory by Wightman (1956) [Wightman 1956], Haag-Kastler (1964) [Haag-Kastler 1964], Streater-Wightman (1964) [Streater-Wightman 1964], and the subsequent algebraic-quantum-field-theory tradition. Local operators at spacelike-separated points commute, and every standard formulation of quantum field theory — Lagrangian QFT [Weinberg 1995], Wightman QFT, algebraic QFT [Haag 1992], and quantum electrodynamics [Feynman 1949; Schwinger 1948; Dyson 1949] — respects this locality axiomatically. The nonlocal correlations were identified by Einstein, Podolsky, and Rosen (1935) [EPR 1935] as a feature they regarded as incompleteness of quantum mechanics, formalized by Bohm (1951) [Bohm 1951] and rigorously analyzed by Bell (1964) [Bell 1964] as a genuine violation of local realistic theories, and experimentally confirmed in the Aspect and subsequent experimental series. The nonlocal correlations are now recognized as a genuine feature of quantum mechanics rather than an artifact of incompleteness.

The two features have been viewed as separate structural aspects of quantum mechanics, each with its own derivational account: local operator algebra is developed in the framework of QFT and establishes microcausality as an axiom; nonlocal correlations are developed in the framework of entanglement theory and established through the Bell inequality apparatus. No prior foundation has supplied both features as theorems of a single deeper principle through disjoint channels of that principle's content.

V.8.2 The Dual-Channel Reading at the Causal/Correlational Level

The McGucken Principle $dx_4/dt = ic$ supplies both features as theorems of its dual-channel content, through the same structure identified at Levels 1–3:

Channel A (algebraic-symmetry content) forces locality and microcausality. The algebraic-symmetry content of $dx_4/dt = ic$ — temporal uniformity, spatial homogeneity, spherical isotropy as symmetry statements, and Lorentz covariance of the rate — is what forces the Minkowski metric and the light-cone causal structure of spacetime (Proposition H.1 in §II). The Minkowski metric in turn forces spacelike-separated events to be causally disconnected, and quantum operators respecting the invariant causal structure of Minkowski spacetime must commute at spacelike separations. Microcausality is thus a theorem of Channel A, derived through the same chain that produced the canonical commutation relation and the Hamiltonian formulation: $dx_4/dt = ic$ → Minkowski metric → Lorentz-invariant causal structure → $[\hat{O}(x), \hat{O}(y)] = 0$ for $(x - y)^2 > 0$. The local operator algebra of standard QFT is the natural formal expression of Channel A's algebraic-symmetry content applied to the spacetime manifold forced by the Principle.

Channel B (geometric-propagation content) forces nonlocality and Bell correlations. The geometric-propagation content of $dx_4/dt = ic$ — spherical expansion at rate c from every spacetime point — is what forces Huygens' Principle, the McGucken Sphere, and the wavefront structure of propagation (Proposition L.1 in §III). The same McGucken Sphere that carries the wave aspect of quantum propagation (§V.6) and the wavefront structure of the Lagrangian path integral (§III) also carries the nonlocal correlations of entanglement. Two particles created at a common spacetime event share a single McGucken Sphere. Because photons are stationary in x_4 ($dx_4/d\tau = 0$ along every null worldline), the photons of an entangled pair remain at the same x_4-coordinate for all subsequent time regardless of their spatial separation, and the four-dimensional interval between them remains null at all times. In the rest frame of the photons themselves, there is no proper time and no proper distance between the two events: the particles have never left each other. The singlet correlation $E(a,b) = -\cos \theta_{ab}$ follows from the SO(3) Haar-measure symmetry of the shared McGucken Sphere, without any local hidden variable [MG-Twistor, Proposition X.6; MG-NonlocCopen §5.5a]. This is the *McGucken Equivalence* [MG-Equiv; MG-Singular §VII]: quantum nonlocality is the three-dimensional shadow of four-dimensional x_4-coincidence on the light cone.

The two features — local operator algebra on one side, nonlocal Bell correlations on the other — are therefore two readings of the same principle through the same two channels that produced the Hamiltonian and Lagrangian formulations, the wave and particle aspects, and the Schrödinger and Heisenberg pictures. The coexistence of locality and nonlocality in quantum mechanics, which has been understood since Bell 1964 as a genuine structural feature requiring reconciliation, is revealed as the fourth appearance of the dual-channel structure of $dx_4/dt = ic$ at a distinct level of quantum-mechanical description.

V.8.3 The Six Senses of Geometric Nonlocality of the McGucken Sphere

The reading of Channel B as generating genuine nonlocality rests on a specific geometric claim: the expanding McGucken Sphere wavefront generated by $dx_4/dt = ic$ is a genuine nonlocal entity whose spatially separated points share a common geometric identity traceable to a single local origin. This claim is established rigorously in [MG-Nonlocality §4] through six independent mathematical frameworks, each of which identifies the expanding wavefront as a geometric locality (and therefore a geometric nonlocality, since its spatially separated points share a common identity) in a distinct sense. The six senses are summarized here as Remarks V.8.3(i)–(vi).

Remark V.8.3(i) — Foliation theory. The family of expanding McGucken Spheres parameterized by time t defines a foliation of three-dimensional space, with each sphere $\Sigma(t)$ being a leaf of the foliation. Spatially separated points on the same leaf share a common foliation-theoretic identity: they are members of the same geometric submanifold, separated by the foliation's transverse structure from points on other leaves. The wavefront's spatially separated points are, in foliation-theoretic terms, a single unified object.

Remark V.8.3(ii) — Level sets of a distance function. The wavefront is the level set $d(x) = ct$ of the distance function from the origin of the expansion. Every point on the wavefront is equidistant from the origin in the induced metric, sharing a common metric-geometric identity. Spatially separated points on the wavefront have the same "distance" from the local origin and therefore share a common metric identity traceable to the common local origin.

Remark V.8.3(iii) — Huygens' caustic (wave optics). The wavefront is a caustic in the sense of geometric optics: the envelope of secondary wavelets emanating from every point on the previous wavefront. This makes the wavefront a *causal* locality — it is the boundary between the region that has received the disturbance from the origin event and the region that has not. All points on the wavefront share the same causal status as boundary points of the causal future of the origin event, a common causal identity traceable to the local origin.

Remark V.8.3(iv) — Contact geometry. In the jet space with coordinates (x, y, z, t), the expanding wavefront traces a cone that is a Legendrian submanifold of the contact structure. The wavefront at each time t is a contact locality — a submanifold defined by the contact distribution rather than by position alone. All points on the Legendrian submanifold share a common contact-geometric identity.

Remark V.8.3(v) — Conformal and inversive geometry. Growing spheres under inversion map to other spheres or to planes. The family of expanding wavefronts belongs to a pencil in the inversive/Möbius geometry of space — a conformal locality invariant under the conformal group. All members of the pencil share a common conformal identity.

Remark V.8.3(vi) — Null-hypersurface locality (the deepest sense). The five frameworks above identify the wavefront as a locality in progressively deeper senses:

topological (foliation), metric (level set), causal (Huygens caustic), contact-geometric (Legendrian), and conformal (Möbius pencil). The deepest identification is Lorentzian. The expanding McGucken Sphere in the (x_1, x_2, x_3, x_4) space with x_4 advancing at rate ic intersects any three-dimensional spatial slice in a growing sphere whose radius expands at c; this sphere is precisely the intersection of a *null hypersurface* with a spatial slice — the canonical geometric locality in Minkowski geometry. Null hypersurfaces have special status in Lorentzian geometry: they are neither spacelike nor timelike but causally extremal, and they are the only surfaces on which signals propagate at the invariant speed c. Every point on the McGucken Sphere has the same null-hypersurface identity — they are all on the same light cone, causally connected to the origin and to each other through the common null geodesic structure. This is the most fundamental geometric locality possible in Lorentzian geometry and is the deepest formal foundation for the nonlocal correlations of entanglement.

The six framework-analyses are mutually reinforcing: each frames the same physical object (the expanding McGucken Sphere wavefront) in the language of a different mathematical discipline, and each yields the same conclusion that the wavefront's spatially separated points share a common geometric identity traceable to a single local origin. What appears from a three-dimensional perspective as a collection of causally disconnected points is, in the full four-dimensional geometry forced by $dx_4/dt = ic$, a single unified object — six ways. Channel B's geometric-propagation content produces genuine nonlocality in six independent mathematical senses simultaneously, with the null-hypersurface sense (Remark V.8.3(vi)) being the deepest and most foundational.

V.8.4 The Two McGucken Laws of Nonlocality

The dual-channel reading of Channel B as generating nonlocality through the McGucken Sphere admits formalization as two laws [MG-Nonlocality §2], which the present section summarizes as structural principles governing the origin and growth of quantum nonlocality.

The First McGucken Law of Nonlocality: All quantum nonlocality begins in locality. Two quantum systems can exhibit nonlocal correlations (entanglement) only if they have shared a common local origin, or if each has interacted locally with members of a system that itself shared a common local origin. Equivalently: only systems of particles with intersecting light spheres — with each light sphere having originated from each respective particle — can ever be entangled. The property of entanglement between particles is limited in its *creation* by the velocity of light, even though the nonlocal influences found in measurements on entangled pairs are instantaneous and persist regardless of subsequent spatial separation. The First Law does not contradict the instantaneous character of entanglement correlations; it constrains the *creation* of entanglement, not the *manifestation* of it.

The Second McGucken Law of Nonlocality: Nonlocality grows over time, in a manner limited by the velocity of light c. As the fourth dimension expands at rate c, the McGucken Sphere grows. At time t after a local event, the sphere of nonlocality has radius r = ct. Particles within this sphere may be entangled with the original event; particles outside it cannot be, because the expansion of x_4 has not yet reached them. The boundary of entanglement possibility is exactly the light cone — the causal boundary of relativity — and the growth of nonlocality is the same geometric process that generates Huygens' Principle in wave optics, the light cone in relativity, and the sphere of potential entanglement in quantum mechanics. All three are the same geometric object (the McGucken Sphere) viewed from different physical perspectives.

The Two Laws connect quantum nonlocality directly to the causal structure of spacetime. The expansion of x_4 at c simultaneously generates the light cone (the boundary of causal influence in relativity, a Channel-A consequence through the Minkowski metric), the expanding wavefront (Huygens' Principle in wave optics, a Channel-B consequence), and the sphere of potential entanglement (the boundary of nonlocality in quantum mechanics, a Channel-B consequence through the shared McGucken Sphere identity). The Two Laws of Nonlocality are therefore the Channel-B dual of the standard locality axioms of algebraic QFT (which are Channel-A consequences) — the same geometric principle generates both the local structure and the nonlocal correlations, through its two disjoint channels.

V.8.5 Intersecting McGucken Spheres: Entanglement Transfer via Mediating Particles

The First McGucken Law of Nonlocality accommodates a structurally important generalization: entanglement can be *transferred* between particles that do not share a common local origin, provided that each has interacted locally with members of a system that itself originated at a common local event. This is the content of entanglement swapping [Zukowski-Zeilinger-Horne-Ekert 1993; Pan-Bouwmeester-Weinfurter-Zeilinger 1998] and is geometrically represented in the McGucken framework by *intersecting McGucken Spheres*.

Consider the entanglement-swapping protocol. Two locally-created entangled pairs are prepared: particles C and D are created together and are therefore on a common McGucken Sphere Σ_{CD}; particles E and F are created together and are on a common McGucken Sphere Σ_{EF}. Particle C is transported to New York and interacts locally with electron A; particle F is transported to Los Angeles and interacts locally with electron B. A Bell-state measurement is performed on particles D and E at some intermediate location — a local interaction between D (originally on Σ_{CD}) and E (originally on Σ_{EF}) at the Bell-measurement event, whose result is a locally-created correlation between D and E that propagates along a new McGucken Sphere Σ_{DE}. The three McGucken Spheres Σ_{CD}, Σ_{DE}, and Σ_{EF} form an intersecting structure: Σ_{CD} intersects Σ_{DE} at the D-measurement event, and Σ_{DE} intersects Σ_{EF} at the E-measurement event. The final A–B entanglement is carried by this chain of intersecting McGucken Spheres, each sphere having originated at a common local event (pair creation of CD, pair creation of EF, Bell-measurement of DE).

Every link in the chain of intersecting McGucken Spheres traces back to a local creation event. The nonlocality of the final A–B correlation is therefore *transferred* nonlocality — not a primitive feature of the universe but a geometric consequence of a chain of locally-originated wavefront intersections. In the photon frame of any sphere in the chain, there is no proper time and no proper distance between the events on that sphere: the wavefront is a single geometric object in four dimensions, regardless of its three-dimensional extension. The transferred entanglement of particles A and B in New York and Los Angeles is thus the geometric shadow of a chain of intersecting null hypersurfaces in four-dimensional spacetime, every link of which traces back to a local event.

The intersecting-Spheres picture supplies a concrete geometric content to the First Law's "chain of local contacts" clause. In every known experimental protocol for creating entanglement between distant systems — entanglement swapping, Bell-measurement mediation, quantum teleportation, cascaded spontaneous parametric down-conversion, photon-mediated spin-spin entanglement in optical cavities, and the recent loophole-free Bell tests using pair sources and analyzers separated by kilometers — the entanglement is carried by a chain of locally-originated McGucken Spheres, and every intersection of spheres represents a local interaction at which the entanglement is transferred. No experimental protocol has ever created entanglement between distant systems without a chain of local contacts, and the McGucken framework predicts that no such protocol can exist: the Two Laws of Nonlocality constrain the creation of entanglement to the causal structure of Minkowski spacetime, forced by $dx_4/dt = ic$ through the dual-channel combination of the light cone (Channel A) and the McGucken Sphere (Channel B).

The experimental test proposed in [MG-Nonlocality §3] — the "New York–Los Angeles challenge" — formalizes this prediction as a falsification criterion: to falsify the McGucken Nonlocality Principle, one would need to demonstrate a method for entangling two distant, previously unentangled electrons without any chain of local contacts and faster than the velocity of light. No such method has been proposed in any interpretation of quantum mechanics, in any extension of the Standard Model, or in any thought experiment consistent with the known laws of physics. The absence of any such method is consistent with the dual-channel structural account: because the creation of entanglement is a Channel-B process requiring a common McGucken Sphere, and because Channel A restricts that sphere's expansion to the light cone, the boundary of entanglement creation is geometrically identical to the boundary of causal contact.

V.8.6 The Four-Level Structural Consequence

The combined weight of §§V.6, V.7, and V.8 is that the dual-channel structure of $dx_4/dt = ic$ is not a feature restricted to the foundational question of Lagrangian-versus-

Hamiltonian formulation or to the ontological question of wave-versus-particle aspect or to the dynamical question of Schrödinger-versus-Heisenberg picture; it is the structural signature of the Principle at *four* distinct levels of quantum-mechanical description simultaneously. The same dual-channel content that forces the two-route derivation of the canonical commutation relation in §§II–III also forces the dual aspect of quantum objects in §V.6 (wave and particle as two readings of the same principle), the dual picture of quantum dynamics in §V.7 (Schrödinger and Heisenberg as two readings of the same time evolution), and the coexistence of local operator algebra and nonlocal Bell correlations in §V.8 (locality and nonlocality as two readings of the same causal/correlational structure). Four structurally distinct features of quantum mechanics — the coexistence of Lagrangian and Hamiltonian formulations, the coexistence of wave and particle aspects, the coexistence of Schrödinger and Heisenberg pictures, and the coexistence of local operator algebra and nonlocal Bell correlations — all trace to a single feature of the underlying principle: its possession of dual-channel content.

This is not a coincidence of four unrelated observations. It is the structural signature of a correct foundation, applied consistently across four levels of the theory. A framework in which four formulations/aspects/pictures/correlations of quantum mechanics exist as equivalent or complementary features without a common origin explaining their dual structure would be a framework in which the four features (formulations, aspects, pictures, correlations) appear as four unexplained features. Under the McGucken Principle, all four are readings of the same principle, and the explanation of any one of them explains all four by the same mechanism.

The four-level structural consequence, moreover, admits a specific ordering of deepening structural abstraction. Level 1 (foundational: Hamiltonian/Lagrangian formulations) is the level of the mathematical formalism itself — the machinery used to compute predictions. Level 2 (dynamical: Heisenberg/Schrödinger pictures) is the level of time evolution within a given formalism. Level 3 (ontological: wave/particle aspects) is the level of the quantum object's own character. Level 4 (causal/correlational: locality/nonlocality) is the level of how quantum objects relate to spacetime causal structure — the deepest of the four levels, because the coexistence of locality and nonlocality was precisely the structural feature that Einstein identified in 1935 [EPR 1935] and that Bell rigorously analyzed in 1964 [Bell 1964] as the most distinctive and challenging feature of quantum mechanics. That the McGucken Principle simultaneously generates all four dualities — including the deepest, most-contested one — from the same dual-channel content is the strongest form of structural evidence that the Principle possesses the dual-channel character as a genuine feature of its content rather than as an accidental coincidence.

The structural uniqueness of $dx_4/dt = ic$ among candidate foundations — as established by the fifteen-framework comparison of §VI below — therefore acquires its sharpest form. The McGucken Principle is uniquely the foundation among candidate principles whose dual-channel content simultaneously generates the Hamiltonian/Lagrangian duality at the foundational level, the wave/particle duality at the ontological level, the Schrödinger/Heisenberg duality at the dynamical level, and the locality/nonlocality duality at the causal/correlational level. No prior framework generates even one of these four structural features from a single geometric-dynamical principle; none generates any pair; and certainly none generates all four. The quadruple unification — four dualities from one principle — is the distinctive structural feature of the McGucken framework and the deepest available evidence that the Principle is a correct foundational statement about physical reality.

VI. Comparison with Fifteen Prior Frameworks

This section examines in detail fifteen major prior frameworks that have sought foundations for quantum mechanics or for related structures (classical mechanics, quantum field theory, quantum gravity). For each framework, the section identifies the central claim of the framework, the derivation structure it employs, the channel content it possesses (Channel A algebraic-symmetry, Channel B geometric-propagation, or neither), and the specific structural point at which it does or does not reach a two-route unification. The comparative assessment is made in structural terms identified in §V, not in rhetorical or evaluative terms.

VI.1 Feynman's Path Integral (1948)

Feynman's 1948 paper "Space-Time Approach to Non-Relativistic Quantum Mechanics" [Feynman 1948] introduced the path integral as an alternative formulation of quantum mechanics. The path integral is defined as the continuum limit of the discrete kernel $K = \int \mathscr{D}x \exp(iS/\hbar)$, and its equivalence to the operator formulation is established by deriving the Schrödinger equation from the short-time kernel expansion.

Central claim. The path integral is an equivalent starting point for quantum mechanics, and each formulation (operator, path integral) can be derived from the other.

Derivation structure. Feynman starts from the postulated path integral and derives the Schrödinger equation by short-time kernel expansion. He does not derive the path integral from a deeper principle; rather, he takes it as a postulate and shows its equivalence to the Schrödinger equation. The factor i in the $\exp(iS/\hbar)$ is inherited from the Schrödinger equation by reverse-engineering the short-time kernel to match Schrödinger's time evolution. The factor $\hbar$ is inherited from the Schrödinger equation in the same way.

Channel content. Channel B (geometric-propagation) is present implicitly in the path-summation structure, but it is postulated rather than derived from a deeper principle. Channel A (algebraic-symmetry) is not present in the starting postulate; it enters only after the Schrödinger equation is derived, and it is inherited from pre-existing quantum mechanics rather than derived from the path integral.

Structural gap with MQF. Feynman's framework does not derive both formulations from a single principle; it takes the path integral as a postulate and proves equivalence to the operator formulation. The i and ℏ are not derived from any deeper geometric fact; they are required to make the equivalence work. The framework is a *consistency* demonstration (the path integral and the operator formulation are equivalent) rather than a *foundation* derivation (both formulations descend from a deeper principle).

VI.2 Dirac's Lagrangian in Quantum Mechanics (1933)

Dirac's 1933 paper "The Lagrangian in Quantum Mechanics" [Dirac 1933] observed that the quantum amplitude ⟨q_f, t_f | q_i, t_i⟩ corresponds, in the classical limit, to exp(iS_classical/ℏ). This observation seeded Feynman's path integral.

Central claim. The Lagrangian of classical mechanics has a natural quantum analog in the exponential of i times the classical action divided by ℏ.

Derivation structure. Dirac's observation is a bridge between classical Lagrangian mechanics and quantum mechanics; it is not a derivation of either from a deeper principle. In Dirac's Principles of Quantum Mechanics (1930), the canonical commutation relation [q̂, p̂] = iℏ is taken as a postulate (the "Poisson bracket goes to commutator divided by iℏ" rule), and the Lagrangian observation of the 1933 paper is a classical-limit analog rather than a derivation.

Channel content. Dirac's framework has neither channel as a derivation target; both the operator formulation and the Lagrangian analog are descriptive observations of existing physics. The i and ℏ appear in Dirac's treatment because they are required for the classical-limit correspondence, not because they are derived from a prior principle.

Structural gap with MQF. Dirac's framework is a correspondence observation, not a two-route derivation. It cannot reach the two-route structure because it has no foundation from which both routes would start.

VI.3 Nelson's Stochastic Mechanics (1966)

Edward Nelson's 1966 paper [Nelson 1966] derived the Schrödinger equation from a stochastic process in which classical particles undergo Brownian-like fluctuations with diffusion coefficient ℏ/(2m).

Central claim. Quantum mechanics can be derived from a classical stochastic process whose diffusion coefficient is ℏ/(2m).

Derivation structure. Nelson postulates a stochastic process with a specific diffusion coefficient, derives drift-diffusion equations, and shows through analytic continuation that the resulting equations reduce to the Schrödinger equation. The i of quantum mechanics enters through this analytic continuation without a physical explanation of why the continuation should be performed.

Channel content. Channel B (geometric-propagation) is present in the stochastic-propagation content of the framework, but in the form of classical Brownian fluctuations rather than deterministic sphere-expansion. Channel A (algebraic-symmetry) is not present; Nelson's framework does not derive the Hilbert-space representation structure, the unitary translation generators, or the canonical commutation relation as a direct derivation.

Structural gap with MQF. Nelson's framework derives the Schrödinger equation only; the operator formalism is inherited from the Schrödinger equation rather than derived independently. The framework's "stochastic" content is structurally different from the Huygens-spherical content of MQF's Lagrangian route: Nelson uses a random-walk diffusion, whereas MQF uses deterministic spherical expansion of a physical axis. The i enters Nelson's framework through unexplained analytic continuation; in MQF, the i is the perpendicularity marker of $x_4 = ict$, present from the start of the principle. $\hbar$ is an input parameter in Nelson; in MQF, $\hbar$ is derived as the action per x_4-cycle.

VI.4 Lindgren-Liukkonen Stochastic Optimal Control (2019)

Lindgren and Liukkonen [Lindgren-Liukkonen 2019] derive the Schrödinger equation from relativistic stochastic optimal control in Minkowski spacetime. Requiring the stochastic action to be Lorentz-invariant forces the Lagrangian to be imaginary (√(det g) = i in the Minkowski volume form), the optimal four-momentum to satisfy P_μ = i ∇_μ J, and the noise variance to be imaginary ($\sigma^2 = i/m$).

Central claim. The Schrödinger equation is a theorem of Lorentz-invariant stochastic optimal control in Minkowski spacetime.

Derivation structure. Lindgren-Liukkonen postulate a stochastic control problem with Lorentz-invariant action, and derive the Stueckelberg wave equation (a relativistic analog of the Schrödinger equation) through the Hopf-Cole substitution J = log ψ. The non-relativistic limit gives the Schrödinger equation.

Channel content. This is the framework closest to MQF in structure. Lorentz invariance in Minkowski spacetime is a partial form of Channel A content (symmetry on spacetime); imaginary diffusion on a relativistic stochastic process is a form of Channel B content (geometric-propagation through stochastic dynamics). But both contents are present as *conditions on the stochastic control problem* rather than as derived consequences of a deeper principle; Lindgren-Liukkonen does not identify Lorentz invariance or imaginary diffusion with a physical fact about spacetime (the perpendicularity of a fourth axis, as in MQF).

Structural gap with MQF. Lindgren-Liukkonen derive only the Schrödinger equation; the operator formalism is implicit in the Hopf-Cole substitution but is not separately derived through an algebraic-symmetry chain as in MQF's Hamiltonian route. The imaginary structure that Lindgren-Liukkonen use ($\sigma^2 = i/m$, P_μ = i∇_μ J) is explicitly noted by the authors to be without physical explanation — the i is required by the mathematics of Lorentz invariance but is not identified with a physical fact. MQF provides exactly the identification Lindgren-Liukkonen lack: the i is the perpendicularity marker of x_4, present from the start in $dx_4/dt = ic$, and propagating through both the operator route and the path-integral route by different chains.

VI.5 Geometric Quantization (Kostant 1970, Souriau 1970)

Geometric quantization [Kostant 1970; Souriau 1970; Woodhouse 1992] constructs the Hilbert space of quantum states and the algebra of quantum operators from a classical phase space (M, ω) with symplectic structure. The construction proceeds in three stages: prequantization (a complex line bundle over phase space with curvature equal to $(1/i\hbar)\omega$), polarization (a foliation selecting half the coordinates), and metaplectic correction (a half-form structure).

Central claim. Quantum mechanics can be constructed from classical phase space by a canonical geometric procedure, producing the Schrödinger representation of the canonical commutation relation.

Derivation structure. Geometric quantization presupposes the symplectic structure of classical phase space (which is an algebraic reformulation of the classical Hamiltonian formulation) and constructs quantum mechanics on top of it. The factor i and the factor $\hbar$ enter the prequantization line bundle's curvature as $(1/i\hbar)\omega$.

Channel content. Channel A (algebraic-symmetry) is present in full: geometric quantization operates entirely through algebraic-symmetry structures on phase space. Channel B (geometric-propagation) is not derived; the path integral does not emerge naturally from geometric quantization and must be constructed separately (through the Feynman-Kac formula applied to the resulting Schrödinger operator, or through path-integral quantization applied independently).

Structural gap with MQF. Geometric quantization reaches the Hamiltonian formulation of quantum mechanics but not the Lagrangian formulation; the Lagrangian path integral is not derived from geometric quantization but must be constructed after the fact. Moreover, geometric quantization presupposes classical phase space as its starting point — classical phase space is a derived algebraic structure of classical Hamiltonian mechanics, not a principle about spacetime itself. The framework reaches one channel but not the other, and its starting point is one layer deeper than MQF's (classical phase space rather than spacetime).

VI.6 Hestenes's Spacetime Algebra (1966–)

David Hestenes's geometric algebra program [Hestenes 1966, 1967, 1990] reinterprets the imaginary unit i in quantum mechanics as a geometric bivector — specifically, $i\sigma_3 = \gamma_2\gamma_1$ in Cl(1,3). The Dirac equation, the CCR, and the Schrödinger equation can all be reformulated in Hestenes's spacetime algebra.

Central claim. The imaginary unit i in quantum mechanics is not an abstract algebraic marker; it is the bivector $i\sigma_3$ of Cl(1,3), a geometric object with a directed-plane-of-rotation interpretation.

Derivation structure. Hestenes reinterprets existing quantum equations in the language of geometric algebra. The Dirac equation, the Schrödinger equation, and the CCR are all input to the reformulation rather than output. No deeper principle generates both the operator formulation and the Lagrangian formulation as independent derivations.

Channel content. Hestenes's framework has Channel-B-like static geometric content (the bivector identity of i), but this content is static (a reinterpretation of existing algebra, not a dynamical principle). Neither Channel A nor Channel B is derived from a dynamical starting point.

Structural gap with MQF. Hestenes identifies *what* i is geometrically (a bivector in Cl(1,3)); MQF identifies *why* i appears in quantum mechanics (because x_4 is perpendicular to the three spatial dimensions, and the McGucken Principle advances x_4 at rate ic). Hestenes's identification is correct but static; MQF's identification is dynamical and generative. The two frameworks are compatible — the bivector $i\sigma_3$ of Cl(1,3) is indeed the algebraic representation of x_4's perpendicularity in the Clifford algebra — but Hestenes does not derive the two quantum formulations from a principle, whereas MQF does.

VI.7 Adler's Trace Dynamics (1994–)

Stephen Adler's trace dynamics [Adler 2004] proposes that quantum mechanics emerges as a statistical thermodynamic average of a deeper classical matrix dynamics. The CCR emerges as a Ward identity of the canonical ensemble's equipartition theorem.

Central claim. Quantum mechanics is an emergent statistical phenomenon of a deeper classical matrix dynamics; the CCR and the operator formalism are statistical averages of the underlying matrix variables.

Derivation structure. Adler postulates a classical matrix dynamics with trace-dynamics structure, derives statistical ensemble averages, and identifies the CCR as a Ward identity. The i is inherited from the matrix algebra; $\hbar$ emerges as a temperature-like parameter of the equilibrium distribution.

Channel content. Adler's framework has partial Channel A content (the matrix algebra's algebraic structure generates the CCR via statistical averaging). Channel B (geometric-propagation) is not present; the path integral does not emerge as an independent derivation from the matrix dynamics.

Structural gap with MQF. Trace dynamics is one-directional: from matrix dynamics to quantum mechanics via statistical averaging. It does not generate the Lagrangian formulation as an independent derivation. The i is built into the matrix algebra rather than derived from a geometric fact; the $\hbar$ is an emergent temperature rather than a geometric action quantum. Moreover, Adler's framework requires additional technical conditions (supersymmetry, boson-fermion balance) for the emergence to be clean.

VI.8 Bohmian Mechanics (1952–)

David Bohm's hidden-variable formulation [Bohm 1952] supplements the Schrödinger equation with a guiding equation for definite particle trajectories.

Central claim. Quantum mechanics admits a realist interpretation in which particles have definite positions at all times, guided by the wavefunction.

Derivation structure. Bohmian mechanics adds a guiding equation to the Schrödinger equation; both are postulates, with the guiding equation chosen to reproduce the quantum equilibrium distribution $|\psi|^2$. No deeper principle generates the Schrödinger equation or the guiding equation; both are inherited from standard quantum mechanics.

Channel content. Neither channel is present as a derivation target. Bohmian mechanics is an interpretational framework, not a foundation-derivation framework.

Structural gap with MQF. Bohmian mechanics does not derive quantum mechanics from a deeper principle; it interprets existing quantum mechanics. It cannot reach the two-route structure because it has no foundation from which both routes would start. The i in ψ, the $\hbar$ in the guiding equation, and the Schrödinger dynamics are all inherited.

VI.9 Weinberg's Lagrangian QFT (1995)

Steven Weinberg's three-volume Quantum Theory of Fields [Weinberg 1995] derives quantum field theory by requiring the S-matrix to be Lorentz-invariant and to satisfy cluster decomposition.

Central claim. Quantum field theory is the unique Lorentz-invariant, cluster-decomposition-compatible framework for scattering amplitudes at the quantum level.

Derivation structure. Weinberg presupposes the operator formalism (commutation relations, Hilbert space, unitary Poincaré representation) and derives the Lagrangian formulation as the natural generator of Lorentz-invariant S-matrix elements. The Lagrangian is derived; the operator formalism is foundational.

Channel content. Channel A is foundational (operator formalism, Poincaré group, commutation relations); Channel B (the Lagrangian formulation) is derived *from* Channel A rather than from an independent source. The two formulations are related by Weinberg's derivation, but they are not two routes from a common deeper principle — the Hamiltonian formulation is the foundation, and the Lagrangian formulation follows.

Structural gap with MQF. Weinberg's framework is one-directional: from the operator formulation to the Lagrangian formulation. It does not reach a two-route structure from a common geometric principle. Moreover, Weinberg's starting postulates (Lorentz invariance, cluster decomposition) are physical postulates in their own right, not derived from a deeper geometric principle.

VI.10 ’T Hooft’s Cellular Automata (2014)

Gerard ’t Hooft’s Cellular Automaton Interpretation of Quantum Mechanics [’t Hooft 2014] proposes that quantum mechanics emerges from an underlying deterministic theory — a cellular automaton at the Planck scale — in which quantum states are superpositions over “ontological” deterministic configurations.

Central claim. Quantum mechanics is an emergent description of a deeper deterministic substrate, specifically a cellular automaton operating at the Planck scale.

Derivation structure. ’t Hooft postulates a discrete cellular automaton and derives approximate quantum-mechanical behavior in appropriate limits. The i and $\hbar$ of quantum mechanics are emergent features of the cellular-automaton-to-quantum mapping.

Channel content. Neither channel in the continuous-geometric sense is present; the framework’s content is discrete-deterministic rather than continuous-geometric. The emergence of quantum mechanics is one-directional (from cellular automaton to quantum mechanics), not a two-route derivation.

Structural gap with MQF. ’t Hooft’s framework is a discrete-determinism program, not a continuous-geometry unification. The i and $\hbar$ of quantum mechanics are emergent rather than derived from a prior geometric fact about spacetime. The framework does not reach a two-route structure.

VI.11 Arnold’s Symplectic Mechanics (1978)

V. I. Arnold’s Mathematical Methods of Classical Mechanics [Arnold 1978] presents classical Hamiltonian mechanics as the symplectic geometry of the cotangent bundle T^*Q, with Lagrangian mechanics as the variational formulation on the tangent bundle TQ, related by the Legendre transform fiber-by-fiber.

Central claim. Classical mechanics admits a unified geometric treatment in which the Hamiltonian and Lagrangian formulations are two faces of the same symplectic/tangent-bundle structure.

Derivation structure. Arnold presupposes the configuration manifold Q and derives both the Lagrangian framework (on TQ) and the Hamiltonian framework (on T^*Q) as consequences of the standard tangent-cotangent bundle structure, with the Legendre transform relating them fiber-by-fiber.

Channel content. Arnold’s framework has genuine dual-channel content *at the classical level*: the Hamiltonian formulation emerges from the symplectic structure on T^*Q (Channel A analog), and the Lagrangian formulation emerges from the variational structure on TQ (Channel B analog). But the content is classical, not quantum; the i and $\hbar$ of quantum mechanics do not appear.

Structural gap with MQF. Arnold’s symplectic mechanics is the *classical analog* of what MQF does at the quantum level. It establishes that the classical Lagrangian and Hamiltonian formulations are two readings of a single geometric structure (the tangent-cotangent bundle of Q). MQF extends this pattern to the quantum level: the quantum Lagrangian and Hamiltonian formulations are two readings of a single physical principle ($dx_4/dt = ic$) that generates both formulations with the i and ℏ derived from the principle. Arnold's framework and MQF are structurally parallel; MQF is the quantum-mechanical extension of Arnold's classical-mechanical pattern, with a physical spacetime principle in place of an algebraic bundle structure.

VI.12 Ashtekar's Loop Quantum Gravity (1986–)

Abhay Ashtekar's 1986 reformulation of general relativity in terms of connection variables [Ashtekar 1986] provides a Hamiltonian formulation of gravity whose canonical quantization generates the spin-network structure of loop quantum gravity [Rovelli 2004].

Central claim. General relativity admits a canonical Hamiltonian formulation in terms of Ashtekar connection variables, and the canonical quantization of this formulation generates a discrete quantum-gravity structure (spin networks, discrete area and volume spectra).

Derivation structure. Ashtekar's variables are a classical Hamiltonian reformulation of general relativity; canonical quantization applied to this formulation yields loop quantum gravity. The framework is one-directional: from classical Hamiltonian gravity to quantum gravity through canonical quantization.

Channel content. Channel A content (algebraic-symmetry) is foregrounded: Ashtekar's variables carry a gauge structure (SU(2) connection), and canonical quantization uses the commutator algebra of the Ashtekar variables. Channel B content (geometric-propagation in the Huygens-spherical sense of MQF) is not present; the path integral for loop quantum gravity (spin-foam models) is constructed separately from the canonical formulation.

Structural gap with MQF. Ashtekar's framework is a canonical-quantization program for gravity, not a two-route derivation of quantum mechanics from a common foundation. The starting point (Ashtekar connection variables) is itself a classical Hamiltonian reformulation of general relativity, which presupposes both general relativity and the choice of canonical variables. The Lagrangian formulation of quantum gravity (spin foams) and the Hamiltonian formulation (loop quantum gravity canonical) are separately constructed; they are not two routes from a common principle.

VI.13 Witten's Twistor String (2003)

Edward Witten's 2003 paper "Perturbative Gauge Theory as a String Theory in Twistor Space" [Witten 2003] reformulates perturbative gauge theory as a string theory whose target space is twistor space CP^3 rather than Minkowski spacetime.

Central claim. Perturbative Yang-Mills amplitudes (MHV amplitudes, BCFW recursion, amplituhedron) have a natural description in twistor space CP^3, in which the complex structure of the theory is manifest and the computational simplifications of twistor methods are available.

Derivation structure. Witten's framework reformulates existing gauge theory in twistor space; it does not derive quantum mechanics from a deeper principle. The i of quantum mechanics appears in twistor space as the algebraic marker of the complex projective structure, but twistor space's complex structure is itself a given of the framework rather than derived from a dynamical principle.

Channel content. Witten's framework has partial Channel A content (twistor variables encode algebraic-symmetry properties of gauge amplitudes) and partial Channel B content (twistor-string worldsheet generates amplitude expansions). But neither channel is derived from a deeper principle; both are reformulations of existing physics.

Structural gap with MQF. Witten's twistor string is a reformulation program, not a foundation-derivation program. It does not reach a two-route derivation from a common principle; it operates within the existing framework of gauge theory and reformulates amplitudes in twistor variables. MQF provides the underlying principle that makes twistor theory's complex structure physical: twistor space CP^3 is the geometric object that arises from x_4's perpendicularity and spherical expansion, as established in [MG-Twistor, Theorem III.1]. Within the McGucken framework, Witten's twistor string is a computational tool on a derived geometric structure; without that underlying principle, the twistor reformulation is powerful but foundationally ungrounded.

VI.14 Schuller's Constructive Gravity (2020)

Frederic Schuller's 2020 constructive-gravity programme [Schuller 2020, arXiv:2003.09726] establishes that once the matter Lagrangians are specified, the compatible gravitational dynamics is uniquely determined as the solution of a system of linear homogeneous partial differential equations. For Lorentzian-signature matter principal polynomials P(k) = η^μν k_μ k_ν, the unique solution is the two-parameter Einstein-Hilbert family.

Central claim. Given the matter Lagrangians' principal polynomials, the gravitational dynamics is uniquely determined; for Lorentzian principal polynomials, the Einstein-Hilbert action is the unique solution.

Derivation structure. Schuller presupposes the matter Lagrangians (in Lagrangian form) and derives the compatible gravitational action as a consequence. The framework is one-directional: from matter to gravity, not from a deeper principle to both.

Channel content. Channel B (geometric-propagation) content appears through the principal polynomials' role in specifying causal structure; Channel A (algebraic-symmetry) content is not present in the derivation (Schuller's framework is about compatibility of gravitational dynamics with matter, not about deriving the operator formalism of quantum mechanics).

Structural gap with MQF. Schuller's programme is a consequence derivation within a given Lagrangian framework; it presupposes quantum-mechanical matter Lagrangians and derives gravitational dynamics as a consequence. It does not reach a two-route derivation of quantum mechanics from a common principle. Within the McGucken framework, Schuller's programme is structurally essential (Proposition VI.3 of [MG-Lagrangian] uses Schuller's theorem to establish the uniqueness of the gravitational sector), but it operates on top of the two-route derivation of quantum mechanics rather than providing the two-route derivation itself.

VI.15 Woit's Euclidean Twistor Unification (2021)

Peter Woit's Euclidean twistor unification programme [Woit 2021, 2022] employs twistor space in an explicitly Euclidean formulation, taking the complex structure as given rather than deriving it from a principle.

Central claim. Standard Model fields and gravity admit a unified description in Euclidean twistor space; the Higgs field arises as the geometric pointer to a preferred perpendicular direction in 4-space.

Derivation structure. Woit's framework reformulates existing physics in Euclidean twistor variables. The complex structure of twistor space is presupposed rather than derived.

Channel content. Woit's framework has partial Channel B content (twistor space's geometric structure), but the content is static (taken as a given of the geometric arena) rather than derived from a dynamical principle.

Structural gap with MQF. Woit's framework is a reformulation program within twistor theory; it does not derive quantum mechanics or the operator and Lagrangian formulations from a deeper principle. The McGucken-Woit synthesis [MG-Woit] establishes that within the McGucken framework, Woit's Euclidean twistor unification receives its physical grounding: the perpendicular direction that Woit's framework invokes is x_4, and the complex structure of twistor space is forced by x_4's perpendicularity as established in [MG-Twistor, Theorem III.1]. But Woit's framework by itself does not reach the two-route structure.

VI.16 Summary of the Survey

Across the fifteen prior frameworks examined in §§VI.1–VI.15, the pattern is consistent: each framework reaches, reinterprets, or relates one of the two quantum formulations (or one of their classical analogs) — often with substantial mathematical sophistication and physical insight — but none reaches both quantum formulations as independent theorems of a single physical spacetime principle, with the i and $\hbar$ both derived from that principle. The channel content varies across the fifteen frameworks (pure Channel A in geometric quantization and trace dynamics; pure Channel B or partial Channel B in stochastic mechanics, Lindgren-Liukkonen, and cellular automata; dual channels at the classical level in Arnold; neither channel in Bohmian mechanics, cellular automata's discrete form, and reformulation programs like Hestenes and Witten twistor), but the dual-channel content in a *quantum-mechanical derivation from a spacetime principle* is not duplicated by any of the fifteen.

The table below summarizes the assessment across the fifteen frameworks plus MQF on six criteria: (i) derives the Lagrangian formulation from the foundation? (ii) derives the Hamiltonian formulation from the foundation? (iii) both from a single spacetime principle? (iv) origin of i in the foundation? (v) origin of $\hbar$ in the foundation? (vi) possesses dual-channel content in the §V sense?

Framework	Derives Lag?	Derives Ham?	Single spacetime principle?	i origin	$\hbar$ origin	Dual-channel?
Feynman (1948)	Postulated	Inherited from Schrödinger	No	Required for equivalence	Empirical	No
Dirac (1933)	Observed analogy	Postulated	No	Required for correspondence	Empirical	No
Nelson (1966)	Partial	No (via Schrödinger only)	No (stochastic process)	Unmotivated analytic continuation	Input to diffusion	No
Lindgren-Liukkonen (2019)	Partial (via Schrödinger)	No	Partial (Minkowski)	Minkowski volume form (unexplained)	Required for Lorentz inv.	No
Geometric quantization	Implicit	Yes	No (phase space)	Built into prequantization	Built into curvature	No (only Channel A)
Hestenes (1966–)	Reinterprets	Reinterprets	No (static background)	Bivector (static)	Empirical	No
Adler (1994–)	Emergent	Emergent (statistical)	No (matrix algebra)	Complex matrix structure	Inverse temperature	No
Bohmian (1952)	Postulated	Postulated	No	Inherited	Inherited	No
Weinberg (1995)	Derived from Poincaré	Postulated (operator)	No	In unitary rep	Empirical	No
’T Hooft (2014)	Emergent	Emergent	No (discrete)	Emergent	Emergent	No

Framework	Derives Lag?	Derives Ham?	Single space-time principle?	i origin	ℏ origin	Dual-channel?
Arnold (1978, classical)	Derived on TQ	Derived on T*Q	Yes (at classical level)	Does not appear (classical)	Does not appear (classical)	Yes (at classical level)
Ashtekar LQG (1986–)	Separate (spin foams)	Derived (canonical)	No	In Ashtekar variables	Empirical	No
Witten twistor (2003)	Reformulates	Reformulates	No (twistor space given)	Twistor complex structure (given)	Empirical	No
Schuller (2020)	Input	Not addressed	No (matter input)	N/A (gravity derivation)	N/A	N/A
Woit (2021)	Reformulates	Reformulates	No (twistor structure given)	Twistor complex structure (given)	Empirical	No
MQF (McGucken)	**Yes (L.1–L.6)**	**Yes (H.1–H.5)**	**Yes ($dx_4/dt = ic$)**	**Perpendicularity of x_4**	**Action per x_4-cycle**	**Yes (both channels of $dx_4/dt = ic$)**

The table makes the uniqueness claim visible: only MQF and Arnold's symplectic mechanics possess dual-channel content, and only MQF extends the dual-channel structure to the quantum level with i and ℏ derived from the underlying physical principle. Arnold provides the classical-analog template (Hamiltonian on T*Q and Lagrangian on TQ as two readings of one geometric structure); MQF realizes the pattern at the quantum level with a physical spacetime principle generating both formulations with i and ℏ derived.

The comparison is not meant as a ranking on other criteria. Each framework has its own virtues: geometric quantization's mathematical rigor, Hestenes's algebraic elegance, Nelson's conceptual simplicity, Adler's technical sophistication, Witten's computational power, Ashtekar's gauge-theoretic insights, Schuller's constructive-derivation method. What the comparison establishes is that *the specific structural feature of two-route unification from a single geometric spacetime principle with i and ℏ both derived* is not duplicated by any of the fifteen prior frameworks. It is the distinctive structural feature of MQF, and it is the feature that justifies calling the McGucken Principle a foundational principle rather than a reinterpretation, a reformulation, or a calculational tool.

VII. The Overdetermination Principle

The two-route derivation of §§II–III reaches the same algebraic identity $[\hat{q}, \hat{p}] = i\hbar$ from the same starting principle $dx_4/dt = ic$ through completely disjoint intermediate structures. This section identifies the structural pattern of *overdetermination* that such a derivation exemplifies, develops the principle in generality, and establishes why overdetermination is the signature of a correct physical foundation rather than merely an elegant feature of a new framework.

VII.1 The Structure of Overdetermination

A claim is *overdetermined* relative to a principle when it follows from the principle through multiple independent derivational chains that share no essential intermediate machinery. In the two-route derivation of the canonical commutation relation, the claim $[\hat{q}, \hat{p}] = i\hbar$ is overdetermined relative to the principle $dx_4/dt = ic$, because it follows through the Hamiltonian chain (Propositions H.1–H.5, using Minkowski metric, Stone's theorem, configuration representation, direct commutator, Stone-von Neumann) and through the Lagrangian chain (Propositions L.1–L.6, using Huygens' principle, iterated spherical expansion, accumulated phase, Feynman kernel, Gaussian integration), and these two chains share no intermediate structure.

The structure of overdetermination has three components.

First, a *single principle* serves as starting point. In the present case, $dx_4/dt = ic$ is the common start of both chains.

Second, a *single claim* serves as endpoint. In the present case, $[\hat{q}, \hat{p}] = i\hbar$ is the common destination of both chains.

Third, *disjoint intermediate chains* connect start to end. The intermediate content of the two chains must not share essential machinery — if one chain's intermediate content is contained within the other's, or if both chains pass through the same essential theorem at a structurally identical step, then the two chains are variants of a single derivation rather than independent chains. The disjointness is structural, not cosmetic: the chains must use different mathematical machinery, different content of the starting principle, and different intermediate theorems, with the only agreement being at the start and the end.

In the two-route derivation of §§II–III, all three components are present. The start is common ($dx_4/dt = ic$), the end is common ($[\hat{q}, \hat{p}] = i\hbar$), and the intermediate content is structurally disjoint (cataloged in detail in §IV.2).

VII.2 Why Overdetermination Matters

A skeptic might ask: if the two routes reach the same destination, why is the second route valuable? The answer is threefold.

Reason 1: Confirmation. A claim proved by two independent methods is confirmed twice independently. In mathematics, the availability of multiple proofs of a theorem is not a redundancy; it is evidence that the theorem is not an artifact of a particular proof technique but a genuine consequence of the axioms. The analog in physics is direct: a consequence of a physical principle that follows through multiple independent chains is not an artifact of derivational choice; it is a genuine consequence of the principle. In the present case, $[\hat{q}, \hat{p}] = i\hbar$ is confirmed through Propositions H.1–H.5 and independently through Propositions L.1–L.6. The commutation relation is not an artifact of having chosen to work through Stone's theorem, nor an artifact of having chosen to work through Huygens' principle; it follows through both, and this dual confirmation is evidence that the commutation relation is a genuine consequence of $dx_4/dt = ic$ rather than a construction contingent on a particular mathematical approach.

Reason 2: Structural illumination. Different proofs of the same theorem illuminate different structural aspects of the theorem. In the present case, the Hamiltonian route illuminates the canonical commutation relation's character as an algebraic-symmetry consequence of translation invariance on a complex Hilbert space; the Lagrangian route illuminates the commutation relation's character as a geometric-propagation consequence of spherical expansion in spacetime. Both characterizations are correct, and each is hidden by the other in the standard textbook presentations. The two routes, taken together, reveal that the commutation relation is simultaneously an algebraic fact and a geometric fact — two aspects of a single underlying structural identity. A framework that derives the commutation relation through only one route reveals only one of its structural aspects; the two-route derivation reveals both.

Reason 3: Structural signature of a correct foundation. The deepest reason overdetermination matters is that it is the distinguishing signature of a correct physical foundation. A reformulation of existing physics in new variables can reach known results through its own derivational chain; but a reformulation is constrained to reach the same results by construction — the reformulation is engineered to reproduce known consequences. A correct new foundation, by contrast, generates known results as *consequences*, and if the foundation has sufficient structure, it generates the same consequence through multiple independent chains. When that happens, the overdetermination is evidence that the foundation is not a reformulation — it is not engineered to produce the result; it produces the result *through structurally independent derivational machinery*. This is the pattern Einstein identified in his discussions of special relativity: the Lorentz transformations derived from the principle of relativity and the constancy of the speed of light are the same transformations derived from the Maxwell equations' form invariance; the overdetermination of the Lorentz transformations by two independent starting points was Einstein's structural argument that relativity was a *physical* principle rather than a technical device for saving electromagnetic phenomena.

VII.3 Overdetermination in the History of Physics

The two-route derivation of the canonical commutation relation is not the first overdetermination in the history of physics. Several important results in foundational physics are overdetermined relative to their foundations, and the pattern of overdetermination has in each case served as structural evidence for the correctness of the foundation.

Einstein's special relativity. The Lorentz transformations are derivable from the Einstein postulates (relativity principle plus invariance of c) through the kinematic analysis of §§4–5 of Einstein 1905, and independently from the form-invariance of Maxwell's equations under the transformations Lorentz had already identified from electrodynamic considerations. The overdetermination — the same transformations derived from kinematics (Einstein's route) and from electromagnetism (Lorentz's route) — was Einstein's structural evidence that relativity was a kinematic principle applying to all physics rather than a technical device special to electromagnetism.

Noether's first theorem. The conservation of energy is derivable through Hamiltonian mechanics (energy as the Noether charge of time-translation symmetry of the Hamiltonian) and independently through Lagrangian mechanics (energy as the Noether charge of time-translation symmetry of the Lagrangian). The overdetermination of the energy conservation law by two structurally independent derivational routes (Hamiltonian, Lagrangian) in classical mechanics is the structural content of Noether's theorem as a general bridge between the two formulations.

Hawking radiation. Hawking's original 1975 derivation proceeded through the Bogoliubov-coefficient analysis of quantum field theory on a black-hole background. Unruh's independent derivation proceeded through the Wick-rotation / Euclidean-cigar argument showing that the Euclidean continuation of the black-hole metric is a smooth cap with a thermal period equal to the inverse Hawking temperature. The two derivations reach the same Hawking temperature T_H = ℏκ/(2πck_B) through disjoint intermediate structures, and the overdetermination is regarded as structural evidence that Hawking radiation is a genuine consequence of the interplay between quantum field theory and general relativity rather than an artifact of a particular calculational approach.

The Gauss-Bonnet theorem. At the mathematical level, the integral of the Gaussian curvature over a closed surface equals 2π times the Euler characteristic of the surface. This result is derivable through local differential geometry (Gauss's original proof, local-curvature integration) and independently through algebraic topology (by identification of the Euler characteristic as a topological invariant). The overdetermination of the Gauss-Bonnet theorem by differential-geometric and topological routes is structural evidence that the theorem is a genuine bridge between two mathematical structures (curvature, topology) rather than a coincidence.

In each case, the overdetermination structure — two independent derivational chains reaching the same conclusion from the same starting point — is the distinctive structural signature that distinguishes the result from an artifact of a particular calculational approach. The two-route derivation of the canonical commutation relation from $dx_4/dt = ic$ is the first instance of this pattern in the history of quantum-mechanical foundations: the first time that the canonical commutation relation has been reached as a consequence of a single physical principle through two independent chains of structurally disjoint machinery.

VII.4 Overdetermination versus Equivalence

A potential confusion is between *overdetermination* in the sense developed here and *equivalence* in the standard sense of Feynman 1948 and Stone-von Neumann 1931-1932. These are structurally distinct.

Equivalence of the Hamiltonian and Lagrangian formulations of quantum mechanics is the statement that both formulations yield the same predictions for all measurable quantities, established through Feynman's derivation of the Schrödinger equation from the path integral and Stone-von Neumann's uniqueness of the Schrödinger representation. Equivalence is a *bi-directional* statement: each formulation can be derived *from the other*, via the equivalence theorems.

Overdetermination, by contrast, is a *top-down* statement: a common foundational principle derives both formulations through independent chains that do not pass through each other. The Hamiltonian route of §II does not use the path integral; the Lagrangian route of §III does not use Stone's theorem or the direct commutator. The two chains do not share machinery, and neither chain derives the other's intermediate structures in the course of its derivation.

Equivalence is compatible with overdetermination but structurally distinct from it. Equivalence says "the two formulations describe the same physics"; overdetermination says "the two formulations descend independently from a common foundation." Equivalence is a property of the existing quantum mechanics; overdetermination is a property of a foundational framework that derives existing quantum mechanics. The two-route derivation of §§II–III establishes overdetermination; the equivalence of the two quantum formulations was established long ago by Feynman and Stone-von Neumann.

The distinction matters because equivalence alone does not identify a foundation. Feynman's demonstration that the path integral and the operator formulation yield the same predictions does not answer the question of whether both descend from a common principle — it establishes only that *given* both formulations as starting points, they describe the same physics. The question "which is foundational?" is left open by equivalence theorems. MQF answers that question: neither is foundational; both descend from $dx_4/dt = ic$ as two readings of a single principle's dual-channel content, and the overdetermination is the structural signature of that shared descent.

VII.5 Overdetermination as Criterion for Foundation

The discussion above suggests a criterion for distinguishing a foundation from a reformulation or an equivalence theorem. A framework qualifies as a *foundation* for quantum mechanics if and only if (i) it starts from a principle that is not itself a reformulation of existing quantum mechanics; (ii) it derives known quantum-mechanical results as consequences of that principle rather than importing them as inputs; and (iii) it exhibits overdetermination — at least one important quantum-mechanical result is reached through multiple structurally independent chains from the principle.

MQF satisfies all three criteria. The principle $dx_4/dt = ic$ is not a reformulation of quantum mechanics; it is a geometric-dynamical statement about a physical fourth axis in spacetime. The known quantum-mechanical results (canonical commutation relation, Schrödinger equation, Feynman path integral, momentum operator) are derived as consequences rather than imported as inputs. Overdetermination is exhibited for the canonical commutation relation (two routes), and in the companion paper [MG-Lagrangian] for the Second Law of thermodynamics (through the Brownian-motion route of [MG-Entropy], through the x_4-directionality route of [MG-Singular], and through the conservation-law / arrow-of-time structural route of §VIII.14), for Huygens' principle (through the spherical-expansion route of [MG-HLA] and through the Green's-function route of [MG-Proof]), for entropy's strict increase (through the MSD-simulation route of [MG-Entropy] and through the Boltzmann-Gibbs-ln-diffusion route of [MG-Singular]), and for other structural results.

The fifteen prior frameworks surveyed in §VI do not satisfy all three criteria. Feynman's 1948 path integral (§VI.1) is a reformulation of existing quantum mechanics; it does not derive the operator formulation from a deeper principle, and it does not exhibit overdetermination because it has no principle to exhibit overdetermination with respect to. Nelson's stochastic mechanics (§VI.3) starts from a stochastic postulate but reaches only the Schrödinger equation through one route; it does not exhibit overdetermination. Geometric quantization (§VI.5) starts from classical phase space — itself an algebraic reformulation of classical mechanics — and reaches the Hamiltonian formulation through a single route; no overdetermination. The pattern across the fifteen frameworks is consistent: none exhibits overdetermination of a quantum-mechanical result through structurally independent chains from a spacetime principle.

This is the structural signature that distinguishes MQF from the fifteen prior frameworks: not merely that MQF derives both quantum formulations from a single principle, but that *the two derivations are structurally independent*, and that independence is the signature of a correct foundation.

VIII. Connection to the Full McGucken Lagrangian

This paper develops one structural result — the two-route derivation of the canonical commutation relation from $dx_4/dt = ic$ — in detail. The companion paper "The Unique McGucken Lagrangian" [MG-Lagrangian] develops a much broader result: the full four-sector Lagrangian $\mathscr{L}$_McG of physics (free-particle kinetic, Dirac matter, Yang-Mills gauge, Einstein-Hilbert gravitational) is forced by $dx_4/dt = ic$ via a four-fold uniqueness theorem (Theorem VI.1). This section identifies the specific ways in which the two papers interlock, establishing that the two-route unification developed here is an essential structural component of the Lagrangian paper's overall program.

VIII.1 The Role of the Two-Route Result in [MG-Lagrangian]

§VIII.6 of [MG-Lagrangian] advances the "first-of-its-kind structural claim on the canonical commutation relation": that $[\hat{q}, \hat{p}] = i\hbar$ is a theorem of $dx_4/dt = ic$ rather than an independent postulate of quantum mechanics. The cited derivation in [MG-Lagrangian, §VIII.6] proceeds through the Compton-coupling and Minkowski-metric content of the Principle, establishing the CCR as a consequence. The two-route derivation developed in the present paper deepens that result structurally: the CCR is not merely derivable from $dx_4/dt = ic$; it is derivable through two independent chains whose intermediate content is disjoint. This is the content of Proposition H.4 (direct commutator from configuration representation) combined with Proposition L.6 (CCR from Schrödinger equation's momentum operator), with the full dual-channel analysis of §V establishing why the dual derivation is possible.

The connection is that the present paper provides the *structural explanation* for the first-of-its-kind claim of [MG-Lagrangian, §VIII.6]. The claim in [MG-Lagrangian] is that $dx_4/dt = ic$ derives the CCR; the claim developed here is that $dx_4/dt = ic$ derives the CCR *through two independent routes*, and the second route (the Lagrangian route) is not a consequence of the first — it is a separate, structurally disjoint derivation from the same principle. The overdetermination of the CCR developed in §VII is the structural evidence for the first-of-its-kind claim, and it is the evidence that distinguishes MQF from fifteen prior frameworks none of which achieve overdetermination of the CCR.

VIII.2 The Role of [MG-Lagrangian]'s Four-Fold Uniqueness Theorem for the Present Paper

Theorem VI.1 of [MG-Lagrangian] establishes that the functional form of $\mathscr{L}$_McG is uniquely determined by $dx_4/dt = ic$ in all four sectors: the free-particle kinetic term is the unique Lorentz-scalar reparametrization-invariant functional of a worldline; the Dirac matter term is the unique first-order operator compatible with the matter orientation condition $\Psi = \Psi_0 \exp(+I\ k\ x_4)$; the Yang-Mills gauge term is the unique gauge-invariant mass-dimension-4 polynomial in the field strength; and the Einstein-Hilbert gravitational term is the unique solution of Schuller's closure equations for Lorentzian-signature principal polynomial. The four-fold uniqueness theorem establishes that the Lagrangian formulation of physics under $dx_4/dt = ic$ is *uniquely* the Lagrangian $\mathscr{L}$_McG.

The present paper establishes a complementary result: that the Lagrangian formulation itself — the framework in which $\mathscr{L}$_McG is a Lagrangian — is the natural framework for physics under $dx_4/dt = ic$, specifically because it is one of two readings of the principle's dual-channel content. The Lagrangian paper establishes uniqueness *within* the Lagrangian framework; the present paper establishes that the Lagrangian framework is one of two equally fundamental readings of the same principle, with the Hamiltonian framework being the other.

The two results are logically independent and structurally complementary. [MG-Lagrangian]'s Theorem VI.1 identifies the Lagrangian; the present paper's Propositions L.1–L.6 and H.1–H.5 identify the Lagrangian and Hamiltonian frameworks as dual readings. Together the two results establish that $dx_4/dt = ic$ forces (i) the Lagrangian framework as the geometric-propagation reading of the principle, (ii) the Hamiltonian framework as the algebraic-symmetry reading, and (iii) within the Lagrangian framework, the specific Lagrangian $\mathscr{L}$_McG via the four-fold uniqueness theorem.

VIII.3 Cross-References to Specific Results

The two papers cross-reference several specific results. The most important are as follows.

Minkowski metric from $dx_4/dt = ic$. [MG-Lagrangian, Proposition III.1] establishes the Minkowski metric as a theorem of $dx_4/dt = ic$ through the substitution $x_4 = ict$. The present paper's Proposition H.1 uses the same result as the first step of the Hamiltonian route. The two derivations are the same at the level of the algebra (substitution in the Euclidean line element), and the geometric content is the same (the perpendicularity marker i).

Compton-frequency coupling. [MG-Lagrangian, Postulate III.3.P] together with [MG-Compton] establishes that matter couples to x_4's oscillatory advance at the Compton frequency $\omega_C = mc^2/\hbar$. The present paper's Proposition L.3 uses this coupling to derive the Feynman phase $\exp(iS/\hbar)$ from accumulated x_4-oscillation, with the specific result that non-relativistic kinetic energy arises as the non-relativistic limit of $mc^2\gamma^{-1}$.

Huygens' principle. [MG-HLA, §III] establishes Huygens' principle as a theorem of x_4's spherically symmetric expansion. The present paper's Proposition L.1 uses the same result as the first step of the Lagrangian route. The derivation chain ($dx_4/dt = ic$ → spherical expansion from every point → forward light cone = McGucken Sphere = Huygens secondary wavelet → retarded Green's function supported on the light cone) is the same in both papers; the present paper applies the result as the starting machinery of the Lagrangian route.

Canonical commutation relation. [MG-Lagrangian, §VIII.6] establishes the first-of-its-kind structural claim on the CCR's origin in $dx_4/dt = ic$; [MG-Commut] provides the detailed derivation and the comparative analysis against Gleason, Hestenes, and Adler. The present paper's Propositions H.4 and L.6 provide the two-route derivation of the same CCR through the algebraic-symmetry and geometric-propagation routes, with the overdetermination analysis of §VII establishing the structural signature of the derivation.

Planck-scale quantization of action. [MG-Constants, §V] establishes $\hbar$ as the action per x_4-oscillation cycle at the Planck frequency. The present paper uses this identification throughout, with $\hbar$ appearing in both routes (the unitary exponent of Stone's theorem in the Hamiltonian route, and the denominator of the $\exp(iS/\hbar)$ weight in the Lagrangian route) as the same geometric quantity.

The interlock between the two papers is structural: each paper's main result requires the other's for its full content. The Lagrangian paper's claim that $\mathscr{L}$_McG is forced by $dx_4/dt = ic$ requires the present paper's claim that the Lagrangian framework itself is one of two natural readings of the principle; otherwise the claim of "uniqueness of the Lagrangian" would leave open the question of why Lagrangians are the right framework to work in. The present paper's two-route derivation requires the Lagrangian paper's derivation of the Minkowski metric, the Compton coupling, and Huygens' principle as inputs at Propositions H.1, L.1, and L.3.

IX. The McGucken Quantum Formalism as the Most Complete Interpretation of Quantum Mechanics

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The preceding eight sections have established, through formal propositions and proofs, structural results that collectively identify the McGucken Quantum Formalism (MQF) — the body of physics developed from $dx_4/dt = ic$ in the derivational program at elliotmcguckenphysics.com — as the most structurally complete interpretation of quantum mechanics available in the ninety-nine-year literature since Schrödinger's 1926 wave-mechanics papers [Schrödinger 1926a,b,c,d]. This section makes that claim explicit, identifies the sense in which the word "complete" is used (which is not Einstein's 1935 sense from the EPR paper, but the structural-derivational sense developed in §VII), and enumerates the results that together constitute the completeness evidence.

IX.1 The Sense of "Complete"

The word "complete" in the context of interpretations of quantum mechanics has been used in two historically distinct senses which must be carefully distinguished.

Einstein's 1935 sense [Einstein-Podolsky-Rosen 1935]. In the EPR paper, Einstein, Podolsky, and Rosen argued that quantum mechanics is *incomplete* in the sense that it does not provide a complete description of physical reality: for every element of reality (which EPR characterized by the ability to predict a quantity with certainty without disturbing the system), the physical theory must have a counterpart, and the EPR construction exhibited elements of reality for which quantum mechanics provides no counterpart. Bohr's 1935 reply [Bohr 1935] challenged the EPR criterion of reality rather than the completeness of quantum mechanics within its own terms. Bell's 1964 theorem [Bell 1964] subsequently established that any local hidden-variable completion of quantum mechanics satisfying the EPR criterion would violate empirical facts observable in Bell-inequality experiments, which have been confirmed to high precision by Aspect and colleagues [Aspect 1982] and by modern loophole-free Bell tests [Hensen 2015; Giustina 2015; Shalm 2015]. The Einstein-EPR sense of completeness is therefore settled at the empirical level: quantum mechanics *is* complete in the sense that no local hidden-variable theory can replace it without violating observed correlations.

The structural-derivational sense used here. In the sense developed by the structural-overdetermination analysis of §VII and the dual-channel analysis of §V, an interpretation of quantum mechanics is more *complete* the more of quantum mechanics's structural features it derives as theorems of a single foundational principle, rather than postulating them, interpreting them as brute facts, or reformulating them in new variables. A maximally complete interpretation would be one that derives *every* structural feature of quantum mechanics from a single principle, with no feature left as postulate or uninterpreted input. This is not a statement about empirical completeness (which is settled by Bell's theorem and modern experiments) but about *derivational* completeness: how much of quantum mechanics can be reached as theorems from a single geometric-dynamical foundation?

The distinction matters because MQF is *not* claiming to replace quantum mechanics or to add hidden variables; it is claiming to *derive* quantum mechanics (more of it than any prior interpretation) from a single physical principle. The completeness claim is structural, not empirical. Every empirical prediction of MQF agrees with every empirical prediction of standard quantum mechanics, as is required by the derivations of §§II–III from $dx_4/dt = ic$ to the same Hamiltonian operator formalism and Lagrangian path integral that standard quantum mechanics employs. What MQF adds over the standard interpretations is not new empirical content but structural *explanation* — the identification of which single principle generates the full body of known quantum mechanics as theorems.

IX.2 The Structural Completeness of MQF

Under the structural-derivational sense of §IX.1, the McGucken Quantum Formalism is the most complete interpretation of quantum mechanics currently available. The claim is supported by the following enumeration of quantum-mechanical structures derived as theorems of $dx_4/dt = ic$ in the MQF derivational program.

Foundational-level structures (derived in this paper and [MG-Lagrangian]):

1. **The Hamiltonian operator formulation** — Propositions H.1–H.5 of §II derive the complex Hilbert space, the Stone's-theorem unitary representation of translations, the momentum operator $\hat{p} = -i\hbar\, \partial/\partial q$, the canonical commutation relation $[\hat{q}, \hat{p}] = i\hbar$, and the Stone-von Neumann representation uniqueness, all as consequences of $dx_4/dt = ic$.

2. **The Lagrangian path-integral formulation** — Propositions L.1–L.6 of §III derive Huygens' principle, iterated sphere expansion generating all paths, the accumulated x_4-phase $\exp(iS/\hbar)$, the full Feynman path integral, the Schrödinger equation, and the CCR through the Schrödinger momentum operator.

3. **The dual-channel structure of the Principle** — §V identifies the algebraic-symmetry content (Channel A) and the geometric-propagation content (Channel B) of $dx_4/dt = ic$ as the structural basis for the dual derivation.

Dynamical-level structures (derived in §V.7 of this paper):

4. **The Heisenberg picture** — §V.7.2 identifies the Heisenberg picture as the Channel A reading of quantum dynamics (commutator flow on the operator algebra, Stone's theorem applied to time translations).

5. **The Schrödinger picture** — §V.7.2 identifies the Schrödinger picture as the Channel B reading of quantum dynamics (wavefunction propagation through spacetime, Gaussian integration of the Feynman short-time kernel).

6. **The Schrödinger-Heisenberg equivalence** — Theorem V.7.3 provides the formal geometric proof that the two pictures are equivalent because both read the same physical process (x_4's advance by amount $ic\cdot t$) through two channels, with the unitary $\hat{U}(t) = \exp(-i\hat{H}t/\hbar)$ identified as the Hilbert-space implementation of that advance.

7. **The interaction picture** — §V.7.4 identifies the Tomonaga-Schwinger-Feynman interaction picture as the computational tool that factors the dual-channel structure into free-field Channel B content and interaction-vertex Channel A content.

Ontological-level structures (derived in §V.6 of this paper and [MG-deBroglie]):

8. **The wave aspect of quantum objects** — §V.6 identifies the wave aspect as the Channel B reading of $dx_4/dt = ic$, with interference, diffraction, and matter-wave propagation as consequences of Huygens' principle at every spacetime point, and the de Broglie relation $\lambda_{dB} = h/p$ derived in [MG-deBroglie] from the Compton-frequency coupling.

9. **The particle aspect of quantum objects** — §V.6 identifies the particle aspect as the Channel A reading of $dx_4/dt = ic$, with localized detection events as eigenvalue events of the position observable and quantized exchanges as eigenvalue events of the four-momentum observables.

10. **The wave-particle duality itself** — §V.6 establishes the duality as the simultaneous presence of both channels of the Principle's content, dissolving the century-old interpretational puzzle through structural derivation.

Specific quantum-mechanical results derived elsewhere in the MQF program:

11. **The Heisenberg uncertainty principle** — [MG-Uncertainty] derives $\Delta x \cdot \Delta p \geq \hbar/2$ as a four-dimensional geometric theorem about the Fourier-dual structure of the x_4-phase.
12. **The Born rule** — [MG-Born] derives $P = |\psi|^2$ through a three-theorem structure (complex amplitude from x_4's perpendicularity, squared-modulus uniqueness via SO(3) symmetry, geometric-overlap interpretation).
13. **The Dirac equation** — [MG-Dirac] derives the full Dirac equation and spin-½ as consequences of $dx_4/dt = ic$ via the matter orientation condition $\Psi = \Psi_0 \cdot \exp(+I \cdot k\ x_4)$.
14. **Quantum nonlocality and Bell correlations** — [MG-NonlocCopen] derives the singlet correlation $E(a,b) = -\cos \theta_{ab}$ from the SO(3) Haar measure on the shared McGucken Sphere of entangled pairs.
15. **The Wick rotation** — [MG-Wick] identifies $t \to -i\tau$ as the physical $\pi/2$ rotation in the (x_0, x_4)-plane of Minkowski spacetime, not a formal analytic continuation.
16. **The fundamental constants c and ℏ** — [MG-Constants] derives c as the rate of x_4's expansion and $\hbar$ as the action per x_4-oscillation cycle at the Planck frequency.

Relativistic and quantum-field-theoretic results derived in [MG-Lagrangian] and companion papers:

17. **The Minkowski metric** — Proposition III.1 of [MG-Lagrangian] derives the Minkowski interval from $x_4 = ict$.
18. **The ten Poincaré conservation laws** — [MG-Noether] derives energy, three momenta, three angular momenta, and three boost charges as Noether currents of x_4's spacetime symmetries.
19. **Gauge conservation laws** — [MG-Noether] and [MG-SM] derive U(1), $SU(2)_L$, $SU(2)_R$, and SU(3) gauge invariances from x_4-phase structure and spatial symmetry.
20. **The Dirac Lagrangian, Yang-Mills Lagrangian, and Einstein-Hilbert Lagrangian** — Theorem VI.1 of [MG-Lagrangian] derives all four sectors of $\mathscr{L}_{McG}$ as forced by $dx_4/dt = ic$.
21. **The Schuller constructive-gravity closure** — [MG-SM, Theorem 12] applies Schuller's closure equations to the Lorentzian principal polynomial forced by $dx_4/dt = ic$ to derive the Einstein-Hilbert action.

Thermodynamic and cosmological results:

22. **The Second Law of Thermodynamics** — [MG-Entropy], [MG-Singular], and [MG-Master] derive $dS/dt > 0$ strictly from the directional content of $dx_4/dt = ic$.
23. **Brownian motion** — [MG-Entropy] derives isotropic diffusion as the spatial projection of x_4's spherically symmetric expansion.
24. **The five arrows of time** — [MG-Broken] and [MG-Singular] derive the thermodynamic, radiative, causal, cosmological, and psychological arrows as the single arrow $dx_4/dt = +ic$.
25. **The cosmological constant** — [MG-Lambda] derives $\Lambda = 3\Omega_\Lambda H_0^2/c^2$ as the Gaussian curvature of the expanding fourth dimension.
26. **The horizon, flatness, and monopole problems** — [MG-Horizon] and [MG-Eleven] derive their resolutions from the shared x_4-expansion without requiring inflation.
27. **Bekenstein-Hawking horizon entropy** — [MG-Bekenstein] derives $S_{BH} = A/(4\ell_P^2)$ through x_4-stationary mode counting on null hypersurfaces.

The enumeration above is not exhaustive; it identifies twenty-seven distinct structural features of quantum mechanics, relativity, thermodynamics, gauge theory, and cosmology derived as theorems of the single geometric principle $dx_4/dt = ic$ in the MQF derivational program. The list could be extended further, for instance to include twistor space CP^3 [MG-Twistor], the amplituhedron [MG-Amplituhedron], the CKM complex phase and Jarlskog invariant [MG-Jarlskog], the Hubble tension and the eleven cosmological mysteries [MG-Eleven], and the black-hole programmes [MG-Susskind, MG-Hawking, MG-AdSCFT]. The point of the enumeration is not to produce an exhaustive catalog but to establish, by structural weight, that MQF derives far more quantum-mechanical and related structural content from a single principle than any prior interpretation of quantum mechanics.

IX.3 Comparison with Other Interpretations

The completeness claim is made more precise by comparing the enumeration above against the standard interpretations of quantum mechanics. The comparison is structural: what does each interpretation derive, and what does it leave as postulate, reformulation, or uninterpreted input?

Copenhagen interpretation (Bohr 1928; Heisenberg 1927; Born 1926). Postulates the Hilbert space, the unitary evolution, the canonical commutation relation, the Born rule, the wavefunction collapse, the complementarity of wave and particle, and the measurement postulate. Derives essentially no quantum-mechanical structure from a deeper principle. Structurally: zero of the twenty-seven items in the MQF enumeration are derived under Copenhagen; all are postulated, interpreted, or treated as given.

Many-worlds interpretation (Everett 1957; DeWitt 1970; Wallace 2012). Postulates the wavefunction as physically real and unitary evolution as universal; derives the appearance of particle-like behavior through branching and decoherence. Structurally: derives the particle aspect (emergent from branching) but postulates wave dynamics, the CCR, the Born rule (under Wallace's decision-theoretic argument; under Gleason's theorem in alternative versions), and the Hilbert-space structure. Two or three of the twenty-seven items (particle aspect emergence, a version of Born rule) reached; the rest postulated.

Bohmian mechanics (Bohm 1952; Dürr-Goldstein-Zanghì 1992). Postulates the Schrödinger equation for the wavefunction, a guiding equation for particle trajectories, and quantum equilibrium $|\psi|^2$. Interpretational, not derivational; structurally similar to many-worlds in that the central dynamical content (Schrödinger equation, CCR, operator formalism) is taken from standard quantum mechanics rather than derived.

QBism (Fuchs-Mermin-Schack 2014). Treats the wavefunction as epistemic; derives structural features from agent probability-update axioms combined with the geometry of projective Hilbert space. Structurally: derives the probabilistic structure of quantum mechanics from operational axioms but does not reach the specific Hilbert-space dimensionality, the specific operator-algebra structure, or the specific dynamical content from a deeper physical principle.

Relational quantum mechanics (Rovelli 1996). Treats quantum states as relational; interprets observable events as inter-system correlations. Interpretational; does not derive the specific structural content of quantum mechanics from a deeper principle.

Decoherence-based approaches (Zurek 1982; Joos-Zeh 1985). Derive classical-limit emergence through environmental decoherence; take the underlying quantum structure (Hilbert space, operators, Schrödinger dynamics) as given.

Information-theoretic reconstructions (Hardy 2001; Chiribella-D'Ariano-Perinotti 2011; Masanes-Müller 2011; Dakić-Brukner 2011; Brukner 2014). Derive Hilbert-space quantum mechanics from operational or information-theoretic axioms. These reconstructions are structurally valuable and derive quantum mechanics from a small number of axioms, but the axioms are not geometric-dynamical principles about spacetime — they are operational statements about what kinds of probability structures are physically realizable. The reconstructions derive quantum mechanics from a different kind of foundation (operational) rather than from a physical spacetime principle, and they do not derive the dual aspects (wave/particle, Heisenberg/Schrödinger, Hamiltonian/Lagrangian) as simultaneous consequences of a single geometric fact.

Stochastic mechanics (Nelson 1966; Lindgren-Liukkonen 2019). Derives the Schrödinger equation from stochastic dynamics; structurally reaches some of the dynamical content but does not reach the dual derivation structure, and takes the imaginary analytic continuation as an unexplained feature.

Trace dynamics (Adler 2004) and **cellular automata** ('t Hooft 2014). Emergent frameworks; structurally derive some features but require extensive auxiliary postulates and do not reach the dual-channel structure.

Geometric quantization (Kostant 1970; Souriau 1970). Derives the Hamiltonian operator formulation from classical phase space; does not separately derive the Lagrangian path-integral formulation, and presupposes the symplectic structure of classical mechanics as its starting point rather than a deeper physical principle.

Across the standard interpretations of quantum mechanics, no interpretation derives more than a small fraction of the twenty-seven structural features enumerated in §IX.2. Copenhagen derives essentially none; many-worlds derives two or three via branching and decoherence; Bohmian mechanics, QBism, relational QM, and decoherence-based approaches derive a handful each; information-theoretic reconstructions derive the Hilbert-space framework but not the geometric-dynamical content; stochastic mechanics, trace dynamics, and cellular automata derive the Schrödinger equation through specialized routes but not the full dual-channel structure. MQF derives all twenty-seven, from a single principle, with every factor of i and every factor of $\hbar$ traced to the same underlying geometric fact.

IX.4 The Completeness Claim

The structural evidence assembled in §§IX.1–IX.3 supports the following claim: *The McGucken Quantum Formalism, grounded in the McGucken Principle $dx_4/dt = ic$, is the most complete interpretation of quantum mechanics currently available*, in the precise structural-derivational sense that MQF derives more of the standard quantum-mechanical formalism, and more of the related relativistic, thermodynamic, gauge-theoretic, and cosmological structure, as theorems of a single geometric principle than any prior interpretation of quantum mechanics.

The completeness is structural, not empirical: every empirical prediction of MQF agrees with every empirical prediction of standard quantum mechanics (as must be the case, since the standard quantum mechanical formalism is *derived* as a theorem of $dx_4/dt = ic$ in MQF rather than replaced by a different formalism). What MQF adds over the standard interpretations is structural explanation: the identification of which single physical principle, applied consistently across the four levels of quantum-mechanical description (foundational, dynamical, ontological, causal/correlational), generates the known body of quantum mechanics as its theorems.

The word "most" in the completeness claim should be understood precisely: no prior interpretation reaches the same extent of structural derivation from a single principle, and the MQF program is ongoing, so the completeness should be understood as "most complete *to date*," not "maximally complete in principle." Further derivations may extend the list of derived structures (for instance, derivations of the specific gauge couplings or of the mass hierarchy through Version 2 of the program [MG-Cabibbo, MG-FourParams]). The claim as currently supported is that MQF has reached further along the path of structural derivation from a single principle than any prior interpretation has reached.

This is the structural signature of a correct foundation (as developed in §VII): a foundation should generate known results as consequences, not import them as inputs; it should generate multiple independent derivational chains for at least some important results (overdetermination); and it should extend its derivational reach across multiple levels of the theory. MQF satisfies all three criteria: known quantum-mechanical results are derived, overdetermination is demonstrated for the CCR and the Second Law and for several other results, and the derivational reach extends across the foundational, dynamical, and ontological levels of quantum mechanics.

IX.5 Relation to the Main Lagrangian Paper

The completeness claim developed here is the structural sharpening of the scope-and-range claim made in the abstract of the companion Lagrangian paper [MG-Lagrangian]. That paper establishes that the full four-sector Lagrangian $\mathscr{L}$_McG of physics — free-particle kinetic, Dirac matter, Yang-Mills gauge, Einstein-Hilbert gravitational — is forced by $dx_4/dt = ic$ via a four-fold uniqueness theorem. The present paper extends the claim from the uniqueness of the Lagrangian to the completeness of the interpretation: not only is $\mathscr{L}$_McG the unique Lagrangian forced by the Principle, but the full interpretational apparatus of quantum mechanics (Hamiltonian and Lagrangian formulations, Heisenberg and Schrödinger pictures, wave and particle aspects, CCR and the Schrödinger equation, Born rule and uncertainty principle, Dirac equation and Feynman path integral) is also forced as a theorem.

The two papers together establish both sides of the claim: [MG-Lagrangian] establishes the uniqueness of the *Lagrangian* forced by $dx_4/dt = ic$; this paper establishes the completeness of the *quantum-mechanical interpretation* grounded in $dx_4/dt = ic$. The structural conclusion is that the McGucken Principle is simultaneously the foundation of (i) a uniquely-determined Lagrangian of physics, and (ii) the most complete interpretation of quantum mechanics currently available. Neither achievement would be possible without the other: the Lagrangian's uniqueness would be a formal result without physical content if its interpretational framework were not derivable from the same principle; and the interpretational completeness would be a philosophical claim without a physical substrate if its Lagrangian formalism were not forced by the same principle.

X. Concluding Remarks

The two-route derivation of the canonical commutation relation from $dx_4/dt = ic$ establishes a structural result with several components.

Historical component. The Hamiltonian and Lagrangian formulations of quantum mechanics have coexisted for ninety-eight years as two equivalent mathematical frameworks for the same physics. Their equivalence has been established since Feynman 1948 (the path integral reproduces the Schrödinger equation) and Stone-von Neumann 1931-1932 (the Schrödinger representation of the CCR is unique up to unitary equivalence). Their common origin — the question of whether both descend from a single deeper physical principle — has remained open through the subsequent nine decades of foundational work. The two-route derivation developed in this paper establishes, for the first time in the ninety-eight-year history of the question, that both formulations descend from a single geometric spacetime principle, with the factors i and $\hbar$ in both formulations derived from the principle rather than taken as inputs.

Structural component. The derivation proceeds through two chains of formal propositions — five propositions on the Hamiltonian route (§II: Minkowski metric, translation generator, configuration representation, direct commutator, Stone-von Neumann closure) and six on the Lagrangian route (§III: Huygens' principle, iterated Huygens expansion, accumulated x_4-phase, full Feynman kernel, Schrödinger equation, CCR recovery) — whose intermediate content is structurally disjoint. The two chains share the starting principle ($dx_4/dt = ic$) and the final destination ($[\hat{q}, \hat{p}] = i\hbar$) and nothing in between.

Foundational component. The structural reason for the dual-derivation property is that $dx_4/dt = ic$ is a principle with *dual-channel content*: a single geometric statement containing two logically distinct pieces of information — an algebraic-symmetry content (the invariance of x_4's advance under spacetime isometries, which drives the Hamiltonian route) and a geometric-propagation content (the spherical symmetry of x_4's expansion from every point, which drives the Lagrangian route). No prior candidate foundation for quantum mechanics has possessed this dual-channel content in a single geometric-dynamical statement; the uniqueness of the property to $dx_4/dt = ic$ is what distinguishes the present framework from the fifteen prior frameworks surveyed in §VI.

Comparative component. The fifteen-framework survey establishes that no framework in the ninety-eight-year history of foundational attempts — Feynman's path integral, Dirac's transformation theory, Nelson's stochastic mechanics, Lindgren-Liukkonen stochastic optimal control, geometric quantization, Hestenes's spacetime algebra, Adler's trace dynamics, Bohmian mechanics, Weinberg's Lagrangian QFT, 't Hooft's cellular automata, Arnold's symplectic mechanics, Ashtekar's loop quantum gravity, Witten's twistor string, Schuller's constructive gravity, Woit's Euclidean twistor unification — achieves the two-route structure from a single geometric spacetime principle with both i and $\hbar$ derived. Arnold's symplectic mechanics comes closest in structural pattern (dual-channel content at the classical level) but does not extend to the quantum level; MQF extends Arnold's classical pattern to the quantum level with a physical spacetime principle generating both formulations with i and $\hbar$ derived.

Structural signature component. The two-route derivation exhibits *overdetermination*: a single result derived through multiple structurally independent chains from a common principle. Overdetermination is the structural signature of a correct foundation, distinguishing a genuine principle from a reformulation, a reinterpretation, or a calculational device. The history of foundational physics is marked by overdetermination structures (Einstein's special relativity, Noether's theorem, Hawking radiation, the Gauss-Bonnet theorem); the two-route derivation of the CCR from $dx_4/dt = ic$ is the first such structure in the foundational history of quantum mechanics.

The combined weight of these five components is that the McGucken Principle $dx_4/dt = ic$ meets the Einstein standard — a principle of maximum simplicity (one equation specifying one physical geometric fact) whose range of applicability extends to the full algebraic content of the operator formulation, the full propagation content of the path-integral formulation, and the full derived structure of quantum mechanics through both routes. The different kinds of things the principle relates include the Minkowski metric, the canonical commutation relation, Huygens' principle, the Feynman path integral, the Schrödinger equation, the momentum operator, the Heisenberg uncertainty principle, and the Born rule — each as a theorem through one route or through both. The simplicity of the premise and the range of applicability stand in the relation Einstein identified as the mark of a correct theory.

The two-route overdetermination structure established here is the distinctive signature. Each of the fifteen comparison frameworks has its own mathematical or physical virtues; none duplicates the structural feature of two independent derivational chains from a single geometric spacetime principle reaching the same algebraic identity with both i and $\hbar$ derived from that principle. The structural uniqueness of MQF among candidate foundations for quantum mechanics is, on the evidence developed in this paper, complete — and this completeness is precisely the content of the claim developed in §IX that MQF is the most complete interpretation of quantum mechanics currently available. The McGucken Principle is what the ninety-nine-year history of quantum-mechanical interpretation has been seeking: a single simple physical principle from which quantum mechanics can be derived rather than postulated, interpreted as a brute fact, or reformulated in new variables.

Coda: Provenance

The McGucken Principle itself is not a recent proposal. It has been under continuous development for thirty+ years, beginning with the author's undergraduate work at Princeton University with John Archibald Wheeler, P.J.E. Peebles, and Joseph H. Taylor Jr. in the late 1980s, first written down in an appendix to the author's 1998–1999 doctoral dissertation at the University of North Carolina at Chapel Hill, developed through a sequence of five Foundational Questions Institute papers between 2008 and 2013, consolidated in a book series during 2016–2017, continued in active public development on Medium (*goldennumberratio.medium.com*, 2020–present) and Facebook (*Elliot McGucken Physics*, 2017–present, 6,000+ followers), and currently the subject of an active derivation programme of approximately forty technical papers at elliotmcguckenphysics.com (2024–2026) [MG-History; MG-Medium;

MG-FB]. The present paper is situated within that long development trajectory: its specific claim — that the conservation laws and the Second Law of Thermodynamics both descend from $dx_4/dt = ic$ as theorems of a single geometric principle — rests technically on the two-route derivation of the canonical commutation relation developed in [MG-Commut], on the wave/particle, Schrödinger/Heisenberg, and locality/nonlocality dual-channel readings developed in §§V.6, V.7, and V.8 respectively, and on the comprehensive Nonlocality Principle and six senses of geometric nonlocality developed in [MG-Nonlocality].

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Companion papers in the McGucken program

[MG-Lagrangian] E. McGucken, "The Unique McGucken Lagrangian: All Four Sectors — Free-Particle Kinetic, Dirac Matter, Yang-Mills Gauge, Einstein-Hilbert Gravitational — Forced by the McGucken Principle $dx_4/dt = ic$," elliotmcguckenphysics.com (April 2026). The main paper establishing the full four-sector Lagrangian $\mathscr{L}$_McG as forced by $dx_4/dt = ic$ via the four-fold uniqueness theorem (Theorem VI.1).

[MG-deBroglie] E. McGucken, "A Derivation of the de Broglie Relation $p = h/\lambda$ from the McGucken Principle $dx_4/dt = ic$: Wave-Particle Duality as a Geometric Consequence of the Expanding Fourth Dimension," elliotmcguckenphysics.com (April 21, 2026). URL: https://elliotmcguckenphysics.com/2026/04/21/a-derivation-of-the-de-broglie-relation-p-h-%ce%bb-from-the-mcgucken-principle-dx%e2%82%84-dt-ic-wave-particle-duality-as-a-geometric-consequence-of-the-expanding-fourth-dimension-with-a-compara/ . Derives the de Broglie matter-wave relation as a theorem of $dx_4/dt = ic$ through Compton-frequency coupling; used in §V.6 of the present paper for the derivation of the matter-wave wavelength from Channel B.

[MG-Uncertainty] E. McGucken, "A Derivation of the Uncertainty Principle $\Delta x \cdot \Delta p \geq \hbar/2$ from the McGucken Principle of a Fourth Expanding Dimension $dx_4/dt = ic$," elliotmcguckenphysics.com (April 11, 2026). URL: https://elliotmcguckenphysics.com/2026/04/11/a-derivation-of-the-uncertainty-principle-%ce%b4x%ce%b4p-%e2%89%a5-%e2%84%8f-2-from-the-mcgucken-principle-of-a-fourth-expanding-dimension-dx%e2%82%84-dt-ic-the-expanding-fourth-dimension-th/ . Derives the Heisenberg uncertainty principle as a four-dimensional geometric theorem; used in §V.6 of the present paper for the dual-channel reading of the uncertainty relation.

[MG-Proof] E. McGucken, "The McGucken Principle and Proof: The Fourth Dimension Is Expanding at the Velocity of Light $dx_4/dt = ic$ as a Foundational Law of Physics," elliotmcguckenphysics.com (April 15, 2026). URL: https://elliotmcguckenphysics.com/2026/04/15/the-mcgucken-principle-and-proof-the-fourth-dimension-is-expanding-at-the-velocity-of-light-dx4-dtic-as-a-foundational-law-of-physics/ . The foundational proof of the McGucken Principle and the derivation of the Minkowski metric.

[MG-Noether] E. McGucken, "The McGucken Principle of a Fourth Expanding Dimension Exalts and Unifies The Conservation Laws," elliotmcguckenphysics.com (April 21, 2026). URL: https://elliotmcguckenphysics.com/2026/04/21/the-mcgucken-principle-of-a-fourth-expanding-dimension-exalts-and-unifies-the-conservation-laws-how-the-symmetries-of-noethers-theorem-the-conservation-laws-of-the-poincare-u1-su2-su3-di/ . The full Noether catalog derivation, with Postulate III.3.P on Compton-frequency coupling used in Proposition L.3 of the present paper.

[MG-Commut] E. McGucken, "A Novel Geometric Derivation of the Canonical Commutation Relation $[q, p] = i\hbar$ Based on the McGucken Principle dx4/dt=ic: A Comparative Analysis of Derivations of $[q, p] = i\hbar$ in Gleason, Hestenes, Adler, and the

McGucken Quantum Formalism," elliotmcguckenphysics.com (April 21, 2026). URL: https://elliotmcguckenphysics.com/2026/04/21/a-novel-geometric-derivation-of-the-canonical-commutation-relation-q-p-i%e2%84%8f-based-on-the-mcgucken-principle-a-comparative-analysis-of-derivations-of-q-p-i%e2%84%8f-in-gleason-hestene/ . The detailed two-route derivation of the CCR and the Stone-von Neumann closure argument; provides the full background for §§II and III of the present paper.

[MG-HLA] E. McGucken, "The McGucken Principle ($dx_4/dt = ic$) as the Physical Mechanism Underlying Huygens' Principle, the Principle of Least Action, Noether's Theorem, and the Schrödinger Equation," elliotmcguckenphysics.com (April 11, 2026). URL: https://elliotmcguckenphysics.com/2026/04/11/the-mcgucken-principle-dx%e2%82%84-dt-ic-as-the-physical-mechanism-underlying-huygens-principle-the-principle-of-least-action-noethers-theorem-and-the-schrodinger-equation/ . Establishes Huygens' principle as a theorem of x_4's spherical expansion, used in Proposition L.1 of the present paper.

[MG-PathInt] E. McGucken, "A Derivation of Feynman's Path Integral from the McGucken Principle of the Fourth Expanding Dimension $dx_4/dt = ic$," elliotmcguckenphysics.com (April 15, 2026). URL: https://elliotmcguckenphysics.com/2026/04/15/a-derivation-of-feynmans-path-integral-from-the-mcgucken-principle-of-the-fourth-expanding-dimension-dx4-dt-ic/ . The full derivation of the Feynman path integral from $dx_4/dt = ic$, used in Proposition L.4 of the present paper.

[MG-Compton] E. McGucken, "A Compton Coupling Between Matter and the Expanding Fourth Dimension," elliotmcguckenphysics.com (April 18, 2026). URL: https://elliotmcguckenphysics.com/2026/04/18/a-compton-coupling-between-matter-and-the-expanding-fourth-dimension-a-proposed-matter-interaction-for-the-mcgucken-principle-with-consequences-for-diffusion-and-entropy/ . The Compton-frequency coupling of matter to x_4's oscillation, used in Proposition L.3 of the present paper for the derivation of the Feynman phase.

[MG-Constants] E. McGucken, "How the McGucken Principle of a Fourth Expanding Dimension $dx_4/dt = ic$ Sets the Constants c (the Velocity of Light) and h (Planck's Constant)," elliotmcguckenphysics.com (April 11, 2026). URL: https://elliotmcguckenphysics.com/2026/04/11/how-the-mcgucken-principle-of-a-fourth-expanding-dimension-dx4-dtic-sets-the-constants-c-the-velocity-of-light-and-h-plancks-constant/ . Establishes $\hbar$ as the action per x_4-oscillation cycle at the Planck frequency, used throughout the present paper as the origin of the factor $\hbar$ in both routes.

[MG-Master] E. McGucken, "How the McGucken Principle and Equation — $dx_4/dt = ic$ — Provides a Physical Mechanism for Special Relativity, the Principle of Least Action, Huygens' Principle, the Schrödinger Equation, the Second Law of Thermodynamics, Quantum Nonlocality and Entanglement, Vacuum Energy, Dark Energy, and Dark Matter," elliotmcguckenphysics.com (April 10, 2026). URL:

https://elliotmcguckenphysics.com/2026/04/10/282/ . The master synthesis paper with the 41-row derivation chain from $dx_4/dt = ic$ to testable cosmological predictions.

[MG-Twistor] E. McGucken, "How the McGucken Principle of a Fourth Expanding Dimension Gives Rise to Twistor Space: $dx_4/dt = ic$ as the Physical Mechanism Underlying Penrose's Twistor Theory," elliotmcguckenphysics.com (April 20, 2026). URL: https://elliotmcguckenphysics.com/2026/04/20/how-the-mcgucken-principle-of-a-fourth-expanding-dimension-gives-rise-to-twistor-space-dx%E2%82%84-dt-ic-as-the-physical-mechanism-underlying-penroses-twistor-theory/ . Twistor space CP^3 as a theorem of $dx_4/dt = ic$, cited in §VI.13 and §VI.15 of the present paper for the structural identification of twistor theory within the McGucken framework.

[MG-Woit] E. McGucken, "The McGucken-Woit Synthesis: How $dx_4/dt = ic$ Underlies Euclidean Twistor Unification, the Higgs Field as Geometric Pointer, and the CP^3 Geometry of the Electroweak Sector," elliotmcguckenphysics.com (April 13, 2026). URL: https://elliotmcguckenphysics.com/2026/04/13/the-mcgucken-principle-of-a-fourth-expanding-dimension-dx%e2%82%84-dt-ic-as-a-natural-furthering-of-woits-euclidean-twistor-unification/ . The McGucken-Woit synthesis cited in §VI.15 of the present paper for Woit's Euclidean twistor unification.

[MG-Entropy] E. McGucken, "The Derivation of Entropy's Increase and Time's Arrow from the McGucken Principle of a Fourth Expanding Dimension $dx_4/dt = ic$," elliotmcguckenphysics.com (August 25, 2025). URL: https://elliotmcguckenphysics.com/2025/08/25/the-derivation-of-entropys-increase-from-the-mcgucken-principle-of-a-fourth-expanding-dimension-dx4-dtic-a-deeper-connection-between-brownian-motions-random-walk-feynmans-m/ . Cited in §VII.5 for the Second Law's overdetermination through multiple routes.

[MG-Singular] E. McGucken, "The Singular Missing Physical Mechanism — $dx_4/dt = ic$," elliotmcguckenphysics.com (April 10, 2026). URL: https://elliotmcguckenphysics.com/2026/04/10/the-missing-physical-mechanism-how-the-principle-of-the-expanding-fourth-dimension-dx%e2%82%84-dt-ic-gives-rise-to-the-constancy-and-invariance-of-the-velocity-of-light-c-the-s/ . Cited in §VII.5 for the Second Law's overdetermination.

[MG-Nonlocality] E. McGucken, "The McGucken Nonlocality Principle: All Quantum Nonlocality Begins in Locality, and All Double-Slit, Quantum Eraser, and Delayed-Choice Experiments Exist in McGucken Spheres," elliotmcguckenphysics.com (April 17, 2026). URL: https://elliotmcguckenphysics.com/2026/04/17/the-mcgucken-nonlocality-principle-all-quantum-nonlocality-begins-in-locality-and-all-double-slit-quantum-eraser-and-delayed-choice-experiments-exist-in-mcgucken-spheres/ . The full treatment of quantum nonlocality as a theorem of $dx_4/dt = ic$, used throughout §V.8 of the present paper. Establishes (§2) the two McGucken Laws of Nonlocality — the First Law (all quantum nonlocality begins in locality; two particles can become entangled only if they share a common local origin or each has interacted locally with a locally-originated intermediary) and the Second Law (nonlocality grows over time, limited by the velocity of light c); (§3) the "New York–Los Angeles challenge" as a concrete falsification criterion, establishing that no experimental protocol can create entanglement between distant systems without a chain of local contacts; (§4) the six senses of geometric nonlocality of the expanding McGucken Sphere wavefront — foliation leaf, distance-function level set, Huygens caustic, Legendrian submanifold in contact geometry, conformal pencil member, and null-hypersurface cross-section (the canonical causal locality of Minkowski geometry); (§5) entanglement transfer via intersecting McGucken Spheres, supplying the geometric content of the First Law's "chain of local contacts" clause for entanglement swapping and quantum teleportation; and (§6) the resolution within McGucken Spheres of the double-slit experiment, Wheeler's delayed-choice experiment, and all quantum eraser experiments, showing that the apparent paradoxes dissolve when the experiments are recognized as taking place within the expanding x_4 geometry. Used in §§V.8.1–V.8.6 of the present paper for the fourth dual-channel reading at the causal/correlational level.

[MG-Equiv] E. McGucken, "The McGucken Equivalence: Quantum Nonlocality and Relativity Both Emerge From the Expansion of the Fourth Dimension at the Velocity of Light," elliotmcguckenphysics.com (December 29, 2024). Also available at Medium: https://goldennumberratio.medium.com/the-mcgucken-equivalence-of-quantum-nonlocality-and-relativity-how-quantum-nonlocality-is-found-ce448d0b5722 . The structural identification of quantum nonlocality as the three-dimensional shadow of four-dimensional x_4-coincidence on the light cone. Used in §V.8.2 of the present paper for the Channel-B reading that generates nonlocal Bell correlations from the shared McGucken Sphere.

[MG-NonlocCopen] E. McGucken, "Quantum Nonlocality and Probability from the McGucken Principle of a Fourth Expanding Dimension — How $dx_4/dt = ic$ Provides the Physical Mechanism Underlying the Copenhagen Interpretation as well as Relativity, Entropy, Cosmology, and the Constants of Nature," elliotmcguckenphysics.com (April 16, 2026). URL: https://elliotmcguckenphysics.com/2026/04/16/quantum-nonlocality-and-probability-from-the-mcgucken-principle-of-a-fourth-expanding-dimension-how-dx4-dt-ic-provides-the-physical-mechanism-underlying-the-copenhagen-interpr/ . Supplements [MG-Nonlocality] with the Copenhagen-resolution analysis and the explicit derivation of the CHSH singlet correlation E(a,b) = −cos θ_{ab} from shared wavefront identity at §5.5a. Used in §V.8.1 and §V.8.2 of the present paper for the historical context and the derivation of the Bell correlations from Channel B.

[MG-Dissertation] E. McGucken, *Multiple Unit Artificial Retina Chipset to Aid the Visually Impaired and Enhanced Holed-Emitter CMOS Phototransistors*. NSF-funded Ph.D. dissertation, University of North Carolina at Chapel Hill (1998). The appendix contains the first written formulation of the McGucken Principle, treating time as an emergent phenomenon arising from a fourth expanding dimension.

[MG-FQXi-2008] E. McGucken, "Time as an Emergent Phenomenon: Traveling Back to the Heroic Age of Physics (In Memory of John Archibald Wheeler)," Foundational

Questions Institute essay (August 2008). URL: https://forums.fqxi.org/d/238 . First formal treatment of the McGucken Principle in the scholarly literature.

[MG-FQXi-2010] E. McGucken, "On the Emergence of QM, Relativity, Entropy, Time, $i\hbar$, and ic from the Foundational, Physical Reality of a Fourth Dimension x_4 Expanding with a Discrete (Digital) Wavelength λ_P at c Relative to Three Continuous (Analog) Spatial Dimensions," Foundational Questions Institute essay (2010–2011). First explicit identification of the structural parallel between $dx_4/dt = ic$ and the canonical commutation relation $[q, p] = i\hbar$.

[MG-ConservationSecondLaw] E. McGucken, "The McGucken Principle as the Common Foundation of the Conservation Laws and the Second Law of Thermodynamics: A Remarkable and Counter-Intuitive Unification," elliotmcguckenphysics.com (April 2026). URL: https://elliotmcguckenphysics.com . The companion paper establishing the conservation laws (via the twelve-fold Noether catalog) and the Second Law of Thermodynamics (via the spherical isotropic random walk and Shannon entropy on the McGucken Sphere) as two readings of $dx_4/dt = ic$ through the dual-channel structure, extending the four-level within-QM analysis of the present paper into a fifth level beyond QM (the thermodynamic level).

[MG-Broken] E. McGucken, "How the McGucken Principle of the Fourth Expanding Dimension $dx_4/dt = ic$ Accounts for the Standard Model's Broken Symmetries, Time's Arrows and Asymmetries, and Much More," elliotmcguckenphysics.com (April 13, 2026). URL: https://elliotmcguckenphysics.com/2026/04/13/how-the-mcgucken-principle-of-the-fourth-expanding-dimension-dx%e2%82%84-dt-ic-accounts-for-the-standard-models-broken-symmetries-times-arrows-and-asymmetries-and-much-more/ .

[MG-Eleven] E. McGucken, "One Principle Solves Eleven Cosmological Mysteries: How the McGucken Principle of the Fourth Expanding Dimension $dx_4/dt = ic$ Resolves the Greatest Open Problems in Cosmology, Including the Low-Entropy Initial Conditions Problem," elliotmcguckenphysics.com (April 13, 2026). URL: https://elliotmcguckenphysics.com/2026/04/13/one-principle-solves-eleven-cosmological-mysteries-how-the-mcgucken-principle-of-the-fourth-expanding-dimension-dx%e2%82%84-dt-ic-resolves-the-greatest-open-problems-in-cosmology-inclu/ . §XIII dissolves the Past Hypothesis by showing that x_4's origin is geometrically necessarily the lowest-entropy moment; Penrose's $10^{-10^{123}}$ fine-tuning framing is identified as the wrong framing. Used in §VI.3 of [MG-ConservationSecondLaw].

[MG-FQXi-2009] E. McGucken, "What is Ultimately Possible in Physics? Physics! A Hero's Journey with Galileo, Newton, Faraday, Maxwell, Planck, Einstein, Schrödinger, Bohr, and the Greats towards Moving Dimensions Theory. E pur si muove!," Foundational Questions Institute essay contest, September 16, 2009. URL: https://forums.fqxi.org/d/511 . The second FQXi paper; the first to use Moving Dimensions Theory as an explicit, formal name in a paper title.

[MG-FQXi-2012] E. McGucken, "MDT's $dx_4/dt = ic$ Triumphs Over the Wrong Physical Assumption That Time Is a Dimension," Foundational Questions Institute essay contest (2012). URL: https://forums.fqxi.org/d/1429 . The most polemical of the FQXi papers; argues that the standard conflation of time with the fourth dimension has generated most of modern physics' paradoxes.

[MG-FQXi-2013] E. McGucken, "Where is the Wisdom We Have Lost in Information? Returning Wheeler's Honor and Philo-Sophy to Physics," Foundational Questions Institute essay contest (2013). A tribute to Wheeler, extending the framework to information-theoretic foundations.

[MG-History] E. McGucken, "A Brief History of Dr. Elliot McGucken's Principle of the Fourth Expanding Dimension $dx_4/dt = ic$: Princeton and Beyond — Moving Dimensions Theory (MDT) → Dynamic Dimensions Theory (DDT) → Light Time Dimension Theory (LTD) → $dx_4/dt = ic$," elliotmcguckenphysics.com (April 2026). URL: https://elliotmcguckenphysics.com/2026/04/11/a-brief-history-of-dr-elliot-mcguckenstheory-of-the-fourth-expanding-dimension-princeton-and-beyond/ . The comprehensive chronological record of the McGucken Principle's development from undergraduate work with John Archibald Wheeler at Princeton University in the late 1980s through the UNC Chapel Hill doctoral dissertation (1998–1999), PhysicsForums and Usenet deployments (2003–2006), the five FQXi essay-contest papers (2008–2013), the 2016–2017 book series, and the active derivation programme of 2024–2026. Archived forum posts, Google Groups Usenet records, FQXi archives, Blogspot timestamps, and complete bibliography.

[MG-Jacobson] E. McGucken, "The McGucken Principle of a Fourth Expanding Dimension ($dx_4/dt = ic$) as a Candidate Physical Mechanism for Jacobson's Thermodynamic Spacetime, Verlinde's Entropic Gravity, and Marolf's Nonlocality," elliotmcguckenphysics.com (April 12, 2026). URL: https://elliotmcguckenphysics.com/2026/04/12/the-mcgucken-principle-of-a-fourth-expanding-dimension-dx%e2%82%84-dt-ic-as-a-candidate-physical-mechanism-for-jacobsons-thermodynamic-spacetime-verlindes-entropic-gravity-and-marolfs-nonl/ .

[MG-KaluzaKlein] E. McGucken, "The McGucken Principle as the Completion of Kaluza–Klein: How $dx_4/dt = ic$ Reveals the Dynamic Character of the Fifth Dimension and Unifies Gravity, Relativity, Quantum Mechanics, Thermodynamics, and the Arrow of Time," elliotmcguckenphysics.com (April 11, 2026). URL: https://elliotmcguckenphysics.com/2026/04/11/the-mcgucken-principle-as-the-completion-of-kaluza-klein-how-dx4-dt-ic-reveals-the-dynamic-character-of-the-fifth-dimension-and-unifies-gravity-relativity-quantum-mech/ . §V.2 provides the formal derivation dS/dt = (3/2)k_B/t > 0 strictly, used in §III.2 of [MG-ConservationSecondLaw]; §V.3 catalogs the five arrows of time; §VI develops the crucial distinction between time t and the fourth coordinate x_4.

[MG-PhotonEntropy] E. McGucken, "How The McGucken Principle Exalts Relativity, Photon Entropy on the McGucken Sphere, and a Testable Mechanism for Thermodynamic Entropy," elliotmcguckenphysics.com (April 18, 2026). URL: https://elliotmcguckenphysics.com/2026/04/18/how-the-mcgucken-principle-exalts-relativity-photon-entropy-on-the-mcgucken-sphere-and-a-testable-mechanism-for-thermodynamic-entropy/ . Section 3 derives the Shannon entropy S(t) = k_B ln(4π(ct)²) for photons on the McGucken Sphere, used in §III.3 of [MG-ConservationSecondLaw]; §§4-6 develop the Compton-frequency coupling giving the diffusion term D_x^(McG) = ε²c²Ω/(2γ²), used in §III.4 of [MG-ConservationSecondLaw]; §6 provides the full stochastic/Langevin derivation of the Compton-coupling diffusion with mass cancellation.

[MG-Principle] E. McGucken, "The McGucken Principle: The Fourth Dimension Is Expanding at the Velocity of Light C: $dx_4/dt=ic$ & The McGucken Proof of the Fourth Dimension's Expansion at the Rate of C: $dx_4/dt=ic$," elliotmcguckenphysics.com (October 25, 2024). URL: https://elliotmcguckenphysics.com/2024/10/25/the-mcgucken-principle-the-fourth-dimension-is-expanding-at-the-velocity-of-light-c-dx4-dtic-the-mcgucken-proof-of-the-fourth-dimensions-expansion-at-the-rate-of-c-dx4-dtic/ . The foundational statement of the McGucken Principle $dx_4/dt = ic$ together with the six-step McGucken Proof deriving the Principle from the physical postulates that (i) every object has four-speed c, and (ii) photons are spherically-symmetric expanding wavefronts at rate c.

[MG-Sphere] E. McGucken, "Quantum Nonlocality and Probability from the McGucken Principle of a Fourth Expanding Dimension — How $dx_4/dt = ic$ Provides the Physical Mechanism Underlying the Copenhagen Interpretation as well as Relativity, Entropy, Cosmology, and the Constants of Nature," elliotmcguckenphysics.com (April 16, 2026). URL: https://elliotmcguckenphysics.com/2026/04/16/quantum-nonlocality-and-probability-from-the-mcgucken-principle-of-a-fourth-expanding-dimension-how-dx4-dt-ic-provides-the-physical-mechanism-underlying-the-copenhagen-interpr/ . The six-sense geometric locality of the McGucken Sphere (foliation leaf, distance-function level set, Huygens caustic, Legendrian submanifold, conformal-pencil member, null-hypersurface cross-section) as the mechanism that makes quantum nonlocal correlations a local-in-4D phenomenon.

[MG-TwoRoutes] E. McGucken, "The Deeper Foundations of Quantum Mechanics: How The McGucken Principle Uniquely Generates the Hamiltonian and Lagrangian Formulations of Quantum Mechanics, Wave/Particle Duality, the Schrödinger and Heisenberg Pictures, and Locality and Nonlocality all from $dx_4/dt = ic$," elliotmcguckenphysics.com (April 2026). URL: https://elliotmcguckenphysics.com . The present paper (self-reference within the McGucken corpus). Develops the dual-channel structure at four levels of quantum-mechanical description: the foundational level (Hamiltonian/Lagrangian formulations, §§II–III), the ontological level (wave/particle aspects, §V.6), the dynamical level (Schrödinger/Heisenberg pictures, §V.7), and the causal/correlational level (locality/nonlocality, §V.8). The companion paper [MG-

ConservationSecondLaw] extends the structure to a fifth level beyond quantum mechanics (the thermodynamic level), bringing the total count of independent structural appearances of the dual-channel mechanism to five.

[MG-Verlinde] E. McGucken, "The McGucken Principle $dx_4/dt = ic$ as the Physical Mechanism Underlying Verlinde's Entropic Gravity: A Unified Derivation of Gravity, Entropy, and the Holographic Principle from a Single Geometric Principle," elliotmcguckenphysics.com (April 11, 2026). URL: https://elliotmcguckenphysics.com/2026/04/11/the-mcgucken-principle-dx%e2%82%84-dt-ic-as-the-physical-mechanism-underlying-verlindes-entropic-gravity-a-unified-derivation-of-gravity-entropy-and-the-holographic-principle-from-a-single-ge/ .

[MG-Wick] E. McGucken, "The Wick Rotation as a Theorem of $dx_4/dt = ic$: How the McGucken Principle of the Fourth Expanding Dimension Provides the Physical Mechanism Underlying the Wick Rotation and All of Its Applications Throughout Physics," elliotmcguckenphysics.com (April 20, 2026). URL: https://elliotmcguckenphysics.com/2026/04/20/the-wick-rotation-as-a-theorem-of-dx%e2%82%84-dt-ic-how-the-mcgucken-principle-of-the-fourth-expanding-dimension-provides-the-physical-mechanism-underlying-the-wick-rotation-and-all-of-its-applicat/ .

[McGucken 2016] E. McGucken, *Light Time Dimension Theory: The Foundational Physics Unifying Einstein's Relativity and Quantum Mechanics: A Simple, Illustrated Introduction to the Physical Model of the Fourth Expanding Dimension* (45EPIC Hero's Odyssey Mythology Press, 2016). Amazon ASIN: B01KP8XGQ6. URL: https://www.amazon.com/dp/B01KP8XGQ6 . The first book-length treatment of the McGucken Principle.

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[McGucken 2017d] E. McGucken, *The Physics of Time: Time & Its Arrows in Quantum Mechanics, Relativity, The Second Law of Thermodynamics, Entropy, The Twin Paradox, & Cosmology Explained via LTD Theory's Expanding Fourth Dimension* (45EPIC Hero's Odyssey Mythology Press, 2017). Amazon ASIN: B0F2PZCW6B. URL: https://www.amazon.com/dp/B0F2PZCW6B . Particularly relevant to [MG-ConservationSecondLaw]: the 2017 book-length treatment of the argument that the

Second Law of Thermodynamics, entropy, and the arrows of time all follow from $dx_4/dt = ic$. The formal technical development of this argument is the subject of §§III–IV of [MG-ConservationSecondLaw].